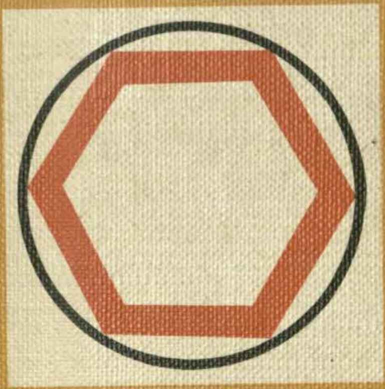
DISCOVERING MATHEMATICS



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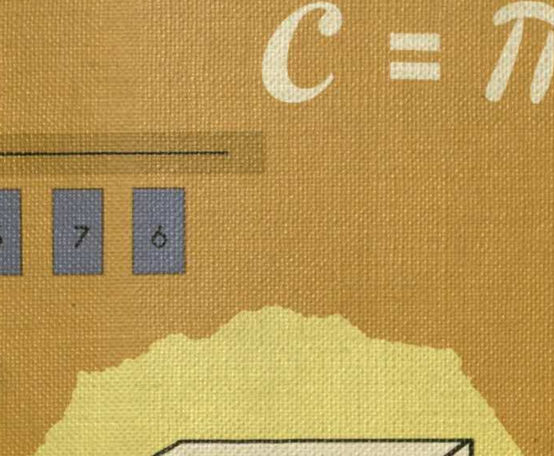

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TEACHER'S EDITION — ANNOTATED AND KEYED

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STATE INSTITUTE OF EDUCATION
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How to Use the TEACHER'S EDITION—Annotated and Keyed

FOR

AMERICAN ARITHMETIC

SECOND EDITION

This Teacher's Edition is designed to be the greatest possible help to the busy teacher. Within one cover, it presents these distinctive features.

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The Teacher's Guide and Key provides a full statement of the aims and detailed practical suggestions for each chapter and each page or group of pages in the pupil's text. Following the discussion of the treatment of each page is the key, working the more involved problems in full. References to pages in the workbook AMERICAN WORKSHOP are provided for those teachers using these workbooks. A list of audio-visual aids is provided at the end of the guide.

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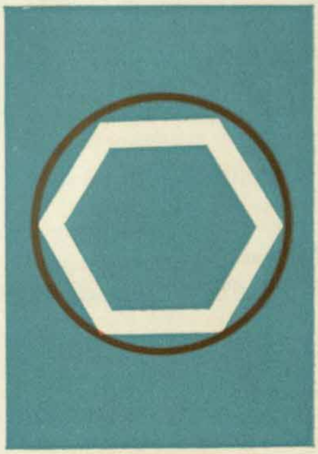

DISCOVERING MATHEMATICS

American Arithmetic

Second Edition

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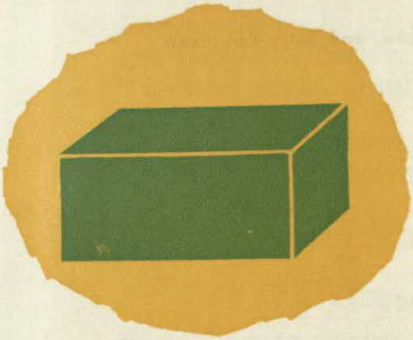

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DISCOVERING MATHEMATICS

American Arithmetic, Second Edition

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TEACHER'S EDITION -- Annotated and Keyed
SL 10 9 8 7 6 5 4 3 2 1

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Before beginning the work in this book, refer to "Suggestions to Teachers," pp. 344-345, and to the Teacher's Guide and Key, Introduction. These sections contain a discussion and explanation of number principles and relationships, long division, improvement tests, an outline of Grade 7 work, and so on, with page references and valuable teaching suggestions.

Review and reteach the four fundamental operations with whole numbers. See Teacher's Guide and Key for specific aims of Chapter 1.

Review reading and writing of large numbers. Have pupils bring to class newspaper clippings with numbers in millions and billions.

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Chapter 1

Our Postal Service

1. The United States Postal Service is one of the largest businesses in the world. In a recent year this service received, transported, and delivered 65,000,000,000 pieces of mail. Read this number.(1) Write in figures: 10 billion;(2) 10 million;(3) 10 thousand.(4) (1) 65 billion; (2) 10,000,000,000; (3) 10,000,000; (4) 10,000

2. There are 35,000 post offices in the United States and more than 582,000 employees at work in our postal service. Read these numbers.(1) Write in figures: 100 thousand;(2) 7 thousand;(3) 80 thousand.(4) (1) 35 thousand, 582 thousand; (2) 100,000; (3) 7000; (4) 80,000

3. In a recent year the amount of money received by the postal services was \$3,423,100,000, and the amount spent was \$4,249,400,000. How much more was spent than received that year? \$826,300,000

4. The growth of our postal service has been very rapid. A half century ago, when the population of the United States was about 92,000,000, the postal service handled 15,000,000,000 pieces of mail per year. Today, when the population is more than 180,000,000, the postal service handles 65,000,000,000 pieces of mail per year. Today's population is how many times that of a half century ago?(1) Today's volume of mail is how many times that of a half century ago?(2) How many additional pieces of mail, on the average, were handled each year during this 50-year period?(3) (1) about 2; (2) $4\frac{1}{3}$, (3) 1,000,000,000

5. In one year 23,000,000,000 postage stamps and 2,000,000,000 postal cards were issued. The number of stamps issued is how many times the number of cards? $11\frac{1}{2}$

Only the numerical part of the answer is given in most cases. Pupils, however, should give the complete answer phrases or sentences.

Reteach rounding off of numbers. Have pupils begin an arithmetic scrapbook to use for various projects, tables, puzzles, and so on.

Round Numbers

1. On page 1 it is stated that there are 35,000 post offices in the United States. In this case 35,000 is a round number, which means that there are about 35,000 post offices in this country. The exact number of post offices is 34,955, which is close to 35,000. It is easier to say 35,000 than 34,955. When you use 35,000 for 34,955, you have rounded off 34,955 to the nearest thousand. You see that 34,955 is closer to 35,000 than it is to 34,000; so 35,000 is the nearest thousand. If the exact number of post offices were 34,186, you would call it 34,000 to the nearest thousand. If the exact number were 34,500, which is halfway between 34,000 and 35,000, you would call it 35,000, just as 37½¢ is called 38¢. Emphasize.

2. Round off these numbers to the nearest thousand:

17,840 18,000 23,178 23,000 210,500 211,000 79,780 80,000 6413 6000

3. In ex. 5 on page 1 the number 23,000,000,000 is also a round number. The exact number of stamps issued by our post offices was 23,001,808,400, but it is more convenient to round this number off to the nearest billion, which is 23,000,000,000. Round off 23,726,472,845 to the nearest billion. 24,000,000,000

4. If 78,179 is rounded off to the nearest thousand, it becomes 78,000; but if it is rounded off to the nearest 10 thousand, it becomes 80,000. Similarly, 73,243 becomes 73,000 to the nearest thousand or 70,000 to the nearest 10 thousand. Round off each number to the nearest thousand (1) and also to the nearest 10 thousand: (2)

(1) 21,867 22,000 15,500 16,000 47,322 47,000 10,865 11,000 9165 9000
(2) 20,000 20,000 50,000 10,000 10,000

5. Round off each number to the nearest million:

19,826,423 6,287,823 7,500,000 3,084,734
20,000,000 6,000,000 8,000,000 3,000,000

6. In ex. 5, round off each number to the nearest 100 thousand; (1) to the nearest 10 thousand. (2) (1) 19,800,000; 6,300,000; 7,500,000; 3,100,000; (2) 19,830,000; 6,290,000; 7,500,000; 3,080,000

7. Round off each number to the nearest thousand:

2,616,000 1,709,000 3,206,000 1,620,000
2,615,813 1,709,116 3,205,500 1,619,927

First give review and practice in rounding off numbers to nearest thousand, hundred, then ten. Emphasize why round numbers are used (ex. 3). Have pupils bring for scrapbooks examples of round numbers found in newspapers.

Teach a short way to write very large round numbers.

What Does 5.7 Billion Mean?

1. In the newspaper Jack read that one year the world wheat crop amounted to 5.7 billion bushels. The number 5.7 is a decimal; 5.7 billion means 5.7 × 1,000,000,000 bu., which equals 5,700,000,000 bu. You see that 5,700,000,000 is a round number, so 5.7 billion is a new way to express a round number. It is easier to read and write 5.7 billion than 5,700,000,000.

Emphasize. This new way of writing round numbers is very convenient when the numbers represent millions or billions. There is no advantage, however, in using it for a smaller number like 274 thousand since it is just as easy to write 274,000.

2. Mary read that the sales (1) in a large department store amounted to $12.3 million. She (2) also read that a mining company had sold 215.7 million pounds of copper. Write each of these numbers in figures. (1) $12,300,000; (2) 215,700,000

3. Write each of these numbers entirely in figures: 6.7 million; $114.3 million; $1.8 billion; 26.4 billion; $2.9 billion.
6,700,000
$114,300,000
$1,800,000,000
26,400,000,000
$2,900,000,000

4. Write each number the new way, rounding it off, if necessary, so there will be only 1 decimal place in the new form: $3,900,000; 6,800,000,000; $27,400,000; 123,600,000,000; 8,732,000; 7,876,000; 5,293,000,000; 2,095,000,000. $3.9 million; 6.8 billion; $27.4 million; 123.6 billion; 8.7 million; 7.9 million; 5.3 billion; 2.1 billion

Stress. **5.** In writing numbers, the figures are separated by commas into groups or **periods** of 3 figures each, beginning at the right. Here are the names of these periods: Have pupils find out names of the next two periods after billions.

Billions	Millions	Thousands	Ones
216,	574,	318,	943

In a 4-figure number like 1958 the comma is often left out, but it is always used when the number has 5 or more figures.

Have pupils show period values in ex. 6.

6. Put commas in each number to separate the number into periods; then read the number: See Guide.

13,148,273,160 5,826,914 16,182,198,000 238,412,143

(1) 13.1 billion 5.8 million 16.2 billion 238.4 million

7. In ex. 6, round off each number and write it the new way.

Discuss ex. 1 thoroughly with class. Use other examples to be sure all pupils understand how to read and write large numbers in the short way. Have pupils bring in newspaper clippings showing examples of these numbers.

Present oral test of pupils' mastery of addition facts.

The 100 Addition Facts

This is a test on the addition facts. You should be able to give all the sums orally within 3 min. If you cannot do this, try to practice so that you can do it by the end of the year. Make a list of any facts that give you trouble and practice on them: Stress.

1.	3	6	4	8	3	4	0	9	8	5
	9	5	0	5	1	5	8	8	4	3
	12	11	4	13	4	9	8	17	12	8
2.	2	3	5	8	4	1	3	9	2	1
	1	7	0	2	4	6	2	3	0	3
	3	10	5	10	8	7	5	12	2	4
3.	6	8	0	2	3	1	5	0	6	8
	2	7	1	5	3	7	4	5	7	1
	8	15	1	7	6	8	9	5	13	9
4.	2	4	6	7	5	1	6	5	0	4
	2	1	8	0	9	8	4	6	3	8
	4	5	14	7	14	9	10	11	3	12
5.	7	6	3	7	4	1	7	8	0	8
	6	0	4	2	9	4	1	6	9	9
	13	6	7	9	13	5	8	14	9	17
6.	0	5	9	4	7	0	9	5	9	7
	2	1	9	3	5	7	6	5	7	3
	2	6	18	7	12	7	15	10	16	10
7.	8	7	2	1	5	3	2	0	1	4
	0	9	9	2	7	5	3	4	9	2
	8	16	11	3	12	8	5	4	10	6
8.	5	9	7	9	2	8	9	1	3	7
	8	0	8	4	7	8	2	1	0	7
	13	9	15	13	9	16	11	2	3	14
9.	6	6	2	8	1	6	2	0	5	4
	9	1	6	3	0	3	8	6	2	7
	15	7	8	11	1	9	10	6	7	11
10.	9	7	3	6	1	2	0	3	4	9
	5	4	6	6	5	4	0	8	6	1
	14	11	9	12	6	6	0	11	10	10

To the Teacher. See the suggestions on page 345 for giving this diagnostic test on the fundamental addition facts.

Some of the better pupils might help you test others. Use various number principles and relationships in reteaching basic facts. See page 5. Retest pupils after a week or two.

$$3+2=2+3$$
$$5+7=7+5$$
$$a+b=b+a$$

Relationships

1. There are interesting and useful relationships between pairs of addition facts. You know that 3 and 2 have the same sum as 2 and 3. $3 + 2 = 2 + 3 = 5$. What is $5 + 7$? 12 $7 + 5$? 12 $9 + 6$? 15 $6 + 9$? 15 **If a and b stand for any two numbers, then $a + b = b + a$.** This is called the **commutative principle for addition.** Tell what this principle means in words.
2. Suppose that you have forgotten $3 + 8$. If you know that $8 + 3 = 11$, the commutative principle tells you that $3 + 8 = 11$. What related fact can help you if you forget $6 + 8$? $7 + 4$? $4 + 7 = 11$ $8 + 6 = 14$
3. The sum of 7 and 8 is 1 more than the sum of 7 and 7. Why? If you know that $7 + 7 = 14$, this relationship tells you that $7 + 8 = 14 + 1 = 15$. You see that the doubles help you with addition facts that are near doubles. What can you do if you forget $5 + 6$? $6 + 7$? $8 + 9$? See Guide.
4. The sum of 9 and 4 is 13, which is the same as the sum of 10 and 3. $9 + 4 = 10 + 3$. If you forget the sum of 9 and 4, subtract 1 from 4 and add 1 to 9. Now you can add 10 and 3, which gives 13, to find the sum of 9 and 4. What can you do if you forget 9 and 6? 7 and 9? 9 and 5? See Guide.
5. It is easy to add 0 to any other number. 6 and 0 are 6. How many are 7 and 0? 7 4 and 0? 4 0 and 5? 5 0 and 0? 0 **If n stands for any number, then $n + 0 = n$.** What does $0 + n$ equal? n Can you tell in words how to find the sum of 0 and any other number?
6. The addition facts help you to add larger numbers. You know that $3 + 4 = 7$. So $13 + 4 = 17$, $23 + 4 = 27$, $33 + 4 = 37$, $43 + 4 = 47$, and so on.
7. You know that $4 + 8 = 12$. Tell how this addition fact helps you find $14 + 8$, 22 $24 + 8$, 32 $34 + 8$, 42 $44 + 8$, 52 and $54 + 8$. 62

Encourage pupils to suggest different ways to recall addition facts. Discuss ways to increase confidence in the answers recalled. Try to develop a smooth and easy performance, as well as a rapid and accurate one.

Point out that the commutative and associative principles are particularly useful in making addition easier and more accurate.

More about Addition

1. You know that you can find the sum $4 + 3 + 6$ in these two ways:

$$(4 + 3) + 6 = 7 + 6 = 13$$
$$4 + (3 + 6) = 4 + 9 = 13$$

You see that the way you group these three numbers when you add them does not affect your answer. Find each of these sums by grouping the numbers two ways: See Guide.
$2 + 5 + 4$; $8 + 1 + 5$; $5 + 8 + 7$; $4 + 9 + 9$.

2. If a, b, and c stand for any three numbers, then $(a + b) + c = a + (b + c)$. This is called the **associative principle for addition.** Tell what this principle means in words. Write each of these sums two ways with parentheses to illustrate the associative principle: See Guide.
$8 + 9 + 7$; $14 + 5 + 3$; $73 + 84 + 7$; $29 + 71 + 18$.

3. When you find $12 + 6$ by adding 2 and 6 and then adding 10 to this sum, you are using the associative principle:
$12 + 6 = (10 + 2) + 6 = 10 + (2 + 6) = 10 + 8 = 18$
When you find $39 + 7$ by subtracting 1 from 7 and adding this 1 to 39, and then adding 40 and 6 you are also using the associative principle:
$39 + 7 = 39 + (1 + 6) = (39 + 1) + 6 = 40 + 6 = 46$

4. You can find the sum of the numbers on the right by adding up the column. When you check your answer by adding down the column you are using both the commutative and the associative principles:

$$\begin{array}{r} 4 \\ 2 \\ \underline{5} \\ 11 \end{array}$$

associative commutative commutative associative
$(5 + 2) + 4 = 4 + (5 + 2) = 4 + (2 + 5) = (4 + 2) + 5$
Which principle is used in each step of the above work?

5. To be successful in column addition you must be skillful in mentally adding a 1-figure number to a 2-figure number.

Add 7 to each number: (1)

(1), (2) 78, 80 91, 93 62, 64 54, 56 73, 75 96, 98 65, 67 70, 72
(3), (4) **6.** 71 77, 79 84 90, 92 55 61, 63 47 53, 55 66 72, 74 89 95, 97 58 64, 66 63 69, 71
31, 33 23, 25 26, 28 60, 62 49, 51 81, 83 56, 58 29, 31
7. 24 30, 32 16 22, 24 19 25, 27 53 59, 61 42 48, 50 74 80, 82 49 55, 57 22 28, 30

8. Add 9 to each number in ex. 6 and 7.(2) Add 6;(3) add 8.(4)

Emphasize sums that are 10. When you add 9, try thinking that you need 1 more to make 10. Two numbers that have a sum of 10 are called complements of each other. The complement of 8 is 2. Experiment with using complements in addition.

Reteach column addition, the terms used in addition, and the method of checking addition.

Column Addition

1. To add the first column in the example at the right, start at the bottom of the column and add up like this: 9 + 6 = 15, 15 + 4 = 19, 19 + 5 = 24, 24 + 8 = 32, 32 + 7 = 39, 39 + 2 = 41. When you add 15 and 4, think "19"; do not think "15 and 4 are 19." As you go up the column, think only these numbers: "15, 19, 24, 32, 39, 41." The answer is 41. Write 1 below the line and carry 4 to the next column. To avoid forgetting the carry number, you may write a small 4 at the bottom of the second column. Add the other columns. The numbers that are added are called **addends.**

$$\begin{array}{r} 842\\ 787\\ 938\\ 565\\ 334\\ 946\\ 659\\ {\scriptstyle 3\,4}\\ \hline 5071 \end{array}\ \textbf{Sum}$$

2. The best way to check column addition is to begin at the top of each column and add down. As you add down the first column, say "9, 17, 22, 26, 32, 41." The sum is 41 again, so the addition of this column is considered correct. Carry the 4 of 41 to the next column and write it as a small figure at the top of the column. Then check the second column. Before you start checking, cross out the carry numbers that you write at the bottom of the example.

3. If you have trouble with addition when the columns are very long, you may break the example into two parts and find the sum of each part. Then add these sums to get the final answer, as shown in the example at the right.

$$\begin{array}{rr} 39 & \\ 64 & \\ 75 & \\ 98 & \\ \underline{47} & 323\\ 86 & \\ 74 & \\ 92 & \\ 69 & \\ \underline{58} & \underline{379}\\ & 702 \end{array}$$

4. Mary went to the store for her mother. She bought these things: bread, 22¢; wax beans, 18¢; butter, 72¢; milk, 26¢; cream, 33¢; cookies, 39¢; eggs, 65¢; honey, 45¢; lettuce, 19¢; oranges, 43¢. How much in all did Mary pay for these items? $3.82

Copy in columns, add, and check:

5. 5 + 7 + 3 + 6 + 3 + 7 + 6 + 0 + 1 + 5 + 8 + 7 58

6. 35 + 48 + 87 + 76 + 40 + 89 + 87 + 70 + 18 + 64 614

More Practice. See ① on page 325.

Have pupils find sums in examples like ex. 1. Then have them do the addition aloud to check for errors or places of difficulty. After they have done ex. 5, have pupils check their answer by adding the numbers horizontally as they are written.

This practice might be given as a contest, with the class divided into teams.

Column Addition

Add up. Check the work by adding down:

1.	878	204	695	543	212	889	751
	621	631	527	571	133	450	705
	706	458	525	750	649	684	314
	749	234	329	793	309	828	405
	170	267	845	523	843	474	194
	3124	1794	2921	3180	2146	3325	2369
2.	518	235	549	789	674	756	647
	710	130	274	779	537	688	638
	641	529	361	847	439	815	384
	930	815	108	298	960	105	274
	701	157	368	109	167	783	799
	287	714	272	366	336	816	821
	346	600	960	568	994	794	458
	259	939	305	336	428	349	225
	4392	4119	3197	4092	4535	5106	4246
3.	7352	3596	1844	6000	8415	9047	
	2678	6903	7089	3543	7534	2672	
	3957	4117	3451	2282	1160	5090	
	7460	2875	3667	5696	1821	1327	
	5848	5206	8789	7969	1579	1937	
	9482	9998	6078	3506	9028	8866	
	5009	4134	1200	2444	4668	2426	
	41,786	36,829	32,118	31,440	34,205	31,365	

Copy, add, and check the work:

4. 84 + 96 + 59 + 99 + 57 + 79 + 89 + 20 + 87 + 96 766
5. 72 + 88 + 65 + 58 + 79 + 59 + 81 + 64 + 47 + 88 701
6. \$13.69 + \$78.83 + \$36.68 + \$5.05 + \$29.56 + \$2.67 \$166.48
7. \$22.45 + \$4.93 + \$63.56 + \$79.47 + \$2.89 + \$14.04 \$187.34
8. \$76.93 + \$2.45 + \$78.25 + \$26.54 + \$2.35 + \$21.83 \$208.35
9. \$73.42 + \$.91 + \$10.51 + \$2.74 + \$59.17 + \$44.22 \$190.97
10. \$54.31 + \$60.64 + \$15.11 + \$4.17 + \$37.09 + \$14.53 \$185.85
11. \$29.00 + \$19.24 + \$.73 + \$2.56 + \$51.00 + \$5.26 \$107.79

More Practice. See ② on page 325.

Most pupils need do only half the exercises on this page. Divide the class into two groups, assigning part of ex. 1-11 to one group, remaining part to others. Use "More Practice" for remedial work, reviews, or tests.

Present one-step problems that require column addition.

Spending Money

1. At the school store Betty spent 10¢ for a pencil, 75¢ for a ball-point pen, 25¢ for a notebook, 98¢ for a book, 35¢ for a notebook cover, 17¢ for ink, and 18¢ for paper. How much in all did Betty spend? $2.78

   ▶ Remember that sums of 100¢ or more should be changed to dollars and cents.

2. For food for a picnic some girls and boys spent 25¢ for bread, 17¢ for butter, 29¢ for sandwich spread, 35¢ for eggs, 19¢ for lettuce, 48¢ for milk, 25¢ for apples, 29¢ for bananas, 35¢ for cookies, and 29¢ for potato chips. How much did all this food cost? $2.91

3. In the school lunchroom Jim spent 25¢ for beef stew, 5¢ for bread, 3¢ for butter, 8¢ for milk, 10¢ for ice cream, and 5¢ for chocolate sauce. How much did Jim spend for his lunch? 56¢

4. Start today to make a list of all the amounts you spend for a week. Then find the total amount. Pupils might like to exhibit lists. Post on bulletin board for reference. Have pupils make up 2 good problems involving column addition and using at least 9 addends (each under $1.00), for class to solve.

Making and Checking Change

1. Susan bought a notebook for 15¢. She gave the clerk $1.00 and received a dime, a quarter, and a half dollar in change. To check the change to see if it was right, Susan did not count the coins to see if she had 85¢; instead she added the coins, one by one, to 15¢, the cost of the notebook, to see if the sum equaled $1.00. Since it did, she knew the change was right.

▶ The steps in adding the change to the cost were as follows: Starting with 15¢ Susan added the dime and said "25¢," next she added the quarter and said "50¢," then she added the half dollar and said "$1.00." In checking change in this way, always begin with the smallest coins.

2. Joe paid $1.37 for a book and gave the clerk $5.00. His change was 3 cents, 1 dime, 1 half dollar, 3 dollars. Check the change as explained in ex. 1. $1.37; $1.40; $1.50; $2.00; $3.00; $4.00; $5.00

3. May paid $.28 for some ribbon and gave the clerk $1.00. Her change was 2 cents, 1 nickel, 1 dime, 2 quarters. Check the change. Is it right? No If not, correct it. 2 pennies, 2 dimes, 2 quarters

4. If Henry bought a flashlight from you for 79¢ and gave you a dollar bill, what coins would you give him as change? How would you count them out? 1 penny, 2 dimes; 79, 80, 90, $1.00

Emphasize that change making and checking change are processes of successive additions, starting with the cost of the item. Make up problems and give each pupil a chance to count change. Stress that the least number of coins is used.

5. A clerk makes change the same way that you check it. If you buy something that costs 31¢ and give him a dollar, he starts with 31¢. Then he adds coins to this amount until he reaches $1.00. As he adds, he says "31, 35, 40, 50, $1.00." Tell what coins he added to 31¢.* These coins are your change. Sometimes the clerk may count each penny; then he would say "31, 32, 33, 34, 35, 40, 50, $1.00." *4 pennies, 1 nickel, 1 dime, 1 half dollar

6. A clerk usually makes change by using the smallest number of coins. In ex. 5, the clerk used 7 coins to make the change. Was that the smallest number he could use? How could he have made the change by using 8 coins? by using 9 coins? 10 coins? 13 coins? 17 coins? 69 coins? See Guide.

See Guide. **Make change from $5.00 if you buy things that cost the following amounts. Use the least number of coins:** Stress.

7. $2.16 $3.75 $2.55 $1.24 $4.85 $1.08

8. $3.05 $0.29 $3.39 $2.12 $0.10 $4.25

9. $1.49 $4.15 $0.46 $3.37 $2.61 $1.83

10. Jack sells tickets at the gate when there is a ball game. To have enough coins to make change he went to the bank today and got $10.00 changed into coins. He asked for two dollars' worth of nickels, three dollars' worth of dimes, and five dollars' worth of quarters. How many coins of each kind did Jack get? 40 nickels, 30 dimes, 20 quarters

11. Ann bought an ice cream cone for 8¢. She gave the man a quarter and 3 pennies. The man gave Ann two dimes in change. Was that the right change? Yes Why did Ann give the man a quarter and 3 pennies instead of giving him only a quarter? *If Ann had given the man a dime and 3 pennies, what change would he have given her? 1 nickel

*She didn't want pennies in change.

12. George gave the clerk a half dollar to pay for a box of bird seed that cost 21¢. The clerk said, "I am short of pennies. Can you give me another cent?" George had a cent and gave it to the clerk. What coins did the clerk give George in change? Give two different answers. 1 nickel, 1 quarter; 3 dimes

Be sure pupils understand why buyers give pennies in ex. 11-12. For more practice, have pupils use prices (under $5.00) of articles in advertisements to count change they would receive from $5.00, if they purchased some items.

Present oral test of pupils' mastery of subtraction facts.

The 100 Subtraction Facts

This is a test on the subtraction facts. You should be able to give all the answers orally within 3 min. If you cannot do this, try to practice so that you can do it by the end of the year. Make a list of any facts that give you trouble and practice on them:

1.	9	7	12	10	16	13	15	9	13	9
	9	6	3	6	8	6	9	7	8	4
	0	1	9	4	8	7	6	2	5	5
2.	16	6	10	3	14	11	5	8	5	14
	9	2	8	0	7	2	5	0	2	6
	7	4	2	3	7	9	0	8	3	8
3.	12	7	13	9	11	12	11	17	8	5
	7	0	9	2	8	6	7	8	6	1
	5	7	4	7	3	6	4	9	2	4
4.	7	9	11	15	4	14	8	10	12	4
	5	1	6	6	3	8	5	3	9	4
	2	8	5	9	1	6	3	7	3	0
5.	9	11	5	7	8	10	2	9	6	1
	6	9	0	7	2	1	0	8	5	0
	3	2	5	0	6	9	2	1	1	1
6.	13	7	18	2	13	6	12	6	14	2
	5	1	9	1	7	4	4	0	5	2
	8	6	9	1	6	2	8	6	9	0
7.	3	1	7	11	8	5	15	10	4	0
	1	1	3	3	8	4	8	5	1	0
	2	0	4	8	0	1	7	5	3	0
8.	16	11	8	12	13	6	9	3	10	6
	7	4	7	5	4	1	5	2	9	3
	9	7	1	7	9	5	4	1	1	3
9.	7	5	9	8	11	3	17	4	9	10
	4	3	0	3	5	3	9	2	3	7
	3	2	9	5	6	0	8	2	6	3
10.	15	10	14	6	8	10	8	12	4	7
	7	4	9	6	4	2	1	8	0	2
	8	6	5	0	4	8	7	4	4	5

To the Teacher. See the suggestions on page 345 for giving this diagnostic test on the fundamental subtraction facts.

See page 4 and Guide for suggestions for giving test and for remedial activities. Pupils might also make small fact cards for practicing the facts of which they are unsure.

Show the relation of the subtraction facts to addition facts; establish subtraction facts involving 0; review terms.

Subtraction

1. The addition facts can help you find the answers to the subtraction facts. Perhaps you cannot remember what 6 from 15 equals. Then think, "6 and what make 15?" If you know that 6 and 9 are 15, then 6 from 15 is 9. Try this way of thinking whenever you forget the answer to a subtraction fact. How should you think if you forget the answer to 7 from 11? $7 + 4 = 11$ the answer to 6 from 14? $6 + 8 = 14$ to 8 from 17? $8 + 9 = 17$

2. The relationship between a subtraction fact and an addition fact can be expressed this way. If you let the letter x represent the number you get when you subtract 4 from 12, then $12 - 4 = x$ and $4 + x = 12$. You can see that x is 8 because $4 + 8 = 12$. So $12 - 4 = 8$. Tell what number x must be in each of these:

$9 + x = 17$ 8 $7 + x = 15$ 8 $x + 4 = 13$ 9 $x + 5 = 13$ 8
$17 - 9 = x$ 8 $15 - 7 = x$ 8 $13 - 4 = x$ 9 $13 - 5 = x$ 8

3. It is easy to subtract 0 from any number. $4 - 0 = 4$. What is $7 - 0$? 7 $5 - 0$? 5 $9 - 0$? 9 **If n stands for any number, then $n - 0 = n$.** Tell what $n - 0 = n$ means in words.

4. It is also easy to subtract a number from itself. $7 - 7 = 0$. What is $8 - 8$? 0 $2 - 2$? 0 **If n stands for any number, then $n - n = 0$.** Tell what $n - n = 0$ means in words.

5. In a subtraction example, the number from which you subtract is called the **minuend,** the number which you subtract is the **subtrahend,** and the answer is the **remainder** or the **difference.**

8000	Minuend
3946	Subtrahend
4054	Remainder

6. To check subtraction, add the remainder to the subtrahend, adding up. The result should equal the minuend.

Copy, subtract, and check your work:

7. 28 − 9 19 57 − 17 40 182 − 90 92 100 − 86 14
8. 32 − 5 27 92 − 36 56 136 − 53 83 200 − 48 152
9. 44 − 8 36 75 − 18 57 145 − 89 56 500 − 37 463

From the check of subtraction, show which parts of a subtraction example can be thought of as addends. Point out that subtraction is the process of finding a missing addend. See Guide for the additive method of subtraction.

Give practice on subtracting larger numbers and present word problems requiring subtraction. Using materials if necessary, redevelop the subtraction process for pupils who show lack of understanding.

Subtraction

Subtract and check your work: Have pupils explain answers.

1.	7260 5459 1801	6485 3892 2593	$714.27 27.59 $686.68	$484.45 326.54 $157.91	$1000.00 992.18 $ 7.82
2.	3111 2999 112	2099 1736 363	$623.41 178.55 $444.86	$904.84 315.47 $589.37	$7687.42 3257.35 $4430.07
3.	9065 438 8627	6422 2466 3956	$842.63 534.86 $307.77	$528.71 282.37 $246.34	$4870.12 748.73 $4121.39
4.	9276 8579 697	7000 6993 7	$924.08 78.34 $845.74	$420.26 35.95 $384.31	$7008.00 6929.40 $ 78.60

More Practice. See ③ on page 326. Use for remedial work.

5. When Mrs. Williams went shopping she had $20.00 in her pocketbook. When she returned home she had $9.65 left. How much did Mrs. Williams spend? $10.35

6. Jim has saved $19.43 all together this year. He wants to buy a radio that costs $39.00. How much less has Jim than he needs to have to buy the radio? $19.57

7. Find the difference between $18.29 and $76.30. $58.01

8. How much more than $83.07 is $90.00? $6.93

9. Some Boy Scouts promised to help in planting 50,000 small trees near the city's water supply. Last Saturday these boys and others planted 22,500 trees. How many trees are there still to be planted? 27,500

Let pupils use numbers in ex. 1–4 in original problems.

Present problems using various forms of subtraction language.

Problems

1. An automobile factory advertised that it needed 3000 new workers in its plant. They have now hired 1275 of these new workers. How many more workers do they need? 1725

2. Mrs. Mason had $183.29 in her checking account at the bank. When she finished writing some checks today she had a balance of $129.65 left in her account. How much money had she paid out by check? $53.64

3. In a recent year the population of Huntington was 28,395 and the population of Bayview was 35,840. How much larger was the population of Bayview than that of Huntington? 7445

4. Mr. North wants to buy a new house. The real estate agent showed him one house for $14,875 and another house for $18,500. What is the difference between the prices of these two houses? $3625

5. The total area of Florida is 58,560 sq. mi. This is made up of 54,262 sq. mi. of land and the rest is the inland water area. How many square miles of inland water are there in Florida? 4298

6. In September of last year the enrollment of Ward College was 8195. In September of this year the enrollment was 10,100. What was the increase in the enrollment? 1905

7. The price of a set of books is $157.95 if you take 6 mo. to pay for them. You can save $23.69 by paying cash. What is the cash price of the set? $134.26

8. Dick is the treasurer of the school paper. Today when he balanced the account he found that he should have $15.33 cash on hand but he had only $14.79. How much less did he have than he should have? $.54

Let pupils do problems independently. Then have different pupils explain their answers, giving reasons why they subtracted. Provide more problems for pupils who had difficulty, and work with them to help develop understanding.

Test pupils' mastery of the multiplication facts. Discuss the uses of the commutative and associative principles in multiplication and compare different ways of finding products.

Relationships in Multiplication

1. The 100 multiplication facts can be written the same way as the 100 addition facts on page 4. Turn to page 4 and try to multiply all the numbers orally within 3 min.

2. Most of the multiplication facts go together in pairs. For example, $3 \times 4 = 12$ and $4 \times 3 = 12$. So $3 \times 4 = 4 \times 3$. What is 6×7? 42 7×6? 42 Does $6 \times 7 = 7 \times 6$? Yes Does $9 \times 5 = 5 \times 9$? Yes **If a and b stand for any two numbers, then $a \times b = b \times a$.** This is called the commutative principle for multiplication. Tell what this principle means.

3. If you forget the answer to 7×8, reverse the numbers and try 8×7. If you know that $8 \times 7 = 56$, the commutative principle tells you that $7 \times 8 = 56$. What related fact can help you if you forget 9×6? 5×8? 7×9? 7×4?
6 x 9 = 54 8 x 5 = 40 9 x 7 = 63 4 x 7 = 28

4. It is easy to multiply 0 by any number. $4 \times 0 = 0$, because $0 + 0 + 0 + 0 = 0$. What is 3×0? 0 7×0? 0 2×0? 0 0×4? 0 **If n stands for any number, then $n \times 0 = 0$.** Tell what $n \times 0 = 0$ means. What does $0 \times n$ equal? 0

5. It is also easy to multiply 1 by any number. $4 \times 1 = 4$, because $1 + 1 + 1 + 1 = 4$. What is 3×1? 3 8×1? 8 **If n stands for any number, then $n \times 1 = n$.** Tell what $n \times 1 = n$ means in words. What does $1 \times n$ equal? n

6. You can find the product $4 \times 2 \times 3$ in these two ways:

$$(4 \times 2) \times 3 = 8 \times 3 = 24$$
$$4 \times (2 \times 3) = 4 \times 6 = 24$$

You see that the way that you group these three numbers when you multiply them does not affect your answer. Find each of these products by grouping the numbers two ways: See Guide.

$2 \times 3 \times 5$ $1 \times 3 \times 2$ $3 \times 6 \times 4$ $5 \times 4 \times 2$

7. If a, b, and c stand for any three numbers, then $(a \times b) \times c = a \times (b \times c)$. This is called the associative principle for multiplication. Tell what this principle means. Write each of these products two ways with parentheses: See Guide.

$3 \times 6 \times 7$ $5 \times 1 \times 8$ $2 \times 3 \times 12$ $15 \times 2 \times 7$

Make a multiplication chart as suggested in the Guide. Find products in it which illustrate the commutative principle. Also illustrate the associative principle, using small factors. For example, 3 x 2 x 4.

Point out that the distributive principle can sometimes be used to make it easier to find a product.

8. Sometimes the commutative and associative principles can be used to make it easy to find a product. For example, $4 \times 7 \times 5$ can be found in this way:

$$\overset{\text{associative}}{4 \times (7 \times 5)} = \overset{\text{commutative}}{4 \times (5 \times 7)} = \overset{\text{associative}}{(4 \times 5) \times 7} = \overset{\text{associative}}{20 \times 7} = 140$$

Tell where the commutative and associative principles are used in the above work.

9. You have learned to find $3 \times (4 + 7)$ by doing the work inside the parentheses first as shown below:

$$3 \times (4 + 7) = 3 \times 11 = 33$$

You can also find $3 \times (4 + 7)$ this way:

$$3 \times (4 + 7) = (3 \times 4) + (3 \times 7) = 12 + 21 = 33$$

Each number in the sum that is inside the parentheses is multiplied by 3, and then these products are added. Find each of these in two ways: See Guide for work.

$2 \times (3 + 4)$ 14 $4 \times (2 + 7)$ 36 $8 \times (6 + 3)$ 72

10. If a, b, and c stand for any three numbers, then $a \times (b + c) = (a \times b) + (a \times c)$. This is called the **distributive principle.** Both addition and multiplication are involved in this principle. Rewrite each of the following with parentheses to illustrate the distributive principle. Find the value of each in two ways. See Guide for work.

$7 \times (4 + 8)$ 84 $9 \times (2 + 5)$ 63 $6 \times (8 + 9)$ 102 $14 \times (2 + 5)$

11. When you multiply 12 by 4, as on the right, you are using the distributive principle, for you really multiply 2 by 4 and 10 by 4 and then add these products. The distributive principle allows you to do this because:

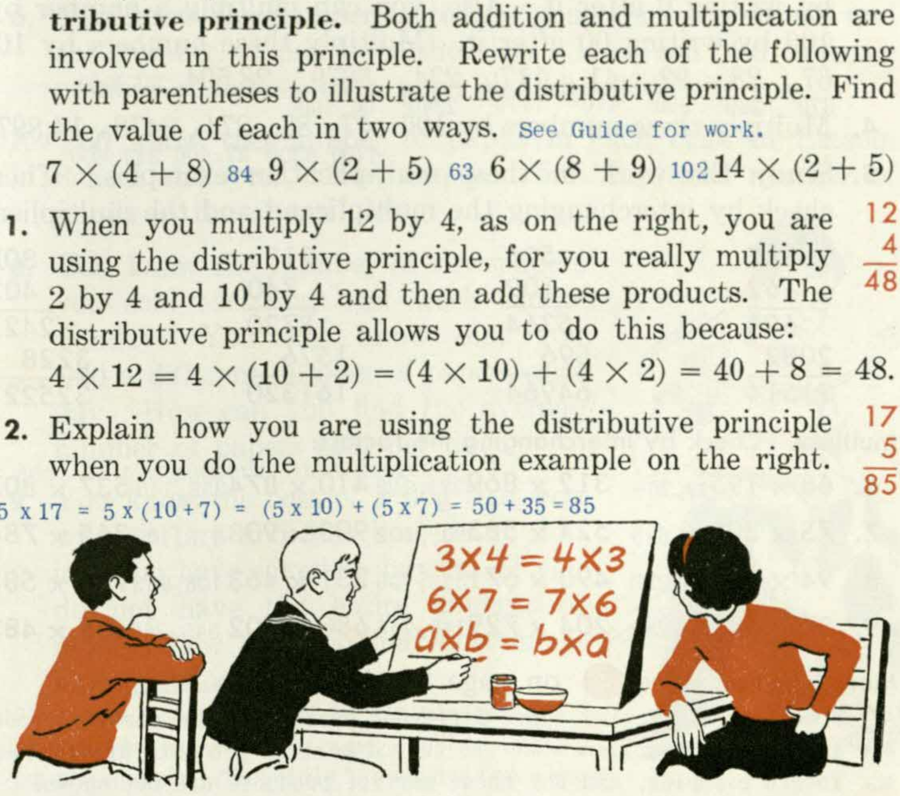
$$\begin{array}{r} 12 \\ \underline{4} \\ 48 \end{array}$$

$$4 \times 12 = 4 \times (10 + 2) = (4 \times 10) + (4 \times 2) = 40 + 8 = 48.$$

12. Explain how you are using the distributive principle when you do the multiplication example on the right.

$$\begin{array}{r} 17 \\ \underline{5} \\ 85 \end{array}$$

$5 \times 17 = 5 \times (10 + 7) = (5 \times 10) + (5 \times 7) = 50 + 35 = 85$



Review multiplication of large numbers, terms used in multiplication, and checking. In discussing ex. 1, emphasize reasons for placement of partial products.

Multiplication

1. In the example at the right, when you multiply 372 by 8, the 6 of 2976 comes under the 8 by which you multiply. Also, the last 8 of 1488 comes under the 4 by which you multiply; and the last 4 of 744 comes under the 2 by which you multiply. Be sure pupils understand that "8," "4," and "2" are 8 ones, 4 tens, 2 hundreds.

		Check
372	Multiplicand	248
248	Multiplier	372
2976	Partial products	496
1488		1736
744		744
92256	Product	92256

You can check a multiplication example by going over the work carefully; you can also interchange the multiplicand and the multiplier and multiply again, as shown above.

2. Numbers which are multiplied together are called **factors** Emphasize. of their product. 7 and 8 are factors of 56. 2, 3, and 5 are factors of 30. 372 and 248 are factors of 92,256.

3. You have learned that you can multiply a number by 10 by writing 0 after it. Also, you can multiply a number by 100 by writing 00 after it. Multiply these numbers by 10: 67, 28, 92, 41, 117, 234, 5679, 28,504.
670 280 920 410 1170 2340 56,790 285,040

4. Multiply these numbers by 100: 47, 83, 274, 3478, 14,897.
4700 8300 27,400 347,800 1,489,700

5. Study the work in these multiplication examples. Then check by interchanging the multiplicand and the multiplier:

347	596	218	807
62	109	740	403
694	5364	8720	2421
2082	596	1526	3228
21514	64964	161320	325221

Multiply. Check by interchanging the factors:

6. 68 × 195 13,260 312 × 869 271,128 410 × 874 358,340 537 × 807 433,359

7. 75 × 209 15,675 524 × 583 305,492 903 × 903 815,409 365 × 784 286,160

8. 94 × 630 59,220 490 × 671 328,790 231 × 463 106,953 907 × 583 528,781

9. 87 × 647 56,289 204 × 729 148,716 680 × 802 545,360 438 × 483 211,554

More Practice. See ④ on page 326. Use for reteaching.

After discussing ex. 1-3, have different pupils explain partial products in ex. 4. Be sure they know what the second partial products are in the second and fourth examples, and why these partial products are not shown.

If pupils seem to have difficulty, suggest that they supply numbers for each problem, solve it, then tell in general terms what they did. Point out that some problems have more than one step.

Problems without Numbers

1. You know the cost of each different article you purchased on a shopping trip and the amount of money you had before you started to shop. How can you find the amount of money you should have left? Add the cost of the articles; then subtract the sum from the amount you had at first.

2. You know the number of gallons of gasoline you buy at one time and the cost of it per gallon. How can you find the cost of all the gasoline? Multiply

3. You know how long it takes you to type one page of a book report. How can you find how long it will take you to type a report of several pages? Multiply

4. You know the number of yards of ribbon in a full roll and the cost of the roll. How can you find the cost of the ribbon per yard? Divide

5. You know the amount of your allowance each week. How can you find your allowance for a year? Multiply

6. You know the number of tickets your class should sell and the number of pupils in the class. How can you find the number of tickets each pupil should sell? Divide

7. You know the number of pupils in each class in Lincoln School this year. How can you find the total number of pupils in the school? Add

8. You know the number of newspapers you sold each day last week except Sunday. You also know that you sold a different number of papers each day. How can you find the average number of papers sold per day?
Add all the numbers; then divide by 6.

9. You know the cost of a dog that you wish to buy and also how many dollars you have saved to buy it. If you do not have the entire amount of money, how can you find how much more you need? Subtract

Give pupils opportunity to explain answers, giving reasons for processes chosen. Let pupils suggest "sensible" numbers for class to use to find numerical answers.

Review problem solving involving multiplication. See Guide.

Problems

1. The high school printed 580 tickets for the school play. If the tickets are sold at $.35 each, how much money will the high school collect from the sale of the tickets? $203.00

2. One kind of airplane can fly 13 mi. on each gallon of gasoline. How far can such an airplane fly on 150 gal.? 1950 mi.

3. Mr. Jackson wants to plant peach trees in a field which he has just bought. He figures that he can plant 18 rows with 24 trees in each row. How many peach trees will Mr. Jackson need to buy? 432

4. Bill wants to know the air distance from his home to Chicago. It takes a plane $3\frac{1}{2}$ hr. to make the trip, traveling at an average speed of 220 mi. per hour. About how far is it from Bill's home to Chicago? 770 mi.

5. Mrs. James bought 3 lb. of meat at $.55 a pound and $\frac{1}{2}$ doz. eggs at $.68 a dozen. How much was her bill? $1.99

6. It costs Mr. Gray an average of $.08 a mile to operate his car. On a recent trip he drove 1465 mi. How much did it cost to operate the car on this trip? $117.20

7. Mr. King paid $2460 for an automobile two years ago. This year he sold it for $\frac{1}{4}$ less than he paid for it. How much did Mr. King get for his car? $1845

8. In one year Maine produced 64,500,000 bu. of potatoes. New Jersey produced about $\frac{1}{5}$ as many. About how many bushels of potatoes did New Jersey produce that year? 12,900,000

9. A bushel of potatoes weighs 60 lb. How many 100-pound bags can a farmer fill if he has 250 bu. of potatoes? 150

After prelesson activity, have pupils complete problems. Instruct them to read problems carefully to determine what facts are known, what the problem asks, what process to use, and how the problem should be written in numerical form.

Test pupils' mastery of division facts (see in Guide suggestions for page 4).

The 90 Division Facts

This is a test on the division facts. You should be able to give all the quotients orally within 3 min.:

1.	$\begin{array}{r}4\\2\overline{)8}\end{array}$	$\begin{array}{r}0\\8\overline{)0}\end{array}$	$\begin{array}{r}2\\4\overline{)8}\end{array}$	$\begin{array}{r}6\\4\overline{)24}\end{array}$	$\begin{array}{r}2\\7\overline{)14}\end{array}$	$\begin{array}{r}8\\2\overline{)16}\end{array}$
2.	$\begin{array}{r}2\\1\overline{)2}\end{array}$	$\begin{array}{r}1\\6\overline{)6}\end{array}$	$\begin{array}{r}0\\1\overline{)0}\end{array}$	$\begin{array}{r}7\\7\overline{)49}\end{array}$	$\begin{array}{r}5\\3\overline{)15}\end{array}$	$\begin{array}{r}7\\6\overline{)42}\end{array}$
3.	$\begin{array}{r}0\\3\overline{)0}\end{array}$	$\begin{array}{r}9\\1\overline{)9}\end{array}$	$\begin{array}{r}8\\6\overline{)48}\end{array}$	$\begin{array}{r}4\\5\overline{)20}\end{array}$	$\begin{array}{r}3\\9\overline{)27}\end{array}$	$\begin{array}{r}5\\4\overline{)20}\end{array}$
4.	$\begin{array}{r}2\\2\overline{)4}\end{array}$	$\begin{array}{r}1\\4\overline{)4}\end{array}$	$\begin{array}{r}9\\7\overline{)63}\end{array}$	$\begin{array}{r}6\\3\overline{)18}\end{array}$	$\begin{array}{r}6\\6\overline{)36}\end{array}$	$\begin{array}{r}6\\9\overline{)54}\end{array}$
5.	$\begin{array}{r}5\\1\overline{)5}\end{array}$	$\begin{array}{r}0\\2\overline{)0}\end{array}$	$\begin{array}{r}5\\5\overline{)25}\end{array}$	$\begin{array}{r}8\\8\overline{)64}\end{array}$	$\begin{array}{r}5\\2\overline{)10}\end{array}$	$\begin{array}{r}3\\5\overline{)15}\end{array}$
6.	$\begin{array}{r}3\\3\overline{)9}\end{array}$	$\begin{array}{r}8\\1\overline{)8}\end{array}$	$\begin{array}{r}4\\9\overline{)36}\end{array}$	$\begin{array}{r}7\\4\overline{)28}\end{array}$	$\begin{array}{r}3\\7\overline{)21}\end{array}$	$\begin{array}{r}5\\6\overline{)30}\end{array}$
7.	$\begin{array}{r}1\\9\overline{)9}\end{array}$	$\begin{array}{r}0\\4\overline{)0}\end{array}$	$\begin{array}{r}9\\2\overline{)18}\end{array}$	$\begin{array}{r}6\\7\overline{)42}\end{array}$	$\begin{array}{r}5\\8\overline{)40}\end{array}$	$\begin{array}{r}4\\4\overline{)16}\end{array}$
8.	$\begin{array}{r}0\\5\overline{)0}\end{array}$	$\begin{array}{r}1\\5\overline{)5}\end{array}$	$\begin{array}{r}2\\8\overline{)16}\end{array}$	$\begin{array}{r}6\\5\overline{)30}\end{array}$	$\begin{array}{r}9\\9\overline{)81}\end{array}$	$\begin{array}{r}4\\3\overline{)12}\end{array}$
9.	$\begin{array}{r}7\\1\overline{)7}\end{array}$	$\begin{array}{r}0\\9\overline{)0}\end{array}$	$\begin{array}{r}7\\9\overline{)63}\end{array}$	$\begin{array}{r}9\\3\overline{)27}\end{array}$	$\begin{array}{r}7\\8\overline{)56}\end{array}$	$\begin{array}{r}2\\5\overline{)10}\end{array}$
10.	$\begin{array}{r}0\\6\overline{)0}\end{array}$	$\begin{array}{r}1\\3\overline{)3}\end{array}$	$\begin{array}{r}6\\2\overline{)12}\end{array}$	$\begin{array}{r}9\\8\overline{)72}\end{array}$	$\begin{array}{r}4\\6\overline{)24}\end{array}$	$\begin{array}{r}4\\8\overline{)32}\end{array}$
11.	$\begin{array}{r}3\\1\overline{)3}\end{array}$	$\begin{array}{r}3\\2\overline{)6}\end{array}$	$\begin{array}{r}7\\5\overline{)35}\end{array}$	$\begin{array}{r}4\\7\overline{)28}\end{array}$	$\begin{array}{r}8\\9\overline{)72}\end{array}$	$\begin{array}{r}8\\3\overline{)24}\end{array}$
12.	$\begin{array}{r}1\\7\overline{)7}\end{array}$	$\begin{array}{r}1\\8\overline{)8}\end{array}$	$\begin{array}{r}8\\4\overline{)32}\end{array}$	$\begin{array}{r}3\\6\overline{)18}\end{array}$	$\begin{array}{r}3\\4\overline{)12}\end{array}$	$\begin{array}{r}8\\7\overline{)56}\end{array}$
13.	$\begin{array}{r}2\\3\overline{)6}\end{array}$	$\begin{array}{r}0\\7\overline{)0}\end{array}$	$\begin{array}{r}6\\8\overline{)48}\end{array}$	$\begin{array}{r}8\\5\overline{)40}\end{array}$	$\begin{array}{r}2\\9\overline{)18}\end{array}$	$\begin{array}{r}7\\2\overline{)14}\end{array}$
14.	$\begin{array}{r}1\\2\overline{)2}\end{array}$	$\begin{array}{r}6\\1\overline{)6}\end{array}$	$\begin{array}{r}5\\7\overline{)35}\end{array}$	$\begin{array}{r}7\\3\overline{)21}\end{array}$	$\begin{array}{r}3\\8\overline{)24}\end{array}$	$\begin{array}{r}9\\6\overline{)54}\end{array}$
15.	$\begin{array}{r}4\\1\overline{)4}\end{array}$	$\begin{array}{r}1\\1\overline{)1}\end{array}$	$\begin{array}{r}9\\5\overline{)45}\end{array}$	$\begin{array}{r}2\\6\overline{)12}\end{array}$	$\begin{array}{r}5\\9\overline{)45}\end{array}$	$\begin{array}{r}9\\4\overline{)36}\end{array}$

To the Teacher. See the suggestions on page 345 for giving this diagnostic test on the fundamental division facts.

Urge pupils to practice on facts of which they are unsure. Retest pupils in a week or two and note further weaknesses. Group pupils for specialized drill on related multiplication and division facts.

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Give pupils oral practice in relating division and multiplication facts. Have them identify the divisor as the known factor and the quotient as the unknown factor.

Relating Division to Multiplication

1. If you forget the answer to the division fact $9\overline{)63}$, ask yourself this question: "How many 9's make 63?" If you know that seven 9's make 63, this tells you that $9\overline{)63}$ is 7. How should you think if you forget $8\overline{)48}$? $6\overline{)54}$? $7\overline{)56}$? See Guide.

2. The relationship between a division fact and a multiplication fact can be expressed this way. If you let the letter y represent the number you get when you divide 18 by 3, then $18 \div 3 = y$ and $y \times 3 = 18$. You can see that y is 6 because $6 \times 3 = 18$. So $18 \div 3 = 6$. Tell what y is in these:

$y \times 2 = 14$ 7 $y \times 6 = 42$ 7 $5 \times y = 45$ 9 $7 \times y = 28$ 4
$14 \div 2 = y$ 7 $42 \div 6 = y$ 7 $45 \div 5 = y$ 9 $28 \div 7 = y$ 4

3. You can use division to find an unknown factor of a number. If you know that one of two factors which have the product 35 is 7, then the division fact $7\overline{)35}$ tells you that the other factor is 5. Find the unknown factor: Stress.

$2 \times$ ..9. $= 18$ ..9 $\times 8 = 72$ $7 \times$ ..3. $= 21$ ..4 $\times 9 = 36$

4. You know that $0 \div 3 = 0$. Give an illustration to show that 0 is a reasonable answer for $0 \div 3$. Does this answer check, using the related multiplication fact $0 \times 3 = 0$? Yes What is $0 \div 7$? 0 $0 \div 2$? 0 **If n stands for any number other than 0, then $0 \div n = 0$.**

5. You know that $5 \div 1 = 5$. What is the related multiplication fact? $5 \times 1 = 5$ What is $3 \div 1$? 3 $7 \div 1$? 7 **If n stands for any number, then $n \div 1 = n$.** Tell what $n \div 1 = n$ means.

6. What is $4 \div 4$? 0 $8 \div 8$? 0 $3 \div 3$? 0 **If n stands for any number other than 0, then $n \div n = 1$.**

Uneven Division Facts

Divide mentally. Give only the quotient and the amount left over. For example, in $3\overline{)16}$, say only "5 and 1 over."

1. $3\overline{)5}$ 1R2 $6\overline{)31}$ 5R1 $4\overline{)13}$ 3R1 $2\overline{)19}$ 9R1 $4\overline{)10}$ 2R2 $3\overline{)23}$ 7R2 $5\overline{)49}$ 9R4
2. $4\overline{)9}$ 2R1 $5\overline{)43}$ 8R3 $2\overline{)17}$ 8R1 $3\overline{)14}$ 4R2 $5\overline{)31}$ 6R1 $6\overline{)19}$ 3R1 $2\overline{)11}$ 5R1
3. $4\overline{)3}$ 0R3 $7\overline{)15}$ 2R1 $5\overline{)24}$ 4R4 $2\overline{)13}$ 6R1 $6\overline{)38}$ 6R2 $3\overline{)29}$ 9R2 $7\overline{)37}$ 5R2

For convenience, answers are written "5 R1" instead of "5 and 1 over."

Review division with 1-figure divisors, checking, and terms used in division.

Division with One-Figure Divisors

1. In division with one-figure divisors you can do the work by writing all the steps in the long form, as shown below at A, or you can do the steps mentally and write only the answer, as shown below at B. If you can get the answers quickly and accurately by using the short form, it will save you time; but if you make mistakes, it is better to use the long form.

A. Long Form	Check	B. Short Form
$\begin{array}{rrl} & 850 & \text{Quotient} \\ \text{Divisor } 7 & \overline{)5954} & \text{Dividend} \\ & \underline{56} & \\ & 35 & \\ & \underline{35} & \\ & 4 & \text{Remainder} \end{array}$	$\begin{array}{r} 850 \\ \underline{7} \\ 5950 \\ \underline{4} \\ 5954 \end{array}$	$\begin{array}{r} 850\text{ R4} \\ 7\overline{)5954} \end{array}$

2. In division, the number that you divide is called the **dividend,** the number by which you divide is the **divisor,** and the answer is the **quotient.** If there is a number left over, it is called the **remainder.** Give pupils practice in identifying these in division examples.

3. On 5 days last week Mr. Westfield drove his automobile 1100 mi. on business. Find the average number of miles he drove his car per day. 220

4. Food for her family last week cost Mrs. Chase $22.05 all together. What was the average cost of the food per day? $3.15

Divide and check by multiplying: Stress checking.

5.	$\overset{1529\text{ R5}}{6\overline{)9179}}$	$\overset{738}{9\overline{)6642}}$	$\overset{4317}{2\overline{)8634}}$	$\overset{7039\text{ R2}}{5\overline{)35197}}$	$\overset{6105}{7\overline{)42735}}$
6.	$\overset{2086}{4\overline{)8344}}$	$\overset{792\text{ R5}}{7\overline{)5549}}$	$\overset{1024}{9\overline{)9216}}$	$\overset{5927}{8\overline{)47416}}$	$\overset{9486}{3\overline{)28458}}$
7.	$\overset{627}{9\overline{)5643}}$	$\overset{842\text{ R3}}{5\overline{)4213}}$	$\overset{1205}{7\overline{)8435}}$	$\overset{9317}{4\overline{)37268}}$	$\overset{4009}{8\overline{)32072}}$
8.	$\overset{834}{7\overline{)5838}}$	$\overset{2958\text{ R1}}{2\overline{)5917}}$	$\overset{1097\text{ R5}}{9\overline{)9878}}$	$\overset{6003\text{ R5}}{8\overline{)48029}}$	$\overset{8734}{6\overline{)52404}}$
9.	$\overset{3007\text{ R2}}{3\overline{)9023}}$	$\overset{1567}{5\overline{)7835}}$	$\overset{1168}{8\overline{)9344}}$	$\overset{6930\text{ R3}}{6\overline{)41583}}$	$\overset{4591}{4\overline{)18364}}$

More Practice. See ⑥ on page 327.

In discussing ex. 1, be sure pupils understand reasons for placement of quotient figures (place value of dividend), and why remainder must be smaller than divisor. Encourage pupils to use short form for small divisors.

Try to retain the element of guided learning by letting pupils discover the generalizations through group exploration.

Casting Out Nines

1. You can tell without dividing that the remainder will be 0 if you divide 764 by 2, because you know that all even numbers are exactly divisible by 2. What is the remainder if 764 is divided by 3? 2 by 4? 0 by 5? 4 by 7? 1 by 9? 8

2. Find the remainders when these numbers are divided by 5:

54 4 64 4 72 2 85 0 91 1 87,619 4 564,962 2

Did you have to do each division to find the remainder?

3. Divide these numbers by 9 and study the remainders:

10 1 100 1 1000 1 20 2 200 2 30 3 300 3 310 4 320 5 330 6

Can you see a relationship between a remainder and the figures or **digits** in the number which was divided?

4. When 221 is divided by 9 the quotient is 24 and the remainder is 5. So 221 = (24 × 9) + 5. You see that 221 contains 5 24 nines. If all the 9's are "cast out" of 221 the remainder or **residue** is 5. Cast out the 9's; observe the residues:

21 3 12 3 35 8 53 8 18 0 81 0 124 7 142 7 421 7 214 7

5. When the 9's are cast out of 32 the residue is 5. The sum of the digits 3 and 2 is also 5. You can find the residue without dividing by adding the digits in the number. Find the residues after the 9's are cast out of these numbers by adding the digits of each number. Then check each residue.

23 5 34 7 71 8 123 6 233 8 413 8 501 6 610 7 2131 7

6. The sum of the digits in 835 is 16, which is greater than 9. If you add the digits in 16 you get 7, which is the residue after all the 9's are cast out of 835. Cast out the 9's:

74 2 85 4 98 8 157 4 438 6 2381 5 8678 2

7. The sum of the digits in 992 is 20. The sum of the digits in 20 is 2. So the residue is 2 if the 9's are cast out of 992. You can get the residue 2 immediately if you skip the two nines when finding the sum of the digits in 992. When casting out nines you can skip any digit which is 9 or any group of digits which has a sum of 9. Cast out the 9's:

191 2 921 3 632 2 275 5 2973 3 12126 3 63163 1

After ex. 7, return to ex. 6 and try casting out 9's as you add the digits. Sometimes one digit can be broken into a sum of two digits to give a 9. In 74 you can think 7 + 2 + 2 and cast out the 7 + 2, leaving a residue of 2.

In using casting out 9's to check addition and multiplication, you do the same operations with the residues that you did with the numbers.

Using Casting Out 9's for Checking

You will usually check column addition by adding down after you have first added up. However, it is interesting to check addition by casting out 9's in this way:

1. Cast the 9's out of each addend. The residues are called the **check numbers** of the addends and are 6, 5, 2 and 4.
2. Add the check numbers of the addends, which gives 17. Then cast the 9's out of 17, which gives 8. Write 8 in parentheses. This is the **final** check number.
3. Cast the 9's out of 2357, which gives a residue of 8. This is the **check number** of the sum. Since this is the same as the final check number, the work is considered correct.

	9's Check
231	6
788	5
371	2
967	4
2357 (8)	17 (8)

You can easily and quickly check multiplication by casting out 9's as shown below:

4. Cast the 9's out of 1532, which gives a residue of 2. This is the check number of the multiplicand.
5. Cast the 9's out of 375. This gives 6 for the check number of the multiplier.
6. Multiply the check number 2 by the check number 6, which gives 12. Then cast the 9's out of 12, which gives 3. This is the final check number.
7. Cast the 9's out of 574,500, which gives 3. This is the check number of the product. Since this is the same as the final check number in step 3, the work is considered correct. When the final check numbers are not the same the work is wrong.

	9's Check
1532	2
375	6
7660	12 (3)
10724	
4596	
574500 (3)	

Multiply. Check by casting out 9's:

8.	583,895 691 × 845	597,432 803 × 744	298,102 749 × 398	3,380,740 635 × 5324
9.	145,222 154 × 943	231,495 253 × 915	242,403 357 × 679	1,776,138 273 × 6506
10.	455,794 653 × 698	456,456 546 × 836	88,963 179 × 497	5,774,442 703 × 8214

Remember that there are some errors which this check will not detect, but all correct answers will check by casting out 9's. See page 26 and the Guide for some of the errors which will not be detected.

More about Casting Out 9's

1. Study these examples carefully:

A.		B.		C.	
338	5	267	6	4253	5
247	4	159	6	1078	7
2366	20 (2)	2403	36 (0)	34024	35 (8)
1552		1335		29771	
676		267		4253	
85486 (4)		42453 (0)		757034 (8)	

2. The final check numbers in A above do not agree. This shows that the work is incorrect. Try to find the mistakes and correct them. See Guide. Then check your work. See Guide.

3. In B the check numbers are both 9; in such cases you "cast out" the 9, which leaves 0. So the final check number is called 0. The largest check number you can have is 8.

4. If you have studied the work in C carefully, you find that it is incorrect although the final check numbers agree. You see that the error is not in the multiplication but in the misplacing of the partial product, 4253; you see that 4253 is written one place too far to the right. This gives the wrong product, but the check of 9's did not detect this error. Whenever a partial product is misplaced, the check of 9's will not detect the error, so it is important to see that each partial product is in the right place before casting out 9's. See Guide for corrected work.

Multiply. Check the work by casting out 9's:

5.	47,970 65 × 738	280,141 457 × 613	3,270,348 428 × 7641	8,570,429 2097 × 4087
6.	63,648 78 × 816	151,682 298 × 509	5,576,627 679 × 8213	28,738,362 3841 × 7482
7.	28,952 94 × 308	260,700 316 × 825	800,935 205 × 3907	35,892,306 5186 × 6921
8.	9234 38 × 243	479,576 604 × 794	534,285 383 × 1395	14,102,000 2750 × 5128
9.	22,356 54 × 414	338,724 582 × 582	856,128 416 × 2058	43,947,516 6239 × 7044

More Practice. See ⑤ on page 326.

Use for remedial work in multiplication and checking. In discussing ex. 3 emphasize that when partial products are misplaced, check of 9's will not disclose this error. Therefore, have pupils check placement of their partial products before using check of 9's.

Reteach division with 2-figure divisors ending in 1 to 5, where Rule I is used. See Guide for discussion of long division.

Division with Two-Figure Divisors

1. Problem Mr. Hall drove his car 5175 mi. in 25 da. How many miles, on the average, did he drive per day? 207

$$\begin{array}{r} 207 \\ 25\overline{)5175} \\ \underline{50} \\ 175 \\ \underline{175} \end{array} \qquad \begin{array}{r} \textbf{Check} \\ 207 \\ \underline{25} \\ 1035 \\ \underline{414} \\ 5175 \end{array}$$

Explanation The first partial dividend is 51. To estimate the first quotient figure, **divide** 5 by 2. This gives 2, which you write over the 1 of 51. **Multiply** 25 by 2, which gives 50. **Subtract** 50 from 51, which gives a remainder of 1. Since 1 is less than the divisor, the quotient 2 is right. **Bring down** 7, which makes the next partial dividend 17. 17 is less than 25, so the next quotient figure is 0.

Ask the reason.

Bring down 5, which gives 175 as the next partial dividend. **Divide** 17 by 2, which gives 8 as the next quotient figure. But when you multiply 25 by 8 you get 200, which is too large to be subtracted from 175. Try 7. Why is 7 right?

Mr. Hall drove an average of 207 mi. per day. To check the work, multiply 207 by 25. The result should equal the dividend. Since the product is 5175, the quotient is correct.

Emphasize Rule I.

When a two-figure divisor ends in 1, 2, 3, 4, or 5, divide by the first figure of the divisor to estimate each figure of the quotient.

Remind pupils that when checking, remainder must be added after multiplying quotient by divisor.

Divide and check:

2.	55 R5 32)1765	59 R5 22)1303	367 85)31195	213 54)11502
3.	326 24)7824	84 23)1932	785 R11 62)48681	375 R13 81)30388
4.	89 63)5607	67 R11 51)3428	296 R16 45)13336	605 R22 43)26037
5.	89 25)2225	308 R5 32)9861	840 R7 33)27727	998 41)40918
6.	92 44)4048	54 35)1890	638 71)45298	406 R13 64)25997

Have pupils explain placement of quotient figures (place value) in ex. 2–6.

More Practice. See ⑦ on page 327. In ex. 1, emphasize the 5 steps necessary for finding each quotient figure (in particular, comparison step). Be sure pupils understand that "51" is 51 hundreds; therefore, "2" is in hundreds place. Do same for other quotient figures.

Give quick oral review, particularly on measures taught in previous grades.

Oral Review

1. Count by 7's from 4 to 60; (1) from 109 to 172. (2) (1) 4, 11, 18, 25, 32, 39, 46, 53, 60; (2) 109, 116, 123, 130, 137, 144, 151, 158, 165, 172

2. Count by 8's from 3 to 99; (1) from 110 to 182. (2) (1) 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, 99; (2) 110, 118, 126, 134, 142, 150, 158, 166, 174, 182

3. Count by 9's from 7 to 88; (1) from 152 to 224. (2) (1) 7, 16, 25, 34, 43, 52, 61, 70, 79, 88; (2) 152, 161, 170, 179, 188, 197, 206, 215, 224

Add 6 to each number. (1) **Add 7;** (2) **add 8;** (3) **add 9:** (4)

(1), (2) 49, 50 33, 34 62, 63 94, 95 101, 102 80, 81 45, 46 68, 69
(3), (4) **4.** 43 51, 52 27 35, 36 56 64, 65 88 96, 97 95 103, 104 74 82, 83 39 47, 48 62 70, 71

100, 101 74, 75 41, 42 85, 86 28, 29 52, 53 63, 64 89, 90
5. 94 102, 103 68 76, 77 35 43, 44 79 87, 88 22 30, 31 46 54, 55 57 65, 66 83 91, 92

6. How many dollars and cents are there in 8 quarters, 6 dimes, 10 nickels, and 13 pennies? $3.23

7. Find the sum of 8, 3, 2, 6, 9, 5, 4, 8, 1, and 7. 53

8. How much change do you get from a half dollar if you spend 37¢? 13¢ if you spend 24¢? 26¢

9. How much change do you get from 1 dollar if you buy 2 notebooks at 19¢ each? $.62

10. How many 4-cent apples can you buy with a quarter? 6, 1 cent over

11. Find the cost of 2 movie tickets at 29¢ each. 58¢

Pupils who do poorly on ex. 12-20 should refer to "Tables of Reference," page 342.

Tell what number should be put in each space:

12. 1 ft. = ... in. 12 6 in. = ... ft. $\frac{1}{2}$ 5 gal. = ... qt. 20
13. 1 mi. = ... ft. 5280 8 qt. = ... pt. 16 45 min. = ... hr. $\frac{3}{4}$
14. 1 yd. = ... ft. 3 $\frac{1}{2}$ lb. = ... oz. 8 15 sec. = ... min. $\frac{1}{4}$
15. 1 lb. = ... oz. 16 2 pk. = ... qt. 16 20 oz. = ... lb. $1\frac{1}{4}$
16. 2 yd. = ... ft. 6 2 ft. = ... yd. $\frac{2}{3}$ $1\frac{1}{2}$ yr. = ... mo. 18
17. 4 yr. = ... mo. 48 6 qt. = ... gal. $1\frac{1}{2}$ 15 pk. = ... qt. 120
18. 2 wk. = ... da. 14 $\frac{1}{4}$ lb. = ... oz. 4 3 tons = ... lb. 6000
19. $\frac{1}{2}$ yd. = ... in. 18 3 hr. = ... min. 180 2 min. = ... sec. 120
20. $\frac{1}{2}$ hr. = ... min. 30 $\frac{1}{2}$ yr. = ... mo. 6 $\frac{1}{2}$ mi. = ... ft. 2640

To the Teacher. Caution pupils never to write in their books to fill in blank spaces.

First have pupils count by 7's, 8's, and 9's by beginning with number itself. Then do ex. 1-3. You might conduct this review as quiz program, letting different pupils act as quizmaster.

Reteach division with 2-figure divisors ending in 6 to 9, when Rule II is used.

More about Long Division

1. Problem Last month Mr. West took in $1277.25 from the sale of 39-cent books. How many books did he sell at this price? 3275

Explanation First write $1277.25 as 127725¢.

```
      3275
39)127725
   117
    107
     78
     292
     273
      195
      195
```

Since the first figure of 39 is 3, use 4 as the trial divisor. You may do this because 39 is almost 40. To find the first quotient figure, think, "12 ÷ 4 = 3." Using 3 as the quotient, you get the remainder 10, which is less than the divisor. So 3 is the right quotient figure.

Tell how you obtain 2 and 7 as the next quotient figures.

For the last quotient figure think, "There are four 4's in 19." Using 4 as the quotient, you get a remainder of 39, which is **equal to** the divisor. This shows that the quotient figure should be 5 instead of 4, since the remainder 39 will contain 39 one more time. The quotient is 3275. Mr. West sold 3275 books.

Emphasize Rule II.

When a two-figure divisor ends in 6, 7, 8, or 9, divide by 1 more than the first figure of the divisor to estimate each figure of the quotient.

Divide and check: Stress checking of each example.

2.	24,889 ÷ 46 541 R3	169,296 ÷ 48 3527	132,300 ÷ 28 4725
3.	89,275 ÷ 28 3188 R11	381,017 ÷ 79 4823	571,444 ÷ 76 7519
4.	32,676 ÷ 37 883 R5	625,350 ÷ 66 9475	324,336 ÷ 87 3728
5.	19,973 ÷ 26 768 R5	253,518 ÷ 87 2914	279,794 ÷ 37 7562
6.	67,392 ÷ 39 1728	124,166 ÷ 49 2534	439,756 ÷ 58 7582
7.	47,927 ÷ 78 614 R35	383,496 ÷ 57 6728	171,028 ÷ 46 3718

More Practice. See ⑧ on page 327. Use to reinforce skill.

Discuss ex. 1 thoroughly with pupils, emphasizing reasons for proper placement of quotient figures (place value). Be sure pupils know why 127 thousands is first partial dividend. Have different pupils explain some quotients in ex. 2–7.

Present more division practice for those who need it.

Problems and Practice

1. Jack is trying to save \$50.00 for a new bicycle. He can save 75¢ a week. So far he has \$37.25 toward the \$50.00. In how many more weeks can he save the rest of the money? 17

2. Our class party cost \$13.16 all together. The 28 pupils in the class shared the cost equally. How much was each one's share of the cost? \$.47

3. Last Saturday Ann drove to Fairfield with her father. The round-trip distance was 237 mi. They went this distance in 6 hr. of driving. What was their average speed in miles per hour on the round trip? $39\frac{1}{2}$

4. From 8:30 A.M. until 1:30 P.M. Miss Black drove her car 190 mi. What was Miss Black's average speed in miles per hour? 38

Remind pupils that a figure must be put in the quotient every time a figure is brought down from the dividend.

Divide and check your work:

5. 6789 ÷ 73 93 　 24,272 ÷ 37 656 　 34,272 ÷ 68 504
6. 4355 ÷ 13 335 　 22,011 ÷ 33 667 　 35,073 ÷ 87 402 R12
7. 9936 ÷ 69 144 　 10,203 ÷ 19 537 　 52,075 ÷ 75 694 R25
8. 7047 ÷ 81 87 　 24,615 ÷ 45 547 　 11,381 ÷ 17 669 R8
9. 4420 ÷ 68 65 　 29,463 ÷ 61 483 　 20,646 ÷ 93 222
10. 6950 ÷ 27 257 R11 　 51,716 ÷ 92 562 R12 　 57,104 ÷ 86 664
11. 6090 ÷ 42 145 　 11,921 ÷ 18 662 R5 　 20,458 ÷ 53 386
12. 9430 ÷ 46 205 　 25,380 ÷ 54 470 　 11,487 ÷ 21 547
13. 7366 ÷ 58 127 　 59,696 ÷ 82 728 　 12,530 ÷ 14 895
14. 9247 ÷ 36 256 R31 　 32,487 ÷ 49 663 　 18,408 ÷ 26 708
15. 8910 ÷ 15 594 　 68,835 ÷ 74 930 R15 　 36,144 ÷ 48 753

Do ex. 1-4 orally with pupils. Assign part of ex. 5-15 to half of the class, remaining examples to the other half. Group pupils who have difficulties to determine causes (estimating quotients, knowledge of facts, placement of figures, etc.).

Review tests of divisibility and use them in changing fractions to lowest terms.

Tests of Divisibility

In long division, remainders are often used to give fractions in the quotients. To change these fractions to lowest terms, the tests of divisibility given below will be helpful. These tests are quick ways to find out whether a number is **exactly divisible** by 2, 3, 5, or 9 **without actually dividing.** Emphasize.

1. *A number is divisible by 3 if the sum of its digits is divisible by 3.*

342 is divisible by 3 because $3 + 4 + 2 = 9$, and 9 is divisible by 3. 521 is not divisible by 3 because $5 + 2 + 1 = 8$, and 8 is not divisible by 3.

2. *A number is divisible by 9 if the sum of its digits is divisible by 9.*

3456 is divisible by 9 because $3 + 4 + 5 + 6 = 18$, and 18 is divisible by 9. 882 is also divisible by 9. 2135 is not divisible by 9 because $2 + 1 + 3 + 5 = 11$, and 11 is not divisible by 9.

3. *A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8.*

528 is divisible by 2 because it ends in 8; but 637 is not divisible by 2 because it ends in 7.

4. *A number is divisible by 5 if it ends in 0 or 5.*

265 and 320 each are divisible by 5, but 214 and 321 are not.

5. Change $\frac{234}{828}$ to lowest terms.

$\frac{234}{828} = \frac{117}{414} = \frac{13}{46}$

Rule 3 above shows that 2 will exactly divide both 234 and 828. Dividing both terms of $\frac{234}{828}$ by 2, you get $\frac{117}{414}$. Rule 2 shows that 9 will exactly divide both 117 and 414. Dividing both terms of $\frac{117}{414}$ by 9, you get $\frac{13}{46}$. Is $\frac{13}{46}$ in its lowest terms? Yes

Change to lowest terms, using the tests of divisibility:

6. $\frac{45}{75}$ $\frac{3}{5}$ $\frac{36}{54}$ $\frac{2}{3}$ $\frac{48}{87}$ $\frac{16}{29}$ $\frac{135}{180}$ $\frac{3}{4}$ $\frac{75}{165}$ $\frac{5}{11}$ $\frac{225}{315}$ $\frac{5}{7}$ $\frac{189}{504}$ $\frac{3}{8}$ $\frac{1296}{1458}$ $\frac{8}{9}$

7. $\frac{42}{57}$ $\frac{14}{19}$ $\frac{48}{72}$ $\frac{2}{3}$ $\frac{84}{96}$ $\frac{7}{8}$ $\frac{45}{120}$ $\frac{3}{8}$ $\frac{108}{144}$ $\frac{3}{4}$ $\frac{270}{324}$ $\frac{5}{6}$ $\frac{243}{297}$ $\frac{9}{11}$ $\frac{1260}{2205}$ $\frac{4}{7}$

8. $\frac{54}{90}$ $\frac{3}{5}$ $\frac{32}{48}$ $\frac{2}{3}$ $\frac{60}{72}$ $\frac{5}{6}$ $\frac{60}{150}$ $\frac{2}{5}$ $\frac{234}{288}$ $\frac{13}{16}$ $\frac{128}{168}$ $\frac{16}{21}$ $\frac{162}{351}$ $\frac{6}{13}$ $\frac{2322}{4158}$ $\frac{43}{77}$

Pupils should be urged to memorize rules after understanding is assured. Demonstrate these rules with other examples also. In ex. 6–8 have pupils tell which rules apply and why.

Reteach division with 3-figure divisors (pages 32-33). In discussing ex. 1, be sure pupils understand why "7" in the quotient is written over "3" in the dividend (since 2553 ones were divided).

Three-Figure Divisors

1. **Problem** An airplane averages 345 mi. an hour. How many hours should it take to travel a distance of 2553 mi.? $7\frac{2}{5}$

Explanation To estimate the quotient figure, divide 25 by 3. This gives 8 as the quotient figure. When you multiply 345 by 8, you get 2760, which is more than 2553. Try 7 as the quotient figure. Multiply 345 by 7, which gives 2415. Subtract 2415 from 2553. This gives a remainder of 138. Why is 7 correct? Write 138 over the divisor, and change the fraction to lowest terms. The quotient is $7\frac{2}{5}$, so $7\frac{2}{5}$ hr. are needed for the trip.

$$\begin{array}{r} 7\frac{138}{345}=7\frac{2}{5} \\ 345\overline{)2553} \\ \underline{2415} \\ 138 \end{array}$$

Check by multiplying 345 by 7 and adding 138.

Emphasize Rule I.

When the second figure of the divisor is 1, 2, 3, 4, or 5, divide by the first figure of the divisor to estimate each quotient figure.

Remind pupils to use divisibility tests in changing fractions in quotients to lowest terms.

Divide and check your work:

2. $\overset{9\frac{3}{8}}{504\overline{)4725}}$ $\overset{124}{716\overline{)88784}}$ $\overset{70}{325\overline{)22750}}$ $\overset{196\frac{11}{53}}{954\overline{)187182}}$

3. $\overset{6\frac{7}{8}}{256\overline{)1760}}$ $\overset{87\frac{1}{2}}{540\overline{)47250}}$ $\overset{54\frac{12}{103}}{412\overline{)22296}}$ $\overset{628}{509\overline{)319652}}$

4. $\overset{9\frac{1}{3}}{225\overline{)2100}}$ $\overset{96}{354\overline{)33984}}$ $\overset{79}{248\overline{)19592}}$ $\overset{259}{627\overline{)162393}}$

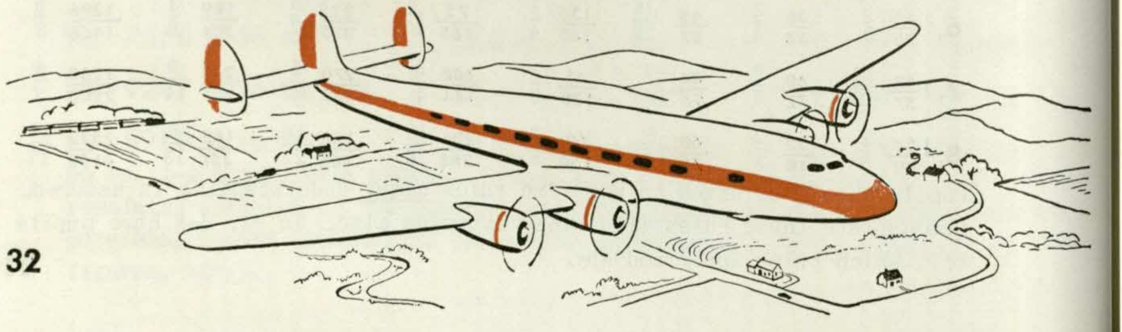
5. $\overset{24}{412\overline{)9888}}$ $\overset{27\frac{11}{93}}{837\overline{)22698}}$ $\overset{75}{741\overline{)55575}}$ $\overset{221\frac{4}{91}}{819\overline{)181035}}$

6. $\overset{25}{338\overline{)8450}}$ $\overset{58}{429\overline{)24882}}$ $\overset{243\frac{5}{9}}{243\overline{)59184}}$ $\overset{588}{406\overline{)238728}}$

7. $\overset{31\frac{3}{7}}{252\overline{)7920}}$ $\overset{39}{608\overline{)23712}}$ $\overset{255}{238\overline{)60690}}$ $\overset{351}{437\overline{)153387}}$

Emphasize comparison step and the reason for the remainder's being smaller than the divisor.

More Practice. See ⑨ on page 328.



More Work in Long Division

1. **Problem** Last month it cost the Star Radio Company $26,048.50 to make 765 automobile radio sets. Find the company's average cost per radio set. $34.05

$$\begin{array}{r} \$34.05\\ 765\overline{)\$26048.50}\\ 2295\\ \hline 3098\\ 3060\\ \hline 38\,50\\ 38\,25\\ \hline 25 \end{array}$$

Explanation Since the second figure of 765 is 6, divide by 8, instead of 7, to estimate each quotient figure. You divide by 8 because 765 is nearer to 800 than it is to 700. Think, "There are three 8's in 26." Write 3 in the quotient. Then multiply 765 by 3 and subtract the product from 2604. This leaves 309, which is less than 765.

To find the second quotient figure, divide 30 by 8. The quotient is 3. Multiply 765 by 3, which gives 2295. Subtract 2295 from 3098, which gives 803; 803 is **larger** than 765. This shows that 3 is **too small** for the quotient figure. Try 4. Why is 4 right?

Tell how to finish the work. The answer is $34.05 with a remainder of $.25. The cost per set was about $34.05.

Emphasize Rule II.

When the second figure of the divisor is 6, 7, 8, or 9, divide by 1 more than the first figure of the divisor to estimate each quotient figure.

Do ex. 2 with pupils. If there are no questions, assign ex. 3-6 as independent work.

Divide. Check the work by multiplying:

2.	$\begin{array}{r}526\\37\overline{)19462}\end{array}$	$\begin{array}{r}982\\76\overline{)74632}\end{array}$	$\begin{array}{r}87\\188\overline{)16356}\end{array}$	$\begin{array}{r}\$\ \ 20.73\\269\overline{)\$5576.37}\end{array}$
3.	$\begin{array}{r}860\ \text{R12}\\58\overline{)49892}\end{array}$	$\begin{array}{r}609\\47\overline{)28623}\end{array}$	$\begin{array}{r}34\ \text{R560}\\672\overline{)23408}\end{array}$	$\begin{array}{r}\$\ \ 2.07\ \text{R\$1.38}\\894\overline{)\$1851.96}\end{array}$
4.	$\begin{array}{r}2457\\26\overline{)63882}\end{array}$	$\begin{array}{r}1847\\19\overline{)35093}\end{array}$	$\begin{array}{r}408\\199\overline{)81192}\end{array}$	$\begin{array}{r}\$\ \ 9.00\ \text{R\$1.04}\\376\overline{)\$3385.04}\end{array}$
5.	$\begin{array}{r}451\\68\overline{)30668}\end{array}$	$\begin{array}{r}1243\ \text{R24}\\27\overline{)33585}\end{array}$	$\begin{array}{r}136\\293\overline{)39848}\end{array}$	$\begin{array}{r}\$\ \ 6.06\\582\overline{)\$3532.74}\end{array}$
6.	$\begin{array}{r}2038\\39\overline{)79482}\end{array}$	$\begin{array}{r}725\ \text{R28}\\46\overline{)33378}\end{array}$	$\begin{array}{r}58\\461\overline{)26738}\end{array}$	$\begin{array}{r}\$\ \ 8.90\ \text{R63¢}\\396\overline{)\$3525.03}\end{array}$

More Practice. See ⑩ on page 328. Use for remedial work.

Discuss ex. 1 carefully with pupils, stressing proper placement of quotient figures and reasons for placement (dividing tens gives tens, and so on). Observe pupils as they do ex. 3-6 to note difficulties and determine causes.

Review and reteach two types of problems with averages.

The School Play

1. Problem The Hudson Junior High School put on a play which was given three times last week. They sold 472 tickets for the first performance, 425 tickets for the second, and 383 tickets for the third. Find the average number of tickets sold per performance. 427

Explanation To find the average, add the three numbers, which gives 1280. Then divide 1280 by 3; this gives $426\frac{2}{3}$, or 427 to the nearest whole number. 427 is called the **average** of the three numbers.

$$\begin{array}{r} 472 \\ 425 \\ \underline{383} \\ 3\overline{)1280} \\ 426\frac{2}{3} \end{array}$$

2. Of all the tickets sold for the play, 683 tickets were sold in advance by 135 pupils. What was the average number of tickets sold per pupil? 5

▶ In this problem the sum 683 is given, so there are no numbers to be added. Divide 683 by 135 to find the average.

3. John was one of the actors in the play. On 4 days before the play was given he practiced his part as follows: 2 hr. the first day, $1\frac{1}{2}$ hr. the second day, $2\frac{1}{2}$ hr. the third day, and 1 hr. the fourth day. What was the average number of hours John practiced per day? $1\frac{3}{4}$

Ex. 1 illustrates 2-step problem in averages (add first, then divide) and ex. 2 illustrates 1-step problem (sum is given, just divide). Be sure pupils understand difference between these types, and give more illustrations to emphasize this.

Stress fact that answers must be sensible (see ex. 4). Give further practice in deciding when to round off answer.

4. In ex. 1, the answer is $426\frac{2}{3}$ tickets; this is not a sensible answer since you cannot sell $\frac{2}{3}$ of a ticket; for this reason the answer is rounded off to 427 tickets. In ex. 3, however, $1\frac{3}{4}$ hr. is a sensible answer, so you keep the fraction instead of rounding off the number.

5. Polly had a very important part in the play. On 5 days before the play she practiced these numbers of hours: $1\frac{3}{4}$ hr., 2 hr., $2\frac{1}{4}$ hr., 1 hr., 1 hr. Find the average number of hours she practiced per day. $1\frac{3}{5}$

6. It was necessary to rent 4 costumes for the play. The charges for renting these costumes were \$2.50 for John's costume, \$3.75 for Polly's, \$1.25 for Fred's, and \$3.50 for Mary's. What was the average charge per costume? \$2.75

7. Last year 1170 tickets in all were sold for the school play and it was given on 3 different days. Find the average number of tickets sold per performance. 390

Have pupils make up problems illustrating the two types, where fraction is kept and where rounded off.

Review division of all types and extend divisors to 4 figures.

Problems and Practice

1. Last week 3568 persons paid $6255.50 to hear a young violinist. What was the average price of the tickets? $1.75 R$11.50, or $1.75

▶ The divisor in this problem has 4 figures. Follow the same rules that are used for 3-figure divisors. See pages 32 and 33.

Review Rules I and II before doing ex. 1 with pupils.

2. On a 25-day trip the speedometer of Mr. Black's car showed he had traveled 4775 mi. Find the average number of miles the car traveled per day. 191

3. In the school store last month they took in $65.74 from the sale of 19-cent notebooks. How many notebooks did they sell at this price? 346

4. The distance by air from New York to San Francisco is 2568 mi. The distance by water, around Cape Horn, is 15,348 mi. The distance by water is how many times as far as the distance by air? Give the answer to the nearest whole number. 5 R 2508, or 6 times

Divide. Check your work by multiplying: Stress.

5. $\begin{array}{r} 304 \\ 73\overline{)22192} \end{array}$ $\begin{array}{r} 528 \\ 46\overline{)24288} \end{array}$ $\begin{array}{r} 38\text{ R4} \\ 537\overline{)20410} \end{array}$ $\begin{array}{r} 183 \\ 9486\overline{)1735938} \end{array}$

6. $\begin{array}{r} 142 \\ 94\overline{)13348} \end{array}$ $\begin{array}{r} 219\text{ R60} \\ 85\overline{)18675} \end{array}$ $\begin{array}{r} 48 \\ 395\overline{)18960} \end{array}$ $\begin{array}{r} 483 \\ 5609\overline{)2709147} \end{array}$

7. $\begin{array}{r} 700\text{ R31} \\ 82\overline{)57431} \end{array}$ $\begin{array}{r} 496 \\ 68\overline{)33728} \end{array}$ $\begin{array}{r} 428 \\ 139\overline{)59492} \end{array}$ $\begin{array}{r} 600\text{ R15} \\ 4576\overline{)2745615} \end{array}$

8. $\begin{array}{r} 827 \\ 37\overline{)30599} \end{array}$ $\begin{array}{r} 783 \\ 24\overline{)18792} \end{array}$ $\begin{array}{r} 51\text{ R7} \\ 458\overline{)23365} \end{array}$ $\begin{array}{r} 619\text{ R13} \\ 3724\overline{)2305169} \end{array}$

Have pupils complete ex. 2-4. Then have volunteers explain solutions at board. If there are no apparent difficulties, assign ex. 5-8. Check pupils' work carefully and plan now for reteaching or specialized review as needed.

Teach casting out 9's as a new check for long division (see Guide for discussion).

A New Way to Check Division

1. Division examples can also be checked by casting out 9's. In the example at the right, first find the check numbers for 186, 563, 17, and 104,735, as was done in checking multiplication. Then multiply the check number 5 (quotient) by the check number 6 (divisor), which gives 30. Add to 30 the check number 8 (remainder); this gives 38, which has the check number 2. Since 2 is the same as the check number for the dividend, consider the work correct.

$$\begin{array}{rrl}
 & (5) & \\
(6) & 563 & \\
186 & \overline{)104735} & (2)\\
 & \underline{930} & \\
 & 1173 & \\
 & \underline{1116} & \\
 & 575 & \\
 & \underline{558} & \\
 & 17 & (8)
\end{array}
\qquad
\begin{array}{rl}
\text{9's Check} & \\
5 & \\
\underline{6} & \\
30 & \\
\underline{8} & \\
38 & (2)
\end{array}$$

Be sure pupils know why 2 is check number (3 + 8 = 11, 1 + 1 = 2).

Remind pupils that we continue to cast out until we have a 1-place check number.

2. Study carefully these examples and their checks:

$$\begin{array}{rrl}
 & (7) & \\
(3) & 457 & \\
75 & \overline{)34298} & (8)\\
 & \underline{300} & \\
 & 429 & \\
 & \underline{375} & \\
 & 548 & \\
 & \underline{525} & \\
 & 23 & (5)
\end{array}
\qquad
\begin{array}{rl}
\text{9's Check} & \\
7 & \\
\underline{3} & \\
21 & \\
\underline{5} & \\
26 & (8)
\end{array}
\qquad\qquad
\begin{array}{rrl}
 & (8) & \\
(2) & 953 & \\
38 & \overline{)35226} & (0)\\
 & \underline{332} & \\
 & 202 & \\
 & \underline{190} & \\
 & 126 & \\
 & \underline{114} & \\
 & 12 & (3)
\end{array}
\qquad
\begin{array}{rl}
\text{9's Check} & \\
8 & \\
\underline{2} & \\
16 & \\
\underline{3} & \\
19 & (1)
\end{array}$$

The second example above does not check because the final check number 1 does not agree with the check number of the dividend, which is 0; so this example contains a mistake. Correct the mistake and then see if the example checks. See Guide.

Divide. Check by casting out 9's:

3. 12,775 ÷ 19 672R7 327,940 ÷ 863 380 305,312 ÷ 329 928

4. 19,450 ÷ 25 778 254,645 ÷ 334 762R137 122,299 ÷ 593 206R141

5. 55,300 ÷ 92 601R8 333,326 ÷ 406 821 161,550 ÷ 718 225

6. 41,173 ÷ 49 840R13 112,948 ÷ 187 604 336,353 ÷ 475 708R53

Discuss ex. 1–2 carefully and have volunteers explain check in ex. 3. You might urge better students to investigate casting out 11's.

Give both oral and written practice on work retaught or reviewed in Chapter 1.

Oral Practice

Divide each number by 5.(1) Divide each by 6:(2)

(1) 5 9R3 6R3 5R3 8 3R3 12 1R3
(2) **1.** 25 4R1 48 8 33 5R3 28 4R4 40 6R4 18 3 60 10 8 1R2

2R2 11 3R4 18 3 7R1 11R3 0R3
2. 12 2 55 9R1 19 3R1 90 15 15 2R3 36 6 58 9R4 3 0R3

Divide mentally:

21 212 320 103 203 70
3. 8)168 4)848 2)640 5)515 3)609 7)490

101 61 62 204 71 20
4. 6)606 9)549 3)186 4)816 8)568 6)120

52 78 100 61 83 76
5. 5)260 2)156 9)900 7)427 4)332 5)380

Find the answers:

6. 7 + 9 + 8 24 2 × 6 × 9 108 (5 × 3) + 2 17 4 × (5 + 4) 36

7. 9 + 6 + 7 22 8 × 7 × 2 112 (6 × 6) + 4 40 5 × (8 + 3) 55

8. 5 + 8 + 8 21 9 × 2 × 7 126 (9 × 6) + 8 62 6 × (8 + 4) 72

Find the unknown factor:

9. 8 × .9. = 72 .5. × 4 = 20 3 × .83. = 249 .48. × 5 = 240

10. 5 × .6. = 30 .1. × 9 = 9 8 × .21. = 168 .61. × 7 = 427

Written Practice Not all pupils need do all these exercises.

11. Multiply by 245: 604 937 280 1986 2753
147,980 229,565 68,600 486,570 674,485
12. Multiply by 307: 289 670 405 3381 7926
88,723 205,690 124,335 1,037,967 2,433,282
13. Multiply by 860: 127 941 665 8602 3978
109,220 809,260 571,900 7,397,720 3,421,080
14. Multiply by 179: 448 192 179 5641 3086
80,192 34,368 34,368 1,009,739 552,394
15. Divide by 250: 16,750 67 97,250 389 356,000 1424

16. Divide by 294: 13,818 47 14,700 50 294,882 1003

17. Divide by 315: 26,460 84 11,025 35 564,795 1793

18. Divide by 567: 10,206 18 26,082 46 175,203 309

Use oral practice to spot-check for class or individual weaknesses. If pupils have difficulties with ex. 11-18, try to determine causes through individual conferences.

Review important problem patterns in division. Discuss problems with pupils and have them explain why division is the process to be used. Then have pupils complete solutions.

More Problems

1. It cost Mr. Green $348 to operate the family car last year. That was an average cost of how much per month? $29

2. The 17 girls in the 4–H Club canned 153 jars of tomatoes. If they share the jars equally, how many jars will each girl receive? 9

3. Mr. James can drive about 18 mi. on each gallon of gasoline. He has to make a trip of 666 mi. About how many gallons of gasoline will he use on the trip? 37

4. On a transcontinental trip an airplane covered 2405 mi. at an average speed of 481 mi. per hour. How many hours did it take to make the trip? 5

5. After driving 30 min. Jack noticed that he had gone 18 mi. How many minutes did it take him to go 1 mi.? $1\frac{2}{3}$

6. A passenger elevator that can carry a total load of 2250 lb. was allowed to carry not more than 15 people. What was the average weight allowed per person? 150 lb.

7. If an airplane uses about $1\frac{1}{4}$ gal. of gasoline per mile and carries a load of 6000 gal. of gasoline, about how many miles can it travel? 4800

8. Mrs. Ray bought some furniture for her new home for $495. If she pays $55 a month, how many months will it take her to pay for the furniture? 9

9. Ann read that 8640 families living in Fairfield own their homes. There are 12,960 families in all who live in that town. What part of the families own their homes? $\frac{2}{3}$

Group for further work pupils who have difficulty in recognizing patterns. Have others make up own problems.

Present summary of work reviewed or retaught in Chapter 1.

Chapter Review

1. Round off each number to the nearest million:
2,093,176 2,000,000 7,728,413 8,000,000 9,994,361 10,000,000

2. Round off each number to the nearest ten million:
60,584,127 60,000,000 25,486,133 30,000,000 89,710,025 90,000,000

3. Round off each number to the nearest billion:
2,000,000,000 8,000,000,000 6,000,000,000
2,189,420,000 7,650,927,164 5,558,607,240

Add 187 to each number; (1) **add 962 to each:** (2)

(1) 3000 6599 9225 5742 8195
(2) **4.** 2813 3775 6412 7374 9038 10,000 5555 6517 8008 8970

Subtract 1284 from each number; (1) **subtract 6099 from each:** (2)

(1) 5997 6716 5000 7719 5771
5. 7281 1182 8000 1901 6284 185 9003 2904 7055 956

Multiply and check by casting out 9's:

6. 253 × 287 72,611 814 × 1728 1,406,592 6400 × 8750 56,000,000

Divide and check by any method:

7. 149,208 ÷ 87 1715R3 756,004 ÷ 938 805R914 1,945,325 ÷ 2636 737R2593

8. Show how a clerk would make change from $5.00 when the article purchased costs: See Guide.
$4.23 $3.90 $2.76 $4.42 $3.15 $1.38

9. Find the average of 6173, 4092, 7384, 5227. 5719

10. Dan sold tickets to the school track meet at $.35 each. He received 2 one-dollar bills, 6 quarters, 11 dimes, 10 nickels, and 15 pennies. How many tickets had Dan sold? 15

Some pupils may need further review as evidenced by errors in ex. 1–18.

Tell the number that should be put in each space:

11. 14 oz. = . 7/8 . lb. 12 min. = . 1/5 . hr. $\frac{1}{2}$ lb. = . . 8 oz.

12. 10 qt. = . 20 pt. 11 gal. = . 44 . qt. $\frac{1}{4}$ bu. = . . 8 qt.

13. 11 ft. = . $3\frac{2}{3}$. yd. 48 sec. = . 4/5 . min. $\frac{3}{4}$ yd. = . 27 . in.

Help pupils analyze causes of errors. Urge pupils to use free time to work on overcoming their difficulties.

A Problem Test

1. Last week the attendance in Miss Allen's class was as follows: 30 pupils on Monday, 25 on Tuesday, 26 on Wednesday, 29 on Thursday, and 24 on Friday. What was the average daily attendance in Miss Allen's class last week? Give the answer to the nearest whole number. 27 pupils

2. Betty and Ann made candy for a fair. They put 24 pieces of candy in each box and had 15 boxes. There were 10 pieces left. How many pieces of candy did the girls make? 370

3. At an average speed of 480 mi. per hour, how far can an airplane travel from 2:07 P.M. until 4:22 P.M.? 1080 mi.

4. Jack wants $6.95 to buy a model airplane set. After working $9\frac{1}{2}$ hr. at $.60 an hour, how much more does he need? $1.25

5. This year the village of Plainfield celebrated the 175th year of its settlement. In what year was the village settled?

6. Balance Tom's account for a week if the amounts he received were $1.50, $.35, $.60, and $.15 and the amounts he spent were $.10, $.45, $.06, $1.00, and $.08. $.91

7. There were 36 pupils who made a trip to the art museum yesterday. The total cost of the trip was $12.60. Find each pupil's share of the cost. $.35

8. Mr. Wood bought 8 gal. of gasoline at 26¢ per gallon. How much change did he receive from a 5-dollar bill? $2.92

9. Last Saturday the attendance at the Yankee ball game was 37,564. There were 6525 reserved seats occupied. How many people did not have reserved seats? 31,039

10. The school store bought a gross of pencils (144 pencils) for $4.50. The pencils were sold at 5¢ each. How much more did the pencils bring than was paid for them? $2.70

SCORE	0–5	6–7	8–9	10
	You need help	Fair	Good	Excellent

Instruct pupils to read problems carefully. Check papers carefully and analyze errors (computational, lack of understanding of problem situation, carelessness). Have pupils keep record of own scores by means of bar graph (see Guide).

Present diagnostic test of skills retaught in Chapter 1, with page references for remedial work. See Guide for discussion of construction of test.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row. Clear up difficulties (reteach or review further) before assigning remedial work.

Add and check:

						Practice Pages
1.	874	787	213	558	245	8
	633	360	327	174	966	
	211	512	557	797	123	
	316	629	285	469	389	
	734	195	755	619	596	
	434	237	291	187	294	
	145	532	865	828	779	
	299	764	148	295	383	
	3646	4016	3441	3927	3775	

Subtract and check:

					Practice Pages
2.	7639	5000	$286.07	$6178.25	14
	5849	1523	37.98	3328.56	
	1790	3477	$ 248.09	$ 2849.69	
3.	8000	9687	$519.24	$9000.00	14
	2716	8037	274.96	2864.35	
	5284	1650	$ 244.28	$ 6135.65	

Multiply and check:

						Practice Pages
4.	706	657	3018	5369	2814	25, 26
	218	496	935	187	607	
	153,908	325,872	2,821,830	1,004,003	1,708,098	
5.	3428	4002	6570	4450	2739	25, 26
	2015	3936	5348	1728	2739	
	6,907,420	15,751,872	35,136,360	7,689,600	7,502,121	

Divide and check:

				Practice Pages
6.	32)18528 579	25)19490 779 R15	82)57431 700 R31	27
7.	46)20010 435	19)15846 834	76)19684 259	29
8.	416)160680 $386\frac{1}{4}$	329)211547 643		32
9.	288)145134 503 R270	264)147312 558		33
10.	2154)7726398 3587	4725)3997350 846		36

Check and analyze pupils' papers to determine kinds of errors. Enter results on progress cards. Help pupils analyze causes of errors.

Present project on train travel. See Guide for specific aims of Chapter 2.

Chapter 2

Traveling by Train

1. Mrs. White lives in Chicago. She wishes to take a railroad trip to Amarillo, Texas, to visit her daughter. At the railroad ticket office she learns that if she travels by coach, the fare one way from Chicago to Amarillo is \$30.84 while the round-trip fare is \$55.55. How much will Mrs. White save by buying a round-trip ticket instead of two one-way tickets? \$6.13

2. Another way to travel by train is to go in a Pullman sleeping car instead of a coach. If Mrs. White travels this way, the railroad fare one way from Chicago to Amarillo is \$39.15, while the round-trip fare is \$70.50. There is also an additional charge of \$13.25 for a lower berth one way in the sleeping car from Chicago to Amarillo. What will the trip cost if Mrs. White buys a round-trip ticket for Pullman travel and has a lower berth each way? \$97.00 How much more will it cost her to travel this way than by coach? \$41.45

3. A roomette one way in a sleeping car from Chicago to Amarillo costs \$18.65. How much more would it cost Mrs. White to have a roomette both ways instead of a lower berth? \$10.80

Have pupils collect fares for different trips from the nearest railroad station. Have them find out what should be done with the return part of a round trip ticket if it is not used.

Reading a Timetable

Miles	Station		Time	
0	Chicago	Lv.	**3:15**	Sun.
234	Shopton	Ar.	**7:00**	"
	Shopton	Lv.	**7:05**	"
449	Kansas City	Ar.	**10:25**	"
	Kansas City	Lv.	**10:45**	"
515	Topeka	Lv.	**11:50**	"
650	Newton	Ar.	2:10	Mon.
	Newton	Lv.	2:15	"
711	Wellington	Ar.	3:50	"
	Wellington	Lv.	3:55	"
1023	Amarillo	Ar.	9:55	"

1. Mrs. White finds that the best train to Amarillo is the one that leaves Chicago at 3:15 P.M., as shown in the timetable at the right. In this table hours in the morning are printed in light type, while hours in the afternoon and evening up to midnight are printed in heavy type. Thus 2:30 means 2:30 A.M., while **7:30** means 7:30 P.M. The abbreviations Sun. and Mon. show how the trip works out if one starts on a Sunday. Any two consecutive days, such as Thur. and Fri., may be substituted for Sun. and Mon. If Mrs. White leaves Chicago at 3:15 P.M. on Wednesday, on what day and at what hour will she arrive in Amarillo? Thursday, 9:55 P. M.

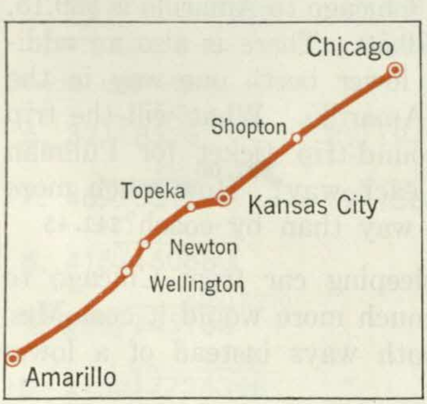
10:25 P. M.

2. At what time does the train arrive at Kansas City? When does it leave Kansas City? How many minutes does the train stop at Kansas City? 20
10:45 P. M.

3. At what time and how long does the train stop at Shopton? at Newton? at Wellington? 3:50 A. M., 5 min.
7:00 P. M., 5 min.
2:10 A. M., 5 min.

11:50 P.M.

4. When is the train due to leave Topeka, Kansas? When is it due to arrive at Amarillo? 9:55 A. M.



5. How many hours and minutes does this train take to make the entire trip from Chicago to Amarillo? 18 hr. 40 min.

6. Try to get one or more railroad timetables and look up trains for trips you would like to take.

Be sure all pupils know how to read timetable shown in ex. 1 and are able to compute various times called for. Try to provide other timetables (bus, plane, train) to give further practice in these skills.

After pupils complete ex. 1-5, use timetables (brought in for page 44) to give further practice in finding time and distances for trips, and average speeds.

How Fast Does the Train Go?

1. Mr. White wanted to know the average speed of the train that his wife plans to take. He looked at the timetable on page 44 to find the total number of miles from Chicago to Amarillo. How many miles is it?(1) Then he found the total time of the trip as you did in ex. 5, page 44. Since he wanted to know only the approximate speed of the train, he called the time 19 hr. How much does this differ from the actual time?(2) Is 19 hr. the time to the nearest hour?(3) Find the average speed of the train to the nearest mile per hour.(4)
(1) 1023 mi.; (2) 20 min.; (3) Yes; (4) 54 mi.

2. After Mr. White found the answer in ex. 1 he noticed that the train makes several stops. How long does the train stop at Kansas City?(1) at Shopton?(2) at Newton?(3) at Wellington?(4) There are other short stops amounting to 20 min. that are not shown on this timetable. Subtracting all of these stops from the total time, what do you get for the actual running time of the train?(5) What is the running time to the nearest hour?(6) Using this time, find the average speed of the train to the nearest mile per hour.(7) (1) 20 min.; (2) 5 min.; (3) 5 min.; (4) 5 min; (5) 17 hr. 45 min; (6) 57 mph

3. How long does it take the train in ex. 1 to go from Chicago to Kansas City?(1) If you use the time to the nearest hour, what is the train's average speed for this part of the trip?(2)
(1) 7 hr. 10 min; (2) 64 mph

4. The timetable gives the distance from Chicago to Kansas City as 449 mi. and the distance from Chicago to Amarillo as 1023 mi. How far is it from Kansas City to Amarillo? 574 mi. Find the approximate speed of the train in ex. 1 for this part of the trip. 52 mph

5. Mr. White noticed in the timetable that a train to Houston, Texas leaves Chicago at 6:00 P.M. as shown at the right. If you take this train on Tuesday, on what day and at what hour will you arrive at Houston? Wed., 7:00 P.M. How many hours will the trip take? 25 hr. Find the average speed of the train on this trip. 55 mph.

Miles	Station		Time	
0	Chicago	Lv.	**6:00**	Sun.
449	Kansas City	Ar.	1:30	Mon.
988	Gainesville	Ar.	11:52	"
1052	Fort Worth	Ar.	**12:55**	"
1368	Houston	Ar.	**7:00**	"

Ex. 1-2 should be discussed with pupils to be sure they understand meaning of average speed, how to compute it, why 19 hr. is used as time, meaning of running time, and so on. Then have them complete ex. 3-5.

Show how to take and score improvement tests (pages 46-50).

Improving Your Skill

1. On page 47 you see 3 addition tests which are called Addition Tests 1a, 1b, and 1c. More addition tests are given on other pages of this book. If you take these tests the **right way**, they will help you greatly to increase your ability to add rapidly and accurately. If you take Test 1a on Monday, do not take Test 1b before Wednesday. Never take more than one test a day; skip a day or two between tests. Directions for taking and scoring these tests are given on page 344. Suppose your score on Test 1a is 8. When you take Test 1b, try to improve this score. If you get 9 on Test 1b, try to get 10 on Test 1c. Always try to equal or raise your previous score. Emphasize that pupil is competing only with himself.

2. After the 3 addition tests are completed on three different days, you will start a set of subtraction tests (see page 57) and later will go on to take a set of multiplication tests (see page 69) and a set of division tests (see page 87). All these tests are called Improvement Tests because they help you to improve your skill in computation. For a complete list of all the Improvement Tests, see the Index of this book.

Refer also to Guide for important discussion of purpose, description, procedure, and records for these tests. The material on this page should be discussed carefully with pupils.

Present first set of improvement tests in addition.

Improving by Practice

Addition Test 1a.

Time: 4 min.

1.	24	94	57	39	54	21	63	86	
	97	36	19	82	95	93	37	45	
	58	87	53	17	47	14	14	11	
	43	60	82	54	15	20	41	82	
	85	75	68	22	61	37	86	58	
	54	24	96	36	23	96	97	90	
	69	88	53	21	78	10	90	69	
	32	39	82	41	65	49	60	16	⑧
	462	503	510	312	438	340	488	457	

Addition Test 1b.

Time: 4 min.

2.	12	90	99	74	52	56	55	90	
	23	61	24	37	31	72	90	45	
	89	70	35	18	90	10	37	77	
	60	86	37	36	16	82	93	63	
	81	35	54	26	77	70	64	80	
	14	73	58	64	89	85	34	53	
	40	44	70	50	60	50	80	25	
	54	78	28	11	14	53	97	24	⑧
	373	537	405	316	429	478	550	457	

Addition Test 1c.

Time: 4 min.

3.	40	20	61	77	98	80	26	50	
	34	57	83	64	86	96	55	47	
	22	21	10	29	72	66	10	42	
	98	14	89	88	14	14	71	74	
	56	79	81	62	73	40	82	40	
	90	98	38	15	30	72	63	24	
	67	23	38	69	67	30	79	19	
	14	77	86	63	90	43	35	31	⑧
	421	389	486	467	530	441	421	327	

To the Teacher. Before giving the Improvement Tests on this page, read pages 48 to 50 and page 344 of this book. If individual pupils get low scores on Improvement Tests in addition and the other processes, such pupils may be assigned extra practice which will be found on pages 325–328.

Note that only one test of a set should be administered on given day. Discuss material on page 46 before giving first test. Group pupils who had errors to help them determine causes (facts, carrying, adding by endings, and so on).

Using Folded Paper

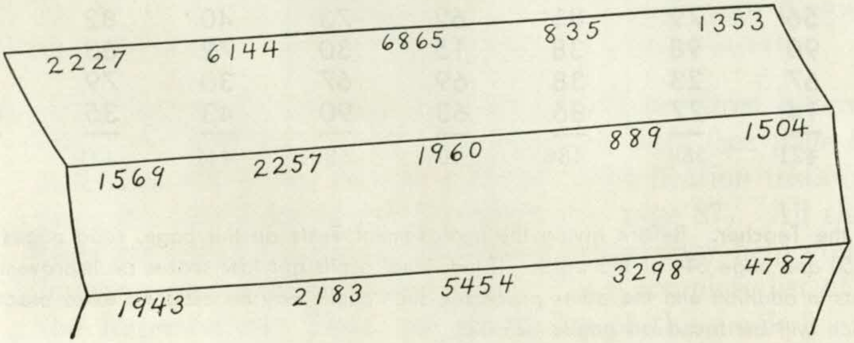
1. When you take an addition test like those on page 47, or a subtraction test like those on page 57, do not copy the examples. Instead, lay a sheet of paper on the book with its edge **under** the top row of examples. Then write the answer to each example on the edge of the paper, as shown at the bottom of this page.

2. When you finish the answers to the first row of examples, fold the answers **under;** then write the answers to the second row along the folded edge, as shown below. Continue in this way for each row of examples. It saves time to fold the paper **before** taking the test. Each fold should be about 1 inch wide.

3. Folded paper cannot be used for tests in multiplication and division like those on pages 69 and 87. In taking such tests you should copy the examples on paper **before** the test begins. In copying, spread the examples over the paper so as to give yourself room enough to work them.

4. Work on each test only the number of minutes given for that test, and stop work **at once at the end of that time.** Stress. You cannot compare your scores on the 3 tests in a set unless you allow exactly the same number of minutes for each one of the tests.

5. If you finish a test before the time to stop, check as many of the examples as you can in the time that is left.



Demonstrate how the paper should be folded and used. As pupils prepare their papers, walk around room and observe pupils to make sure that it is done correctly.

Using a Scoring Table

NUMBER OF EXAMPLES IN TEST	NUMBER OF EXAMPLES RIGHT															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	0	3	5	8	10										B	
5	0	2	4	6	8	10										
6	0	2	3	5	7	8	10									
7	0	1	3	4	6	7	9	10								
8	0	1	3	4	5	6	8	9	10							
9	0	1	2	3	4	6	7	8	9	10						
10	0	1	2	3	4	5	6	7	8	9	10					
12	0	1	2	3	3	4	5	6	7	8	8	9	10		C	
A → 15	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10

1. When a test contains 10 examples, it is easy to score it; if you get 9 examples right, your score is 9. When a test contains 15 examples, like the tests on page 57, your score is 10 if you get 15 examples right; but what is your score if you get 14 examples right, or only 12 right? The scoring table above answers these questions.

2. Suppose you take a test containing 15 examples and you get only 12 examples right. You find your score in the table like this: Since the test had 15 examples, look down the first column on the left side of the table until you come to the row marked 15 (shown by Arrow A). Then look along that row until you reach the column marked 12 at the top (shown by Arrow B) and you will find the number 8 (shown by Arrow C). This means that your score is 8 on that test.

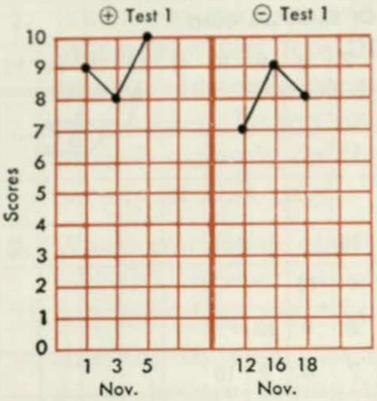
3. In the third row of Subtraction Test 1a on page 57, you see a number in a circle like this: ⑮. This tells you that this test contains 15 examples, so it is not necessary to count to see how many examples the entire test contains.

4. The scoring table shown above is also printed on page 345 of this book.

Discuss material on this page thoroughly with pupils. Then give them further practice, as in ex. 2, in using scoring table. Have pupils find own scores for tests on page 47.

See Guide for suggestions for making Score Book. Have pupils record their scores for tests on page 47.

Recording Your Scores



Record of Joe's Scores

1. The picture at the left shows Joe's scores on some tests that he took. This picture is called a **line graph.**

2. The top of the first graph is marked "⊕ Test 1." This line graph shows Joe's scores for Addition Tests 1a, 1b, and 1c, on page 47. These three tests make up Test 1. ⊕ means "addition." You see three heavy dots on the graph. The first dot is on the horizontal line marked 9; this means that Joe's score on Test 1a was 9. The first dot is also on the vertical line marked 1 at the bottom of the graph; this means that Joe took Test 1a on Nov. 1. The second dot shows Joe's score on Test 1b; his score was 8 and he took the test on Nov. 3. What was his score on Test 1c and on what date did he take that test? 10; Nov. 5 The dots on a line graph are connected by drawing lines between them.

3. The second graph headed "⊖ Test 1" gives Joe's scores on the 3 subtraction tests on page 57. ⊖ means "subtraction." What were his scores on these three tests and on what dates did he take them? 7 on Nov. 12; 9 on Nov. 16, 8 on Nov. 18

4. You should make a **Score Book** in which you can make a line graph of your scores for each set of tests. It will be a record of your scores on all the Improvement Tests that you take this school year. This book can be easily made from a few sheets of squared paper. Always watch your scores to see whether you are improving or not. Emphasize.

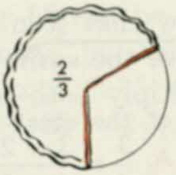
5. When you find your score on a test, count only the examples that are entirely right. An example that is partly finished does not count.

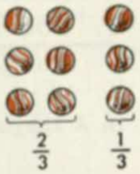
Before discussing ex. 1-5 review functions of graphs and ask pupils if they know of other kinds. As material is explained in ex. 2-3, have pupils actually locate dates and scores on the graphs.

Review the three meanings of fractions and special vocabulary connected with fractions.

Meaning of Fractions

1. This picture shows $\frac{2}{3}$ of a pie. In this case the fraction $\frac{2}{3}$ represents a **part of one thing.** The picture also shows $\frac{2}{3}$ of 9 marbles. Here $\frac{2}{3}$ stands for a **part of a group** of 9 things. You see that the fraction $\frac{2}{3}$ may have either of these two meanings.





2. Draw a picture showing $\frac{3}{4}$ of a cake. Also draw a picture showing $\frac{3}{4}$ of 16 apples.

3. A fraction can also indicate division. The fraction $\frac{2}{3}$ means $2 \div 3$; likewise, $\frac{7}{8}$ means $7 \div 8$ and $\frac{1}{4}$ means $1 \div 4$.

Pount out that numerator tells number of parts being used.

4. In the fraction $\frac{5}{8}$, 5 is the **numerator** of the fraction and 8 is the **denominator.** The numerator and denominator of a fraction are called the **terms** of the fraction.

Point out that denominator names the parts and tells number of equal parts in a whole or grou

5. Fractions like $\frac{2}{3}$ and $\frac{5}{8}$, in which the numerator is smaller than the denominator, are called **proper fractions.** A proper fraction is less than 1 whole unit.

6. Fractions like $\frac{6}{6}$ and $\frac{8}{3}$, in which the numerator is equal to or larger than the denominator, are **improper fractions.** An improper fraction is equal to or larger than 1 whole unit. Draw 2 circles and show that $\frac{5}{3}$ is larger than 1 unit.

7. Draw 2 circles and divide each circle into 4 equal parts. What part of a circle is one part? Show on these circles that $\frac{7}{4}$ is more than 1; show that $\frac{5}{4}$ is more than 1. Also show that $\frac{4}{4}$ equals 1 and that $\frac{3}{4}$ is less than 1.

8. Numbers like $5\frac{2}{3}$ and $4\frac{3}{4}$, which consist of a whole number and a fraction, are called **mixed numbers.**

9. Tell which of the fractions below are proper (P) fractions and which are improper (I) fractions:

$\frac{6}{5}$ I $\frac{3}{10}$ P $\frac{4}{5}$ P $\frac{12}{12}$ I $\frac{11}{3}$ I $\frac{5}{6}$ P $\frac{10}{7}$ I $\frac{8}{8}$ I $\frac{1}{3}$ P

10. Using 5, 3, and 8, or any two of them, write a proper fraction, an improper fraction, and a mixed number.

Use chalkboard diagrams to illustrate meaning of proper and improper fractions and mixed numbers. Have pupils perform all activities suggested in ex. 2, 7, 10. Give further practice similar to that in ex. 7 and 9.

Reteach fundamental principle of fractions. Demonstrate with diagrams that $\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$, $\frac{1}{4} = \frac{2}{8} = \frac{4}{16}$, and so on (also use 6ths and 12ths). Slower pupils should use fractional kits (whole circles or squares and fractional parts).

Changing Fractions

1. You know that $\frac{3}{4}$ in. is equal to $\frac{6}{8}$ in. This means that $\frac{3}{4}$ and $\frac{6}{8}$ have the **same value.** A quick way to change $\frac{3}{4}$ to $\frac{6}{8}$ is to multiply both terms of $\frac{3}{4}$ by 2, as shown at A. Point out that in A the size of the equal parts is made smaller; in B they are made larger.

Emphasize.

$$\text{A. } \frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8} \qquad \text{B. } \frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

You can change $\frac{6}{8}$ back to $\frac{3}{4}$ by dividing both terms of $\frac{6}{8}$ by 2, as shown above at B. When you change $\frac{6}{8}$ to $\frac{3}{4}$, you are changing $\frac{6}{8}$ to **lowest terms.** A fraction can be changed to lowest terms by dividing both terms by the largest whole number that will divide both of them exactly. In changing fractions, the tests of divisibility on page 31 are helpful. Stress.

Demonstrate and emphasize principle of fractions.

If you multiply or divide both terms of a fraction by the same number, you get another fraction having the same value.

Point out and show that only form of fraction is changed.

Change each fraction to lowest terms and tell by what number you divide both terms: (2) Remind pupils that fraction is in lowest terms when there is no number, except 1, by which both terms can be divided evenly.

2. $\frac{9}{12}$ $\frac{3}{4}$, 3 $\frac{14}{28}$ $\frac{1}{2}$, 14 $\frac{20}{25}$ $\frac{4}{5}$, 5 $\frac{9}{27}$ $\frac{1}{3}$, 9 $\frac{12}{32}$ $\frac{3}{8}$, 4 $\frac{20}{24}$ $\frac{5}{6}$, 4 $\frac{10}{30}$ $\frac{1}{3}$, 10 $\frac{15}{60}$ $\frac{1}{4}$, 15 $\frac{75}{100}$ $\frac{3}{4}$, 25

3. $\frac{15}{20}$ $\frac{3}{4}$, 5 $\frac{12}{18}$ $\frac{2}{3}$, 6 $\frac{18}{20}$ $\frac{9}{10}$, 2 $\frac{18}{36}$ $\frac{1}{2}$, 18 $\frac{16}{48}$ $\frac{1}{3}$, 16 $\frac{64}{72}$ $\frac{8}{9}$, 8 $\frac{30}{80}$ $\frac{3}{8}$, 10 $\frac{5}{70}$ $\frac{1}{14}$, 5 $\frac{50}{100}$ $\frac{1}{2}$, 50

4. $\frac{9}{15}$ $\frac{3}{5}$, 3 $\frac{24}{36}$ $\frac{2}{3}$, 12 $\frac{45}{54}$ $\frac{5}{6}$, 9 $\frac{15}{45}$ $\frac{1}{3}$, 15 $\frac{16}{96}$ $\frac{1}{6}$, 16 $\frac{20}{60}$ $\frac{1}{3}$, 20 $\frac{40}{50}$ $\frac{4}{5}$, 10 $\frac{12}{20}$ $\frac{3}{5}$, 4 $\frac{36}{100}$ $\frac{9}{25}$, 4

5. $\frac{12}{42}$ $\frac{2}{7}$, 6 $\frac{6}{24}$ $\frac{1}{4}$, 6 $\frac{14}{35}$ $\frac{2}{5}$, 7 $\frac{20}{32}$ $\frac{5}{8}$, 4 $\frac{14}{21}$ $\frac{2}{3}$, 7 $\frac{12}{48}$ $\frac{1}{4}$, 12 $\frac{18}{90}$ $\frac{1}{5}$, 18 $\frac{35}{40}$ $\frac{7}{8}$, 5 $\frac{35}{100}$ $\frac{7}{20}$, 5

Tell what number should be put in the numerator of the second fraction so that both fractions will be equal:

6. $\frac{4}{5} = \frac{?}{10}$ 8 $\frac{5}{8} = \frac{?}{24}$ 15 $\frac{11}{12} = \frac{?}{36}$ 33 $\frac{3}{10} = \frac{?}{60}$ 18 $\frac{4}{25} = \frac{?}{100}$ 16

7. $\frac{3}{8} = \frac{?}{16}$ 6 $\frac{7}{10} = \frac{?}{40}$ 28 $\frac{5}{16} = \frac{?}{48}$ 15 $\frac{11}{20} = \frac{?}{80}$ 44 $\frac{19}{20} = \frac{?}{100}$ 95

8. $\frac{2}{3} = \frac{?}{12}$ 8 $\frac{7}{8} = \frac{?}{24}$ 21 $\frac{7}{10} = \frac{?}{70}$ 49 $\frac{4}{15} = \frac{?}{45}$ 12 $\frac{9}{10} = \frac{?}{100}$ 90

9. It is 660 ft. around a circular running track. What part of a mile is it around the track? 5280 ft. = 1 mi. $\frac{660}{5280} = \frac{1}{8}$

10. It takes Mary 20 min. to walk to school. What part of an hour is this? $\frac{20}{60} = \frac{1}{3}$

Superior pupils should understand why principle of fractions is valid (since you are really multiplying or dividing by 1, that is, $\frac{3}{3} = 1$, $\frac{2}{2} = 1$, and so on). Make class chart of unlike, equivalent fractions.

Reteach changing improper fraction to mixed number and mixed number to improper fraction. Use diagrams to illustrate ex. 1, and other examples if needed. Do all work orally.

Mixed Numbers

1. An improper fraction like $\frac{6}{3}$, $\frac{12}{5}$, or $\frac{16}{12}$ can be changed to a whole number, or to a mixed number, by dividing the numerator by the denominator. For example, $\frac{6}{3} = 6 \div 3 = 2$; $\frac{12}{5} = 12 \div 5 = 2\frac{2}{5}$; and $\frac{16}{12} = 16 \div 12 = 1\frac{4}{12}$, or $1\frac{1}{3}$.

Change each fraction to a whole number or to a mixed number. Change the fraction in the mixed number to lowest terms: Do orally, and have some pupils prove answers with diagrams or subtraction.

2. $\frac{10}{3}$ $3\frac{1}{3}$ $\frac{25}{8}$ $3\frac{1}{8}$ $\frac{6}{4}$ $2\frac{1}{2}$ $\frac{12}{6}$ 2 $\frac{18}{12}$ $1\frac{1}{2}$ $\frac{30}{5}$ 6 $\frac{25}{20}$ $1\frac{1}{4}$ $\frac{11}{2}$ $5\frac{1}{2}$ $\frac{21}{9}$ $2\frac{1}{3}$

3. $\frac{15}{10}$ $1\frac{1}{2}$ $\frac{30}{12}$ $2\frac{1}{2}$ $\frac{9}{8}$ $1\frac{1}{8}$ $\frac{13}{4}$ $3\frac{1}{4}$ $\frac{15}{8}$ $1\frac{7}{8}$ $\frac{14}{14}$ 1 $\frac{20}{5}$ 4 $\frac{19}{10}$ $1\frac{9}{10}$ $\frac{35}{20}$ $1\frac{3}{4}$

4. You know that $\frac{9}{1} = 9 \div 1 = 9$. This shows that a whole number can be written as a fraction by using the denominator 1. Likewise, 6 can be written $\frac{6}{1}$. Emphasize.

5. Mrs. Case has $3\frac{1}{2}$ grapefruit. To how many people can she serve $\frac{1}{2}$ grapefruit each? 7

▶ You must change $3\frac{1}{2}$ grapefruit to halves of a grapefruit. There are 2 halves in 1 whole grapefruit, so in 3 grapefruit there are 6 halves. This shows that $3 = \frac{6}{2}$. Then $3\frac{1}{2} = \frac{6}{2} + \frac{1}{2}$, or $\frac{7}{2}$. An easy way to change $3\frac{1}{2}$ to $\frac{7}{2}$ is to multiply 3 by 2 and add 1, which gives 7. Then write 7 over 2, which gives $\frac{7}{2}$.

Demonstrate and emphasize. →

To change a mixed number to an improper fraction, multiply the whole number by the denominator, add the numerator, and place the result over the denominator.

Pupils should understand why whole number is multiplied by denominator

Change these mixed numbers to improper fractions:

(since denominator tells number of parts in 1 whole).

6. $6\frac{1}{2}$ $\frac{13}{2}$ $2\frac{3}{4}$ $\frac{11}{4}$ $5\frac{1}{8}$ $\frac{41}{8}$ $4\frac{2}{3}$ $\frac{14}{3}$ $6\frac{1}{5}$ $\frac{31}{5}$ $3\frac{7}{8}$ $\frac{31}{8}$ $6\frac{3}{8}$ $\frac{51}{8}$ $1\frac{3}{10}$ $\frac{13}{10}$

7. $2\frac{5}{8}$ $\frac{21}{8}$ $4\frac{1}{6}$ $\frac{25}{6}$ $8\frac{2}{3}$ $\frac{26}{3}$ $7\frac{1}{4}$ $\frac{29}{4}$ $3\frac{1}{7}$ $\frac{22}{7}$ $5\frac{1}{2}$ $\frac{11}{2}$ $9\frac{2}{5}$ $\frac{47}{5}$ $2\frac{7}{12}$ $\frac{31}{12}$

8. $3\frac{1}{3}$ $\frac{10}{3}$ $5\frac{4}{5}$ $\frac{29}{5}$ $3\frac{5}{6}$ $\frac{23}{6}$ $9\frac{1}{2}$ $\frac{19}{2}$ $8\frac{3}{4}$ $\frac{35}{4}$ $2\frac{3}{8}$ $\frac{19}{8}$ $8\frac{1}{2}$ $\frac{17}{2}$ $3\frac{11}{16}$ $\frac{59}{16}$

9. $5\frac{2}{7}$ $\frac{37}{7}$ $6\frac{1}{3}$ $\frac{19}{3}$ $1\frac{7}{8}$ $\frac{15}{8}$ $2\frac{5}{6}$ $\frac{17}{6}$ $8\frac{1}{3}$ $\frac{25}{3}$ $3\frac{3}{5}$ $\frac{18}{5}$ $4\frac{3}{4}$ $\frac{19}{4}$ $6\frac{1}{12}$ $\frac{73}{12}$

Brighter pupils should be encouraged to prove ex. 1 by subtraction. In ex. 5 use addition first to show there are 6 halves ($\frac{2}{2} + \frac{2}{2} + \frac{2}{2}$) in 3 wholes. Lead pupils to suggest multiplication ($3 \times \frac{2}{2} = \frac{6}{2}$, $\frac{6}{2} + \frac{1}{2} = \frac{7}{2}$). Then present short cut.

Reteach addition of fractions with like denominators. First demonstrate at chalkboard addition of 4 apples and 7 apples, 2 eighths and 3 eighths, then $\frac{2}{5}$ and $\frac{1}{5}$.

Adding Fractions

1. **Problem** Mrs. Smith bought $4\frac{1}{8}$ yd. of cloth for a coat, $2\frac{3}{8}$ yd. of the same cloth for a skirt, and $\frac{5}{8}$ yd. of it for a scarf and hat. How many yards did she buy? $7\frac{1}{8}$

Explanation Since the fractions all have the same denominator, you add the fractions by adding the numerators. The sum is $\frac{9}{8}$. Change $\frac{9}{8}$ to $1\frac{1}{8}$; then add $1\frac{1}{8}$ to 6, which gives $7\frac{1}{8}$. Mrs. Smith bought $7\frac{1}{8}$ yd. of cloth.

$$\begin{array}{r} 4\frac{1}{8} \\ 2\frac{3}{8} \\ \frac{5}{8} \\ \hline 6\frac{9}{8} = 7\frac{1}{8} \end{array}$$

2. Study these addition examples: Have pupils explain them.

$$\begin{array}{r} \frac{2}{3} \\ \frac{2}{3} \\ \hline \frac{4}{3} = 1\frac{1}{3} \end{array} \qquad \begin{array}{r} 6\frac{1}{8} \\ 4\frac{3}{8} \\ \hline 10\frac{4}{8} = 10\frac{1}{2} \end{array} \qquad \begin{array}{r} 7\frac{1}{6} \\ 1\frac{5}{6} \\ \hline 8\frac{6}{6} = 9 \end{array} \qquad \begin{array}{r} 3\frac{7}{12} \\ 1\frac{7}{12} \\ \hline 4\frac{14}{12} = 5\frac{2}{12} = 5\frac{1}{6} \end{array}$$

While you work with slower pupils, have brighter ones use numbers in ex. 3-5 in original problems about distance, time, weight.

Add. Check by going over your work:

3.

$$\begin{array}{r} 8\frac{1}{6} \\ 8\frac{5}{6} \\ \hline 17 \end{array} \qquad \begin{array}{r} 4\frac{7}{8} \\ 3\frac{5}{8} \\ \hline 8\frac{1}{2} \end{array} \qquad \begin{array}{r} 4\frac{3}{8} \\ 5\frac{5}{8} \\ \hline 10 \end{array} \qquad \begin{array}{r} 2\frac{5}{6} \\ 7\frac{5}{6} \\ \hline 10\frac{2}{3} \end{array} \qquad \begin{array}{r} 8\frac{3}{4} \\ 2\frac{3}{4} \\ \hline 11\frac{1}{2} \end{array} \qquad \begin{array}{r} 1\frac{1}{10} \\ 4\frac{7}{10} \\ \hline 5\frac{4}{5} \end{array} \qquad \begin{array}{r} 4\frac{9}{16} \\ 7\frac{9}{16} \\ \hline 12\frac{1}{8} \end{array}$$

4.

$$\begin{array}{r} 4\frac{1}{3} \\ 2\frac{1}{3} \\ \hline 6\frac{2}{3} \end{array} \qquad \begin{array}{r} 1\frac{5}{6} \\ 1\frac{5}{6} \\ \hline 3\frac{2}{3} \end{array} \qquad \begin{array}{r} 4\frac{1}{2} \\ 4\frac{1}{2} \\ \hline 9 \end{array} \qquad \begin{array}{r} 9\frac{5}{8} \\ 3\frac{5}{8} \\ \hline 13\frac{1}{4} \end{array} \qquad \begin{array}{r} 6\frac{1}{6} \\ 1\frac{1}{6} \\ \hline 7\frac{1}{3} \end{array} \qquad \begin{array}{r} 4\frac{7}{12} \\ 6\frac{5}{12} \\ \hline 11 \end{array} \qquad \begin{array}{r} 1\frac{7}{10} \\ 8\frac{9}{10} \\ \hline 10\frac{3}{5} \end{array}$$

5.

$$\begin{array}{r} 2\frac{3}{4} \\ 5\frac{3}{4} \\ 1\frac{3}{4} \\ \hline 10\frac{1}{4} \end{array} \qquad \begin{array}{r} 8\frac{7}{8} \\ 1\frac{3}{8} \\ 2\frac{3}{8} \\ \hline 12\frac{5}{8} \end{array} \qquad \begin{array}{r} 3\frac{5}{6} \\ 6\frac{5}{6} \\ 1\frac{5}{6} \\ \hline 12\frac{1}{2} \end{array} \qquad \begin{array}{r} 4\frac{1}{4} \\ 5\frac{3}{4} \\ 7\frac{1}{4} \\ \hline 17\frac{1}{4} \end{array} \qquad \begin{array}{r} 4\frac{2}{3} \\ 4\frac{2}{3} \\ 1\frac{2}{3} \\ \hline 11 \end{array} \qquad \begin{array}{r} 2\frac{3}{16} \\ 2\frac{7}{16} \\ 2\frac{9}{16} \\ \hline 7\frac{3}{16} \end{array} \qquad \begin{array}{r} 5\frac{11}{12} \\ 2\frac{7}{12} \\ 6\frac{1}{12} \\ \hline 14\frac{7}{12} \end{array}$$

More Practice. See ⑪ on page 328. Use for further reteaching.

Bring out fact (through above demonstration) that we can add numerators of fractions that have same name (denominator). Slower students may need to use materials first to visualize process. Group them for this work.

Reteach addition of unlike fractions. Show first that 2 quarters and 3 dimes can be added only by changing both to cents. Similarly, fractions with unlike denominators must be changed to ones having same denominator.

Adding Fractions

1. **Problem** Last Saturday some boys walked $1\frac{1}{2}$ mi. to Green Lake, $2\frac{3}{4}$ mi. around the lake, and $2\frac{1}{8}$ mi. to get home. How many miles in all did the boys walk? $6\frac{3}{8}$

Explanation Before you can add these fractions you must change them to fractions with like denominators. You can change them all to **eighths** as shown at the right. The sum of the fractions is $\frac{11}{8}$. Change $\frac{11}{8}$ to $1\frac{3}{8}$; then add $1\frac{3}{8}$ to 5, which gives $6\frac{3}{8}$. The boys walked $6\frac{3}{8}$ mi.

$$\begin{array}{r}
1\frac{1}{2} = 1\frac{4}{8} \\
2\frac{3}{4} = 2\frac{6}{8} \\
\underline{2\frac{1}{8}} = \underline{2\frac{1}{8}} \\
5\frac{11}{8} = 6\frac{3}{8}
\end{array}$$

Emphasize. →

To add fractions with unlike denominators, first change them to fractions with like denominators.

2. Study these addition examples: Let different pupils explain them.

$$\begin{array}{r}
4\frac{3}{16} = 4\frac{3}{16} \\
\underline{5\frac{3}{8}} = \underline{5\frac{6}{16}} \\
9\frac{9}{16}
\end{array}
\qquad
\begin{array}{r}
3\frac{1}{6} = 3\frac{1}{6} \\
\underline{2\frac{1}{3}} = \underline{2\frac{2}{6}} \\
5\frac{3}{6} = 5\frac{1}{2}
\end{array}
\qquad
\begin{array}{r}
1\frac{3}{4} = 1\frac{9}{12} \\
\underline{1\frac{7}{12}} = \underline{1\frac{7}{12}} \\
2\frac{16}{12} = 3\frac{4}{12} = 3\frac{1}{3}
\end{array}$$

Add. Check by going over your work: Have volunteers explain solutions to ex. 3–5 at chalkboard.

3.	$2\frac{3}{4}$	$3\frac{2}{3}$	$5\frac{1}{2}$	$7\frac{2}{3}$	$6\frac{3}{10}$	$3\frac{1}{6}$	$3\frac{3}{8}$
	$2\frac{3}{8}$	$6\frac{1}{6}$	$\frac{2}{5}$	$3\frac{5}{6}$	$4\frac{1}{5}$	$5\frac{7}{12}$	$3\frac{1}{16}$
	$5\frac{1}{8}$	$9\frac{5}{6}$	$5\frac{9}{10}$	$11\frac{1}{2}$	$10\frac{1}{2}$	$8\frac{3}{4}$	$6\frac{7}{16}$
4.	$6\frac{3}{4}$	$7\frac{1}{2}$	$2\frac{5}{6}$	$\frac{5}{6}$	$9\frac{7}{10}$	$2\frac{11}{12}$	$5\frac{1}{4}$
	$3\frac{5}{8}$	$2\frac{1}{3}$	$3\frac{1}{2}$	$5\frac{2}{3}$	$5\frac{1}{2}$	$1\frac{1}{3}$	$9\frac{1}{12}$
	$10\frac{3}{8}$	$9\frac{5}{6}$	$6\frac{1}{3}$	$6\frac{1}{2}$	$15\frac{1}{5}$	$4\frac{1}{4}$	$14\frac{1}{3}$
5.	$7\frac{1}{8}$	$4\frac{5}{6}$	$2\frac{1}{2}$	$4\frac{5}{8}$	$1\frac{7}{12}$	$8\frac{9}{10}$	$5\frac{1}{16}$
	$5\frac{1}{2}$	$2\frac{2}{3}$	$6\frac{3}{4}$	$2\frac{3}{4}$	$1\frac{2}{3}$	$2\frac{7}{10}$	$2\frac{1}{8}$
	$3\frac{1}{4}$	$2\frac{1}{2}$	$4\frac{3}{4}$	$5\frac{1}{8}$	$1\frac{5}{6}$	$4\frac{3}{5}$	$1\frac{1}{16}$
	$15\frac{7}{8}$	10	14	$12\frac{1}{2}$	$5\frac{1}{12}$	$16\frac{1}{5}$	$8\frac{1}{4}$

More Practice. See ⑫ on page 329. Use to reinforce skill.

After prelesson demonstration, discuss ex. 1–2 with pupils. Brighter students should have no difficulties with ex. 3–5, but slower ones may need diagrams. Work with them to assure understanding. Have the others work on original problems.

Reteach subtraction of mixed numbers where fraction in minuend is larger than that in subtrahend. Use materials and/or diagrams to demonstrate a few more examples first.

Subtracting Fractions

1. Problem Joe and Bob started to walk $5\frac{1}{2}$ mi. to Swan Lake. After they had gone $1\frac{1}{4}$ mi. Mr. Wells came along in his car and asked the boys to ride with him to Swan Lake. How many miles did Mr. Wells take them? $4\frac{1}{4}$

Explanation You cannot subtract **fourths** from **halves** since the denominators are not alike. Change $\frac{1}{2}$ to $\frac{2}{4}$ and subtract $\frac{1}{4}$ from $\frac{2}{4}$. Also subtract 1 from 5. Mr. Wells took the boys $4\frac{1}{4}$ mi. Check the work.

$$\begin{array}{r} 5\frac{1}{2} = 5\frac{2}{4} \\ \underline{1\frac{1}{4} = 1\frac{1}{4}} \\ 4\frac{1}{4} \end{array}$$

2. From a piece of ribbon $3\frac{7}{8}$ yd. long Betty cut off a piece $1\frac{1}{2}$ yd. long. How many yards were left in the other piece? $2\frac{3}{8}$

Subtract. Check the work by going over it: Have pupils show work as in ex. 1.

3.	$5\frac{3}{8}$	$6\frac{5}{6}$	$7\frac{3}{4}$	$9\frac{4}{5}$	$6\frac{3}{4}$	$3\frac{5}{6}$	$7\frac{7}{8}$
	$1\frac{1}{4}$	$3\frac{1}{6}$	$6\frac{1}{4}$	$6\frac{3}{10}$	5	$2\frac{1}{3}$	$4\frac{1}{16}$
	$4\frac{1}{8}$	$3\frac{2}{3}$	$1\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{1}{2}$	$13\frac{13}{16}$
4.	$1\frac{2}{3}$	$4\frac{1}{3}$	$5\frac{3}{4}$	$8\frac{9}{10}$	$5\frac{2}{3}$	$6\frac{1}{2}$	$7\frac{9}{16}$
	$1\frac{1}{2}$	$2\frac{1}{6}$	$2\frac{1}{4}$	$6\frac{2}{5}$	$4\frac{1}{4}$	$1\frac{1}{6}$	$5\frac{1}{8}$
	$\frac{1}{6}$	$2\frac{1}{6}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{5}{12}$	$5\frac{1}{3}$	$2\frac{7}{16}$
5.	$3\frac{1}{2}$	$4\frac{2}{3}$	$9\frac{3}{4}$	$7\frac{7}{12}$	$6\frac{3}{4}$	$5\frac{7}{8}$	$8\frac{11}{12}$
	$1\frac{1}{8}$	$1\frac{1}{6}$	$2\frac{1}{8}$	$2\frac{1}{3}$	$4\frac{5}{8}$	$3\frac{3}{8}$	$1\frac{1}{12}$
	$2\frac{3}{8}$	$3\frac{1}{2}$	$7\frac{5}{8}$	$5\frac{1}{4}$	$2\frac{1}{8}$	$2\frac{1}{2}$	$7\frac{5}{6}$
6.	$2\frac{3}{4}$	$3\frac{7}{8}$	$9\frac{1}{2}$	$6\frac{9}{10}$	$7\frac{3}{8}$	$1\frac{3}{4}$	$4\frac{7}{10}$
	$1\frac{1}{5}$	$\frac{3}{8}$	7	$2\frac{1}{2}$	$1\frac{3}{8}$	$\frac{1}{2}$	$3\frac{3}{5}$
	$1\frac{11}{20}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{2}{5}$	6	$1\frac{1}{4}$	$1\frac{1}{10}$
7.	$7\frac{1}{2}$	$8\frac{2}{3}$	$4\frac{5}{6}$	$9\frac{9}{10}$	$9\frac{2}{3}$	$7\frac{2}{3}$	$8\frac{11}{12}$
	$3\frac{1}{5}$	$2\frac{1}{4}$	$\frac{1}{2}$	$3\frac{1}{2}$	$6\frac{2}{5}$	$\frac{1}{2}$	$3\frac{5}{6}$
	$4\frac{3}{10}$	$6\frac{5}{12}$	$4\frac{1}{3}$	$6\frac{2}{5}$	$3\frac{4}{15}$	$7\frac{1}{6}$	$5\frac{1}{12}$
8.	$3\frac{3}{4}$	$8\frac{2}{5}$	$9\frac{3}{4}$	$4\frac{7}{12}$	$8\frac{2}{3}$	$5\frac{1}{4}$	$9\frac{7}{12}$
	$2\frac{3}{8}$	$4\frac{2}{5}$	$8\frac{1}{2}$	$2\frac{7}{12}$	7	$5\frac{1}{4}$	$5\frac{1}{4}$
	$1\frac{3}{8}$	4	$1\frac{1}{4}$	2	$1\frac{2}{3}$	0	$4\frac{1}{3}$
9.	$9\frac{1}{2}$	$8\frac{1}{2}$	$5\frac{5}{6}$	$8\frac{5}{12}$	$9\frac{7}{8}$	$3\frac{7}{8}$	$6\frac{7}{16}$
	$4\frac{1}{3}$	$8\frac{1}{2}$	$3\frac{2}{3}$	5	$1\frac{3}{4}$	$2\frac{3}{4}$	$4\frac{1}{16}$
	$5\frac{1}{6}$	0	$2\frac{1}{6}$	$3\frac{5}{12}$	$8\frac{1}{8}$	$1\frac{1}{8}$	$2\frac{3}{8}$

More Practice. See ⑬ on page 329.

In ex. 1 explanation bring out fact that fraction can always be changed to higher terms ($\frac{1}{2} = \frac{2}{4}$) but not always to lower terms. Emphasize that in order to subtract fractions denominators must be the same.

Present first set of improvement tests in subtraction.

Improving by Practice

Subtraction Test 1a.

Time: $3\frac{1}{2}$ min.

1.	6283	3716	9900	7325	9497	
	5799	2223	8592	2532	4498	
	484	1493	1308	4793	4999	
2.	8640	3925	7171	7529	9230	
	2999	2346	6294	4963	5993	
	5641	1579	877	2566	3237	
3.	4869	6316	9857	7201	8755	
	2290	4397	7647	346	7413	⑮
	2579	1919	2210	6855	1342	

Subtraction Test 1b.

Time: $3\frac{1}{2}$ min.

4.	6590	9710	3246	7162	8365	
	3692	7753	2560	5497	3478	
	2898	1957	686	1665	4887	
5.	5896	6842	2799	8272	9035	
	5011	4379	2220	8016	8479	
	885	2463	579	256	556	
6.	6204	2684	5500	5014	5103	
	4240	1159	1501	1667	2870	⑮
	1964	1525	3999	3347	2233	

Subtraction Test 1c.

Time: $3\frac{1}{2}$ min.

7.	9387	9812	7371	9148	4637	
	3111	4192	2601	4799	4089	
	6276	5620	4770	4349	548	
8.	4056	9118	8213	6934	5567	
	3877	679	2587	1049	88	
	179	8439	5626	5885	5479	
9.	4124	4140	3744	4101	3967	
	377	1248	1996	2470	3705	⑮
	3747	2892	1748	1631	262	

To the Teacher. The Improvement Tests above are to be given to the pupils on three different days. See page 344 for the procedure to follow.

See pages 46–50 and Guide for procedure and suggestions. Through conferences with pupils, try to determine causes of errors. Plan remedial work after difficulties are cleared up. Remind pupils to record scores (page 50).

Reteach subtraction of mixed numbers where fraction in subtrahend is larger than that in minuend.

Subtracting Fractions

1. **Problem** Bill started to ride his bicycle to New City, which is $8\frac{1}{4}$ mi. from his house. After he had been riding for some time he saw a sign marked, "New City $2\frac{1}{2}$ mi." How far had Bill gone? $5\frac{3}{4}$ mi.

Explanation You must change $2\frac{1}{2}$ to $2\frac{2}{4}$ so that both fractions will have the same denominator. But you cannot subtract $\frac{2}{4}$ from $\frac{1}{4}$, so you must also change $8\frac{1}{4}$ to $7\frac{5}{4}$. Now subtract $2\frac{2}{4}$ from $7\frac{5}{4}$. Bill had gone $5\frac{3}{4}$ mi.

$$\begin{array}{r} 8\frac{1}{4} = 7\frac{5}{4} \\ \underline{2\frac{1}{2} = 2\frac{2}{4}} \\ 5\frac{3}{4} \end{array}$$

2. Study these subtraction examples: Discuss these with pupils.

$$\begin{array}{r} 1 = \frac{3}{3} \\ \underline{\frac{2}{3} = \frac{2}{3}} \\ \frac{1}{3} \end{array} \qquad \begin{array}{r} 9 = 8\frac{4}{4} \\ \underline{4\frac{3}{4} = 4\frac{3}{4}} \\ 4\frac{1}{4} \end{array} \qquad \begin{array}{r} 7\frac{1}{8} = 6\frac{9}{8} \\ \underline{4\frac{3}{8} = 4\frac{3}{8}} \\ 2\frac{6}{8} = 2\frac{3}{4} \end{array} \qquad \begin{array}{r} 9\frac{5}{16} = 8\frac{21}{16} \\ \underline{7\frac{7}{8} = 7\frac{14}{16}} \\ 1\frac{7}{16} \end{array}$$

Emphasize reason for exchange of whole for particular unit: if halves are to be subtracted, whole must be changed to $\frac{2}{2}$, and so on.

Subtract. Check by going over your work:

3.	$6\frac{1}{2}$	9	1	$8\frac{1}{12}$	$4\frac{1}{16}$	$6\frac{3}{10}$	$3\frac{1}{12}$
	$2\frac{3}{4}$	$2\frac{1}{2}$	$\frac{7}{10}$	$2\frac{5}{6}$	$3\frac{7}{16}$	$4\frac{9}{10}$	$2\frac{5}{6}$
	$3\frac{3}{4}$	$6\frac{1}{2}$	$\frac{3}{10}$	$5\frac{1}{4}$	$\frac{5}{8}$	$1\frac{2}{5}$	$\frac{1}{4}$
4.	$7\frac{1}{8}$	$9\frac{1}{8}$	8	$6\frac{1}{16}$	$5\frac{1}{12}$	$4\frac{5}{12}$	$7\frac{3}{16}$
	$6\frac{3}{4}$	$8\frac{5}{8}$	$4\frac{3}{16}$	$2\frac{3}{4}$	$4\frac{7}{12}$	$1\frac{2}{3}$	$1\frac{5}{8}$
	$\frac{3}{8}$	$\frac{1}{2}$	$3\frac{13}{16}$	$3\frac{5}{16}$	$\frac{1}{2}$	$2\frac{3}{4}$	$5\frac{9}{16}$
5.	5	$5\frac{1}{6}$	$5\frac{1}{12}$	$8\frac{5}{12}$	$9\frac{3}{10}$	$8\frac{3}{16}$	$8\frac{3}{10}$
	$1\frac{1}{8}$	$3\frac{1}{2}$	$2\frac{1}{4}$	$5\frac{11}{12}$	$3\frac{1}{2}$	$4\frac{3}{8}$	$6\frac{3}{5}$
	$3\frac{7}{8}$	$1\frac{2}{3}$	$2\frac{5}{6}$	$2\frac{1}{2}$	$5\frac{4}{5}$	$3\frac{13}{16}$	$1\frac{7}{10}$
6.	$7\frac{5}{8}$	6	$9\frac{3}{16}$	$5\frac{1}{10}$	$6\frac{1}{12}$	$4\frac{1}{10}$	$6\frac{5}{12}$
	$5\frac{3}{4}$	$3\frac{2}{3}$	$8\frac{9}{16}$	$2\frac{7}{10}$	$2\frac{1}{3}$	$3\frac{3}{5}$	$4\frac{1}{2}$
	$1\frac{7}{8}$	$2\frac{1}{3}$	$\frac{5}{8}$	$2\frac{2}{5}$	$3\frac{3}{4}$	$\frac{1}{2}$	$1\frac{11}{12}$

More Practice. See ⑭ on page 330.

Use chalkboard diagrams to give visual demonstration first ($3\frac{1}{4} = 2\frac{5}{4}$, $4\frac{1}{4} = 3\frac{5}{4}$). Give pupils further practice on these changes ($2\frac{5}{8} = 1\frac{?}{8}$, $5\frac{3}{16} = 4\frac{?}{16}$, and so on). Then follow development as given. Have pupils explain answers to ex. 3-6.

Reteach most difficult case of subtraction of mixed numbers where fraction in subtrahend is larger than that in minuend.

More Subtraction

Tell what numbers go in the numerators:

1. $6\frac{1}{2} = 6\frac{?}{4} \overset{2}{=} 5\frac{?}{4}{}^{6}$ $2\frac{3}{4} = 2\frac{?}{12} \overset{9}{=} 1\frac{?}{12}{}^{21}$ $3\frac{1}{3} = 3\frac{?}{12} \overset{4}{=} 2\frac{?}{12}{}^{16}$

2. $8\frac{2}{3} = 8\frac{?}{6} \overset{4}{=} 7\frac{?}{6}{}^{10}$ $9\frac{5}{6} = 9\frac{?}{12} \overset{10}{=} 8\frac{?}{12}{}^{22}$ $6\frac{3}{4} = 6\frac{?}{16} \overset{12}{=} 5\frac{?}{16}{}^{28}$

3. $4\frac{1}{4} = 4\frac{?}{8} \overset{2}{=} 3\frac{?}{8}{}^{10}$ $7\frac{1}{2} = 7\frac{?}{10} \overset{5}{=} 6\frac{?}{10}{}^{15}$ $4\frac{3}{4} = 4\frac{?}{12} \overset{9}{=} 3\frac{?}{12}{}^{21}$

4. $9\frac{1}{2} = 9\frac{?}{6} \overset{3}{=} 8\frac{?}{6}{}^{9}$ $4\frac{5}{8} = 4\frac{?}{16} \overset{10}{=} 3\frac{?}{16}{}^{26}$ $5\frac{4}{5} = 5\frac{?}{10} \overset{8}{=} 4\frac{?}{10}{}^{18}$

5. **Problem** Ann needs $3\frac{1}{2}$ yd. of cloth for a new dress. She saw one piece of material she liked but it contained only $2\frac{7}{8}$ yd. How much too short was the piece? $\frac{5}{8}$ yd.

Explanation You must change $3\frac{1}{2}$ to $3\frac{4}{8}$ so that both fractions have the same denominator. But you cannot subtract $\frac{7}{8}$ from $\frac{4}{8}$, so you must also change $3\frac{4}{8}$ to $2\frac{12}{8}$. Then subtract $2\frac{7}{8}$ from $2\frac{12}{8}$. The piece of material was $\frac{5}{8}$ yd. too short.

$$\begin{array}{r} 3\frac{1}{2} = 3\frac{4}{8} = 2\frac{12}{8} \\ \underline{2\frac{7}{8} = 2\frac{7}{8} = 2\frac{7}{8}} \\ \frac{5}{8} \end{array}$$

Have pupils do all work in ex. 6-10 as shown in ex. 5.

Subtract. Check your work by going over it:

6.	$2\frac{1}{2}$ $1\frac{7}{8}$ $\frac{5}{8}$	$4\frac{1}{2}$ $1\frac{5}{6}$ $2\frac{2}{3}$	$8\frac{3}{4}$ $5\frac{15}{16}$ $2\frac{13}{16}$	$5\frac{1}{2}$ $2\frac{7}{12}$ $2\frac{11}{12}$	$6\frac{1}{4}$ $1\frac{15}{16}$ $4\frac{5}{16}$	$3\frac{2}{3}$ $2\frac{11}{12}$ $\frac{3}{4}$	$7\frac{3}{4}$ $2\frac{13}{16}$ $4\frac{15}{16}$
7.	$5\frac{1}{4}$ $3\frac{2}{3}$ $1\frac{7}{12}$	$9\frac{1}{3}$ $4\frac{5}{6}$ $4\frac{1}{2}$	$6\frac{1}{3}$ $1\frac{7}{12}$ $4\frac{3}{4}$	$8\frac{3}{8}$ $7\frac{15}{16}$ $\frac{7}{16}$	$4\frac{3}{5}$ $1\frac{7}{10}$ $2\frac{9}{10}$	$8\frac{1}{4}$ $6\frac{11}{16}$ $1\frac{9}{16}$	$5\frac{3}{5}$ $3\frac{9}{10}$ $1\frac{7}{10}$
8.	$9\frac{2}{3}$ $6\frac{5}{6}$ $2\frac{5}{6}$	$8\frac{1}{2}$ $5\frac{5}{8}$ $2\frac{7}{8}$	$7\frac{2}{5}$ $2\frac{7}{10}$ $4\frac{7}{10}$	$4\frac{1}{6}$ $2\frac{5}{12}$ $1\frac{3}{4}$	$7\frac{1}{8}$ $4\frac{9}{16}$ $2\frac{9}{16}$	$9\frac{1}{2}$ $1\frac{9}{10}$ $7\frac{3}{5}$	$4\frac{1}{2}$ $3\frac{11}{12}$ $\frac{7}{12}$
9.	$6\frac{1}{4}$ $4\frac{7}{8}$ $1\frac{3}{8}$	$4\frac{1}{3}$ $1\frac{1}{2}$ $2\frac{5}{6}$	$5\frac{5}{6}$ $2\frac{11}{12}$ $2\frac{11}{12}$	$9\frac{1}{5}$ $5\frac{9}{10}$ $3\frac{3}{10}$	$6\frac{3}{4}$ $3\frac{11}{12}$ $2\frac{5}{6}$	$6\frac{1}{2}$ $5\frac{11}{16}$ $\frac{13}{16}$	$8\frac{5}{8}$ $1\frac{15}{16}$ $6\frac{11}{16}$
10.	$8\frac{1}{2}$ $4\frac{7}{8}$ $3\frac{5}{8}$	$2\frac{1}{4}$ $1\frac{5}{8}$ $\frac{5}{8}$	$9\frac{1}{4}$ $3\frac{5}{12}$ $5\frac{5}{6}$	$6\frac{1}{2}$ $2\frac{15}{16}$ $3\frac{9}{16}$	$8\frac{1}{2}$ $2\frac{7}{10}$ $5\frac{4}{5}$	$5\frac{1}{2}$ $1\frac{15}{16}$ $3\frac{9}{16}$	$7\frac{1}{3}$ $4\frac{11}{12}$ $2\frac{5}{12}$

More Practice. See ⑮ on page 330.

Do some other simple examples, like those in ex. 1-4, and have pupils verify answers with diagrams. Then follow development as given. Discuss ex. 5 carefully, and do ex. 6 with pupils also. Then assign ex. 7-10. Use "More Practice" for further reteaching.

Give quick oral review of preceding work.

Oral Review

1. Which fraction is larger, $\frac{1}{2}$ or $\frac{1}{3}$? $\frac{1}{2}$ $\frac{3}{4}$ or $\frac{5}{8}$? $\frac{3}{4}$
2. Find the sum of $\frac{5}{12}$, $\frac{1}{12}$, $\frac{7}{12}$, and $\frac{5}{12}$. $1\frac{1}{2}$
3. Add $\frac{1}{8}$ and $\frac{5}{8}$ and subtract the sum from 2. $1\frac{1}{4}$
4. Add $4\frac{1}{4}$ and $2\frac{3}{4}$; 7 $6\frac{1}{3}$ and $2\frac{1}{3}$; $8\frac{2}{3}$ $1\frac{1}{5}$ and $1\frac{1}{10}$. $2\frac{3}{10}$
5. How much is $\frac{5}{6} - \frac{1}{6}$? $\frac{2}{3}$ $\frac{7}{8} - \frac{3}{8}$? $\frac{1}{2}$ $\frac{9}{10} - \frac{1}{2}$? $\frac{2}{5}$
6. How many dimes can you get by changing 3 quarters and 3 nickels into dimes? 9
7. A taxi ride costs 45¢ for the first mile and 25¢ for each mile after the first. Find the cost of riding 6 mi. in a taxi. $1.70
8. Add 12 to each number: 42, 36, 29, 58, 30, 67.
   54 48 41 70 42 79
9. How many minutes are there in 1 hr. 20 min.? 80
10. Read these numbers: MDCCXLII, MCML, DCXXV.
    1742 1950 625
11. Jack can earn 50¢ an hour. How many hours must he work to earn $1.00? $2.00? $1.50? $4.00?
    2 hr. 4 hr. 3 hr. 8 hr.
12. How many 4-cent stamps can I buy for $1.00? 25 How many 4-cent and 3-cent stamps can I buy for $3.50 so that I get an equal number of each kind? 50 of each
13. Find the sum of 6, 2, 9, 3, 8, 4, 7, 6, 5, 9. 59
14. Change each fraction to a mixed number:
    $\frac{12}{5}$ $2\frac{2}{5}$ $\frac{10}{3}$ $3\frac{1}{3}$ $\frac{13}{4}$ $3\frac{1}{4}$ $\frac{18}{4}$ $4\frac{1}{2}$ $\frac{20}{9}$ $2\frac{2}{9}$ $\frac{11}{6}$ $1\frac{5}{6}$ $\frac{20}{8}$ $2\frac{1}{2}$ $\frac{22}{7}$ $3\frac{1}{7}$ $\frac{15}{6}$ $2\frac{1}{2}$
15. Tell the fractions that are equal: $\frac{3}{4}$, $\frac{15}{20}$, $\frac{7}{8}$, $\frac{9}{12}$. $\frac{3}{4}$, $\frac{15}{20}$ and $\frac{9}{12}$
16. Change to lowest terms: $\frac{10}{15}$, $\frac{2}{3}$ $\frac{8}{12}$, $\frac{2}{3}$ $\frac{6}{8}$, $\frac{3}{4}$ $\frac{14}{16}$, $\frac{7}{8}$ $\frac{12}{15}$ $\frac{4}{5}$
17. Henry stayed at football practice from 3:25 P.M. until 5:10 P.M. How many hours and minutes did Henry practice? 1hr. 45 min.
18. From a board $5\frac{1}{2}$ ft. long Sam cut off 1 ft. 6 in. How many feet of board were left in the piece? 4
19. Count the change as a clerk might count it when you pay for a 79-cent book with $1.00. $.79, $.80, $.90, $1.00

Spot-check for class or individual weaknesses. Review particular concept or skill if many pupils evidence difficulty.

Reteach and extend method of finding least common denominator (pages 61-62). Review meaning of common denominator (same denominator).

Least Common Denominator

1. **Problem** Find the sum of $\frac{1}{4}$ and $\frac{1}{6}$. $\frac{5}{12}$

Explanation To add $\frac{1}{4}$ and $\frac{1}{6}$ you must change them to fractions having a **common denominator.** You can find a common denominator for $\frac{1}{4}$ and $\frac{1}{6}$ by multiplying the denominators 4 and 6 together. This gives 24, as shown at A. The answer $\frac{10}{24}$ can be changed to $\frac{5}{12}$.

$$\text{A.}\quad \begin{array}{r} \frac{1}{4} = \frac{6}{24} \\ \underline{\frac{1}{6} = \frac{4}{24}} \\ \frac{10}{24} = \frac{5}{12} \end{array} \qquad\qquad \text{B.}\quad \begin{array}{r} \frac{1}{4} = \frac{3}{12} \\ \underline{\frac{1}{6} = \frac{2}{12}} \\ \frac{5}{12} \end{array}$$

You can also add $\frac{1}{4}$ and $\frac{1}{6}$ as shown at B. Here the common denominator is 12; 12 is called the **least common denominator** because it is the smallest number that contains as a factor both 4 and 6. The answer $\frac{5}{12}$ is in lowest terms; it is the same as the answer at A. Review meaning of factor.

Add. Find the least common denominator:

$$\text{2.}\quad \begin{array}{r} 4\frac{5}{6} \\ 7\frac{1}{4} \\ \hline 12\frac{1}{12} \end{array} \quad \begin{array}{r} 9\frac{3}{4} \\ 3\frac{3}{10} \\ \hline 13\frac{1}{20} \end{array} \quad \begin{array}{r} 7\frac{5}{6} \\ 8\frac{3}{8} \\ \hline 16\frac{5}{24} \end{array} \quad \begin{array}{r} 2\frac{7}{10} \\ 8\frac{5}{6} \\ \hline 11\frac{8}{15} \end{array} \quad \begin{array}{r} 6\frac{1}{8} \\ 7\frac{1}{12} \\ \hline 13\frac{5}{24} \end{array} \quad \begin{array}{r} 8\frac{1}{6} \\ 3\frac{3}{4} \\ \hline 11\frac{11}{12} \end{array} \quad \begin{array}{r} 5\frac{9}{10} \\ 6\frac{1}{6} \\ \hline 12\frac{1}{15} \end{array}$$

$$\text{3.}\quad \begin{array}{r} 1\frac{1}{8} \\ 6\frac{1}{6} \\ \hline 7\frac{7}{24} \end{array} \quad \begin{array}{r} 1\frac{1}{4} \\ 1\frac{1}{10} \\ \hline 2\frac{7}{20} \end{array} \quad \begin{array}{r} 2\frac{3}{4} \\ \frac{5}{6} \\ \hline 3\frac{7}{12} \end{array} \quad \begin{array}{r} 3\frac{11}{12} \\ 1\frac{3}{8} \\ \hline 5\frac{7}{24} \end{array} \quad \begin{array}{r} 1\frac{1}{6} \\ 5\frac{3}{10} \\ \hline 6\frac{7}{15} \end{array} \quad \begin{array}{r} 6\frac{7}{8} \\ 2\frac{1}{6} \\ \hline 9\frac{1}{24} \end{array} \quad \begin{array}{r} 4\frac{7}{10} \\ 3\frac{1}{4} \\ \hline 7\frac{19}{20} \end{array}$$

Subtract. Find the least common denominator:

$$\text{4.}\quad \begin{array}{r} 6\frac{1}{4} \\ 3\frac{1}{6} \\ \hline 3\frac{1}{12} \end{array} \quad \begin{array}{r} 2\frac{1}{4} \\ 1\frac{3}{10} \\ \hline \frac{19}{20} \end{array} \quad \begin{array}{r} 9\frac{1}{8} \\ 7\frac{5}{6} \\ \hline 1\frac{7}{24} \end{array} \quad \begin{array}{r} 4\frac{5}{6} \\ 3\frac{3}{10} \\ \hline 1\frac{8}{15} \end{array} \quad \begin{array}{r} 5\frac{3}{4} \\ 3\frac{1}{6} \\ \hline 2\frac{7}{12} \end{array} \quad \begin{array}{r} 3\frac{1}{12} \\ 2\frac{3}{8} \\ \hline \frac{17}{24} \end{array} \quad \begin{array}{r} 8\frac{3}{8} \\ 5\frac{3}{10} \\ \hline 3\frac{3}{40} \end{array}$$

$$\text{5.}\quad \begin{array}{r} 7\frac{1}{6} \\ 4\frac{3}{8} \\ \hline 2\frac{19}{24} \end{array} \quad \begin{array}{r} 4\frac{7}{10} \\ 2\frac{1}{4} \\ \hline 2\frac{9}{20} \end{array} \quad \begin{array}{r} 3\frac{1}{4} \\ \frac{5}{6} \\ \hline 2\frac{5}{12} \end{array} \quad \begin{array}{r} 9\frac{3}{4} \\ 5\frac{3}{10} \\ \hline 4\frac{9}{20} \end{array} \quad \begin{array}{r} 6\frac{5}{6} \\ 1\frac{3}{4} \\ \hline 5\frac{1}{12} \end{array} \quad \begin{array}{r} 5\frac{1}{6} \\ 4\frac{3}{10} \\ \hline \frac{13}{15} \end{array} \quad \begin{array}{r} 1\frac{9}{10} \\ \frac{5}{6} \\ \hline 1\frac{1}{15} \end{array}$$

$$\text{6.}\quad \begin{array}{r} 9\frac{5}{6} \\ 4\frac{7}{8} \\ \hline 4\frac{23}{24} \end{array} \quad \begin{array}{r} 8\frac{9}{10} \\ 3\frac{5}{8} \\ \hline 5\frac{11}{40} \end{array} \quad \begin{array}{r} 5\frac{3}{4} \\ 1\frac{5}{6} \\ \hline 3\frac{11}{12} \end{array} \quad \begin{array}{r} 2\frac{5}{8} \\ 1\frac{7}{12} \\ \hline 1\frac{1}{24} \end{array} \quad \begin{array}{r} 4\frac{5}{8} \\ 4\frac{1}{6} \\ \hline \frac{11}{24} \end{array} \quad \begin{array}{r} 7\frac{1}{10} \\ 5\frac{3}{4} \\ \hline 1\frac{7}{20} \end{array} \quad \begin{array}{r} 3\frac{11}{12} \\ 1\frac{7}{8} \\ \hline 2\frac{1}{24} \end{array}$$

More Practice. See ⑯ on page 331. ↖Use for individual assistance.

Discuss ex. 1 carefully with pupils. Be sure all understand why 12 is called least common denominator. Do ex. 2 with pupils, letting different ones explain how they found least common denominator.

Least Common Denominator

1. Problem Add $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{2}{3}$. $1\frac{7}{8}$

Explanation There is an easy way to find the least common denominator for the fractions shown above. First try the largest denominator, which is 8, and see whether it contains as a factor both 6 and 3; 8 **does not** contain either 6 or 3. Next multiply 8 by 2, which gives 16, and see whether 16 contains as a factor both 6 and 3; 16 **does not** contain them. Then multiply 8 by 3, which gives 24; 24 **does** contain as a factor both 6 and 3, so 24 is the least common denominator. Tell how to complete the work.

$$\begin{array}{r} \frac{5}{6} = \frac{20}{24} \\ \frac{3}{8} = \frac{9}{24} \\ \underline{\frac{2}{3} = \frac{16}{24}} \\ \frac{45}{24} = 1\frac{21}{24} = 1\frac{7}{8} \end{array}$$

Emphasize.

To find the least common denominator, see if the largest denominator contains as factors the other denominators. If not, multiply it in turn by 2, 3, 4, and so on, until you get a number that contains as a factor each denominator.

Have pupils show work as in ex. 1.

Add. Find the least common denominator as shown above: Emphasize.

2.	$6\frac{1}{3}$	$9\frac{3}{8}$	$4\frac{1}{4}$	$2\frac{11}{16}$	$1\frac{5}{8}$	$1\frac{5}{6}$	$1\frac{1}{2}$
	$2\frac{1}{8}$	$1\frac{7}{12}$	$4\frac{3}{10}$	$1\frac{1}{6}$	$\frac{2}{3}$	$3\frac{1}{2}$	$\frac{7}{8}$
	$5\frac{1}{6}$	$3\frac{3}{16}$	$2\frac{2}{5}$	$2\frac{5}{8}$	$1\frac{7}{12}$	$4\frac{3}{5}$	$2\frac{1}{6}$
	$13\frac{5}{8}$	$14\frac{7}{48}$	$10\frac{19}{20}$	$6\frac{23}{48}$	$3\frac{7}{8}$	$9\frac{14}{15}$	$4\frac{13}{24}$
3.	$4\frac{1}{8}$	$2\frac{4}{5}$	$7\frac{11}{12}$	$4\frac{3}{8}$	$9\frac{3}{4}$	$1\frac{2}{3}$	$4\frac{1}{6}$
	$8\frac{3}{4}$	$4\frac{3}{4}$	$2\frac{1}{3}$	$3\frac{9}{10}$	$4\frac{3}{5}$	$8\frac{1}{4}$	$7\frac{1}{8}$
	$3\frac{5}{6}$	$\frac{9}{10}$	$6\frac{5}{8}$	$5\frac{1}{4}$	$2\frac{7}{10}$	$2\frac{5}{8}$	$3\frac{1}{2}$
	$16\frac{17}{24}$	$8\frac{9}{20}$	$16\frac{7}{8}$	$13\frac{21}{40}$	$17\frac{1}{20}$	$12\frac{13}{24}$	$14\frac{19}{24}$
4.	$2\frac{5}{6}$	$1\frac{5}{8}$	$8\frac{1}{4}$	$2\frac{1}{3}$	$4\frac{3}{8}$	$2\frac{1}{3}$	$1\frac{3}{8}$
	$3\frac{3}{8}$	$2\frac{2}{3}$	$2\frac{3}{10}$	$2\frac{7}{10}$	$5\frac{5}{12}$	$5\frac{1}{8}$	$8\frac{1}{2}$
	$1\frac{1}{2}$	$6\frac{7}{12}$	$3\frac{2}{5}$	$3\frac{1}{2}$	$1\frac{1}{6}$	$1\frac{1}{6}$	$4\frac{2}{3}$
	$7\frac{17}{24}$	$10\frac{7}{8}$	$13\frac{19}{20}$	$8\frac{8}{15}$	$10\frac{23}{24}$	$8\frac{5}{8}$	$14\frac{13}{24}$

More Practice. See ⑰ on page 331.

Do more examples like ex. 1 to demonstrate how to find least common denominator. Then have pupils do ex. 2-4, using method of ex. 1. Check pupils as they work to be sure all have clear understanding.

Reteach multiplication of fraction and whole number (page 63) and mixed number and whole number (page 64).

Multiplying Fractions

1. **Problem** Ann uses $\frac{2}{3}$ cup of orange juice for each cup of fruit drink that she makes. How many cups of orange juice does she need for 8 cups of fruit drink? $5\frac{1}{3}$

Explanation To find the number of cups of orange juice Ann will need, you must multiply 2 **thirds** by 8. 8 × 2 thirds = 16 thirds, or $\frac{16}{3}$. This can be changed to $5\frac{1}{3}$. Ann needs $5\frac{1}{3}$ cups of orange juice.

$$8 \times \frac{2}{3} = \frac{8 \times 2}{3} = \frac{16}{3} = 5\frac{1}{3}$$

2. **Problem** Cupcakes cost 55¢ a dozen. What will $\frac{3}{4}$ doz. cupcakes cost? 42¢

Explanation In this example, you find $\frac{3}{4}$ of 55 but you write it as shown here. The sign × means "of" when the multiplier is a fraction. The cupcakes cost $41\frac{1}{4}$¢, which is called 42¢. In most stores any fraction of a cent is called another cent. Emphasize.

$$\frac{3}{4} \times 55 = \frac{165}{4} = 41\frac{1}{4}$$

Emphasize after demonstration. →

To multiply a fraction and a whole number, first multiply the numerator of the fraction and the whole number; then divide the result by the denominator.

Find the answers: Have pupils show work as in ex. 1–2.

3. $\frac{3}{4} \times 17$ $12\frac{3}{4}$ $\frac{5}{6} \times 18$ 15 $\frac{7}{12} \times 36$ 21 $7 \times \frac{4}{5}$ $5\frac{3}{5}$ $9 \times \frac{3}{10}$ $2\frac{7}{10}$

4. $\frac{2}{3} \times 10$ $6\frac{2}{3}$ $\frac{3}{4} \times 19$ $14\frac{1}{4}$ $\frac{9}{10} \times 13$ $11\frac{7}{10}$ $9 \times \frac{5}{6}$ $7\frac{1}{2}$ $7 \times \frac{5}{12}$ $2\frac{11}{12}$

5. $\frac{3}{8} \times 12$ $4\frac{1}{2}$ $\frac{1}{2} \times 47$ $23\frac{1}{2}$ $\frac{3}{16} \times 40$ $7\frac{1}{2}$ $6 \times \frac{3}{5}$ $3\frac{3}{5}$ $3 \times \frac{7}{16}$ $1\frac{5}{16}$

6. $\frac{4}{5} \times 11$ $8\frac{4}{5}$ $\frac{1}{6} \times 13$ $2\frac{1}{6}$ $\frac{11}{12} \times 11$ $10\frac{1}{12}$ $9 \times \frac{1}{5}$ $1\frac{4}{5}$ $9 \times \frac{9}{10}$ $8\frac{1}{10}$

7. $\frac{5}{8} \times 15$ $9\frac{3}{8}$ $\frac{7}{8} \times 16$ 14 $\frac{3}{10} \times 17$ $5\frac{1}{10}$ $3 \times \frac{7}{8}$ $2\frac{5}{8}$ $5 \times \frac{11}{12}$ $4\frac{7}{12}$

8. $\frac{3}{5} \times 13$ $7\frac{4}{5}$ $\frac{1}{4} \times 17$ $4\frac{1}{4}$ $\frac{9}{16} \times 12$ $6\frac{3}{4}$ $5 \times \frac{3}{4}$ $3\frac{3}{4}$ $3 \times \frac{15}{16}$ $2\frac{13}{16}$

More Practice. See ⑱ on page 332.

Use addition ($\frac{2}{3}$ added 8 times) and diagrams to supplement ex. 1 explanation. Before discussing ex. 3, demonstrate some other examples ($\frac{3}{4}$ x 12), using diagrams. Emphasize meaning of x sign when multiplier is a fraction.

Working after School

1. Problem Every day after school Bill works $1\frac{3}{4}$ hr. in his father's store. How many hours does Bill work in 5 da? $8\frac{3}{4}$

Explanation First multiply $\frac{3}{4}$ by 5, which gives $3\frac{3}{4}$. Then multiply 1 by 5. Place the products as shown at the right. Then add the products, which gives $8\frac{3}{4}$. Bill works $8\frac{3}{4}$ hr. in 5 da.

$$\begin{array}{r} 1\frac{3}{4} \\ 5 \\ \hline 3\frac{3}{4} \\ 5 \\ \hline 8\frac{3}{4} \end{array}$$

2. Problem Bill earns \$.60 an hour when he works after school. How much does he earn in $8\frac{3}{4}$ hr.? \$5.25

Explanation First multiply \$.60 by $\frac{3}{4}$. Then multiply \$.60 by 8. Add the products and put in the decimal point. The answer is \$5.25.

$$\begin{array}{r} \$.60 \\ 8\frac{3}{4} \\ \hline 45 \\ 4\,80 \\ \hline \$5.25 \end{array}$$

Find the answers. Check by going over your work:

3. $6 \times 5\frac{1}{2}$ 33 $14 \times 7\frac{2}{5}$ $103\frac{3}{5}$ $2\frac{3}{4} \times 11$ $30\frac{1}{4}$ $3\frac{5}{6} \times \$.24$ \$.92

$\$3.23\frac{3}{4}$, or \$3.24

4. $9 \times 3\frac{2}{5}$ $30\frac{3}{5}$ $10 \times 8\frac{1}{3}$ $83\frac{1}{3}$ $5\frac{1}{2} \times 15$ $82\frac{1}{2}$ $9\frac{1}{4} \times \$.35$

$\$3.83\frac{1}{3}$ or \$3.84

5. $7 \times 3\frac{1}{8}$ $21\frac{7}{8}$ $15 \times 6\frac{1}{2}$ $97\frac{1}{2}$ $6\frac{3}{4} \times 17$ $114\frac{3}{4}$ $7\frac{2}{3} \times \$.50$

$\$1.29\frac{3}{8}$ or \$1.30

6. $4 \times 8\frac{2}{3}$ $34\frac{2}{3}$ $19 \times 4\frac{3}{8}$ $83\frac{1}{8}$ $3\frac{3}{8} \times 24$ 81 $2\frac{7}{8} \times \$.45$

More Practice. See ⑲ on page 332.

Follow development as given. Show multiplication of $\frac{3}{4}$ by 5 and \$.60 by $\frac{3}{4}$ as done in ex. 1-2 on page 63. Stress proper placement of partial products. Assign ex. 3-6 as independent work if there are no difficulties.

Reteach multiplication of two fractions and show how cancellation by division shortens the work (pages 65–66).

Fractions in Multiplication

1. **Problem** How much is ½ of ½ in.? $\frac{1}{4}$ in.

Explanation You see on your ruler that ½ of ½ in. is ¼ in. You can find ½ of ½ without a ruler, as shown here. This is done by multiplying the numerators of the two fractions and also the denominators.

$$\frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

Emphasize. →

To multiply a fraction by a fraction, first multiply the numerators; then multiply the denominators.

2. **Problem** How much is ⅓ of ¾ in.? $\frac{1}{4}$ in.

Explanation You can do the work in two different ways. In the first way, shown at A, multiply the numerators and denominators as usual, which gives $\frac{3}{12}$. Then change $\frac{3}{12}$ to ¼ by dividing 3 and 12 by 3. The answer is ¼. Be sure that pupils understand that cancel means to divide both numerator and denominator by same number.

A. $\frac{1}{3} \times \frac{3}{4} = \frac{1 \times 3}{3 \times 4} = \frac{3}{12} = \frac{1}{4}$

B. $\frac{1}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{4} = \frac{1 \times 1}{1 \times 4} = \frac{1}{4}$

In the second way, shown at B, first cancel 3 from the denominator 3 of the first fraction and the numerator 3 of the second fraction. Then the answer is ¼. You divided the first denominator by 3 and the second numerator by 3; this is the same as dividing both terms of the answer by 3.

In ex. 3, point out that 2 was cancelled in second example, not 10 or 8.

3. Study these examples carefully: Also emphasize showing results of cancellation immediately (writing 1 above 2, and so on).

$\frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{5} = \frac{3}{10}$

$10 \times \frac{5}{8} = \frac{\overset{5}{\cancel{10}}}{1} \times \frac{5}{\underset{4}{\cancel{8}}} = \frac{25}{4} = 6\frac{1}{4}$

4. How much is ½ of ⅞ in.? $\frac{7}{16}$ in. ⅔ of $\frac{9}{16}$ in.? $\frac{3}{8}$ in.

When you cancel, you divide a numerator and a denominator each by the same number.

In ex. 1, ask pupils why product is smaller than original fraction (you are finding a part of a part). In discussing ex. 2–3 emphasize that cancellation by division is merely application of principle of fractions (page 52).

Fractions in Multiplication

1. Andy lives $\frac{3}{4}$ mi. from school. He walks that distance 10 times each week. How many miles does Andy walk each week going to and from school? $7\frac{1}{2}$

2. Bill lives $\frac{9}{10}$ mi. from school but his father drives Bill $\frac{2}{3}$ of that distance when he goes to business. What part of a mile does Bill's father drive him on the way to school? $\frac{3}{5}$

3. How much is $\frac{2}{3}$ of $\frac{3}{4}$ mi.? $\frac{1}{2}$ mi. How much is $\frac{4}{5}$ of $\frac{3}{4}$ mi.? $\frac{3}{5}$ mi. How much is $\frac{1}{2}$ of $\frac{3}{4}$ mi.? $\frac{3}{8}$ mi.

Multiply. Check your work by going over it:

4. $\frac{3}{4} \times \frac{3}{4}$ $\frac{9}{16}$ $6 \times \frac{3}{4}$ $4\frac{1}{2}$ $\frac{2}{3} \times \frac{3}{8}$ $\frac{1}{4}$ $\frac{3}{10} \times 6$ $1\frac{4}{5}$ $\frac{3}{16} \times 12$ $2\frac{1}{4}$

5. $\frac{5}{6} \times \frac{3}{8}$ $\frac{5}{16}$ $8 \times \frac{5}{6}$ $6\frac{2}{3}$ $\frac{1}{2} \times \frac{4}{5}$ $\frac{2}{5}$ $\frac{9}{16} \times \frac{2}{3}$ $\frac{3}{8}$ $\frac{9}{10} \times \frac{5}{12}$ $\frac{3}{8}$

6. $\frac{1}{2} \times \frac{3}{8}$ $\frac{3}{16}$ $\frac{1}{2} \times \frac{5}{6}$ $\frac{5}{12}$ $\frac{5}{6} \times 2$ $1\frac{2}{3}$ $\frac{11}{12} \times 4$ $3\frac{2}{3}$ $18 \times \frac{7}{12}$ $10\frac{1}{2}$

7. $\frac{3}{5} \times \frac{5}{6}$ $\frac{1}{2}$ $\frac{3}{8} \times \frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4} \times \frac{2}{3}$ $\frac{1}{2}$ $\frac{9}{10} \times \frac{5}{6}$ $\frac{3}{4}$ $\frac{3}{16} \times \frac{1}{12}$ $\frac{1}{64}$

8. $6 \times \frac{2}{3}$ 4 $9 \times \frac{1}{6}$ $1\frac{1}{2}$ $\frac{1}{3} \times \frac{4}{5}$ $\frac{4}{15}$ $\frac{3}{16} \times \frac{1}{6}$ $\frac{1}{32}$ $\frac{7}{10} \times 15$ $10\frac{1}{2}$

More Practice. See ⑳ on page 332.

Do problems with class. Then, if there are no apparent difficulties, assign ex. 4–8 as independent work. Group pupils who had errors in ex. 4–8 to help determine causes and clear up difficulties. Have others work on problems using numbers from ex. 4–8.

Reteach multiplication of mixed number by fraction or mixed number.

Multiplying Mixed Numbers

1. **Problem** Mrs. Hill uses 3 eggs and $2\frac{1}{4}$ cups of flour in a recipe for a large cake. If she uses only 2 eggs for a smaller cake, she must use only $\frac{2}{3}$ as much flour. How many cups of flour is this? $1\frac{1}{2}$

Explanation Find $\frac{2}{3}$ of $2\frac{1}{4}$. To do this change $2\frac{1}{4}$ to $\frac{9}{4}$. Then multiply $\frac{9}{4}$ by $\frac{2}{3}$. Divide both 9 and 3 by 3; divide both 2 and 4 by 2. The answer is $1\frac{1}{2}$. Mrs. Hill needs $1\frac{1}{2}$ cups of flour when she uses 2 eggs.

$$\frac{2}{3}\times 2\frac{1}{4} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} = \frac{3}{2} = 1\frac{1}{2}$$

2. Study these examples carefully:

$$2\frac{2}{3}\times\frac{\cancel{3}}{\cancel{8}} = \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{3}}{\underset{1}{8}} = 1 \qquad 3\frac{1}{8}\times 1\frac{1}{5} = \frac{\overset{5}{\cancel{25}}}{\underset{4}{\cancel{8}}}\times\frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{5}}} = \frac{15}{4} = 3\frac{3}{4}$$

When a factor of a product is a mixed number, you often change it to an improper fraction before you multiply.

Emphasize.

Multiply. Cancel when you can. ✓ **Check the work by going over it:**

3. $2\frac{2}{3}\times 1\frac{1}{2}$ 4 $\quad 1\frac{1}{8}\times 1\frac{1}{3}$ $1\frac{1}{2}$ $\quad \frac{1}{3}\times 6\frac{3}{4}$ $2\frac{1}{4}$ $\quad 2\frac{1}{2}\times\frac{1}{2}$ $1\frac{1}{4}$
4. $1\frac{2}{5}\times 1\frac{1}{2}$ $2\frac{1}{10}$ $\quad 1\frac{1}{4}\times 1\frac{1}{5}$ $1\frac{1}{2}$ $\quad \frac{2}{3}\times 7\frac{1}{2}$ 5 $\quad 2\frac{5}{8}\times\frac{2}{3}$ $1\frac{3}{4}$
5. $1\frac{1}{2}\times 1\frac{1}{2}$ $2\frac{1}{4}$ $\quad 4\frac{1}{2}\times 2\frac{2}{3}$ 12 $\quad \frac{2}{3}\times 4\frac{1}{5}$ $2\frac{4}{5}$ $\quad 1\frac{1}{2}\times\frac{2}{3}$ 1
6. $6\frac{1}{4}\times 2\frac{2}{5}$ 15 $\quad 2\frac{1}{3}\times 1\frac{1}{2}$ $3\frac{1}{2}$ $\quad \frac{3}{4}\times 5\frac{1}{3}$ 4 $\quad 5\frac{1}{4}\times\frac{2}{3}$ $3\frac{1}{2}$
7. $1\frac{1}{5}\times 3\frac{1}{3}$ 4 $\quad 2\frac{1}{4}\times 3\frac{1}{3}$ $7\frac{1}{2}$ $\quad \frac{2}{5}\times 1\frac{7}{8}$ $\frac{3}{4}$ $\quad 5\frac{1}{2}\times\frac{2}{5}$ $2\frac{1}{5}$
8. $2\frac{1}{2}\times 1\frac{3}{5}$ 4 $\quad 2\frac{3}{5}\times 3\frac{1}{2}$ $9\frac{1}{10}$ $\quad \frac{3}{5}\times 1\frac{1}{4}$ $\frac{3}{4}$ $\quad 4\frac{2}{3}\times\frac{1}{2}$ $2\frac{1}{3}$
9. $3\frac{1}{2}\times 1\frac{1}{4}$ $4\frac{3}{8}$ $\quad 5\frac{1}{3}\times 1\frac{1}{8}$ 6 $\quad \frac{1}{2}\times 1\frac{1}{6}$ $\frac{7}{12}$ $\quad 2\frac{2}{5}\times\frac{5}{6}$ 2
10. $1\frac{3}{4}\times 2\frac{2}{3}$ $4\frac{2}{3}$ $\quad 1\frac{4}{5}\times 4\frac{1}{6}$ $7\frac{1}{2}$ $\quad \frac{3}{10}\times 5\frac{5}{6}$ $1\frac{3}{4}$ $\quad 3\frac{3}{4}\times\frac{2}{3}$ $2\frac{1}{2}$
11. $1\frac{2}{3}\times 2\frac{1}{2}$ $4\frac{1}{6}$ $\quad 1\frac{1}{3}\times 1\frac{1}{2}$ 2 $\quad \frac{9}{10}\times 6\frac{2}{3}$ 6 $\quad 1\frac{1}{5}\times\frac{3}{4}$ $\frac{9}{10}$
12. $3\frac{3}{4}\times 1\frac{3}{5}$ 6 $\quad 3\frac{1}{3}\times 1\frac{3}{4}$ $5\frac{5}{6}$ $\quad \frac{15}{16}\times 3\frac{1}{5}$ 3 $\quad 3\frac{3}{5}\times\frac{1}{6}$ $\frac{3}{5}$
13. $1\frac{3}{5}\times 1\frac{1}{8}$ $1\frac{4}{5}$ $\quad 3\frac{1}{8}\times 7\frac{1}{5}$ $22\frac{1}{2}$ $\quad \frac{5}{12}\times 1\frac{1}{3}$ $\frac{5}{9}$ $\quad 2\frac{2}{5}\times\frac{5}{8}$ $1\frac{1}{2}$

More Practice. See ㉑ on page 332.

Discuss model examples in ex. 1-2 thoroughly with class. Emphasize that each mixed number is first changed to an improper fraction. Stress importance of showing results of cancellation by division immediately.

Present summary of all types of multiplication retaught on pages 63–67.

Problems and Practice

1. Jack's rate of walking is about $3\frac{1}{4}$ mi. per hour. How far can he walk in $1\frac{1}{2}$ hr.? in 2 hr.? in $2\frac{1}{2}$ hr.? $8\frac{1}{8}$ mi.
$4\frac{7}{8}$ mi. $6\frac{1}{2}$ mi.

2. The cost of electricity for an electric iron is $3\frac{1}{3}$¢ an hour in the town where Mrs. Jackson lives. How much does it cost to use the iron for $1\frac{1}{2}$ hr.? to use the iron for $3\frac{3}{4}$ hr.? for $\frac{1}{2}$ hr.? for $4\frac{1}{2}$ hr.? 5¢ $12\frac{1}{2}$¢
$1\frac{2}{3}$¢ 15¢

3. On a hot day the large electric fan in Mr. White's office ran for 8 hr. It costs $2\frac{1}{4}$¢ for electricity to run that fan for an hour. How much did it cost to run it 8 hr.? 18¢

4. There are $7\frac{1}{2}$ gal. in 1 cu. ft. of water. How many gallons are there in $2\frac{1}{2}$ cu. ft. of water? $18\frac{3}{4}$

5. A gallon of water weighs about $8\frac{1}{3}$ lb. How much do the 24 gal. of water in a large fish tank weigh? 200 lb.

Multiply. Check the work by going over it:

6.	$3\frac{5}{6} \times 3$ $11\frac{1}{2}$	$24 \times \frac{5}{8}$ 15	$\frac{3}{4} \times \frac{5}{12}$ $\frac{5}{16}$	$5\frac{1}{5} \times 7\frac{1}{2}$ 39
7.	$12 \times \frac{3}{4}$ 9	$3\frac{1}{3} \times 5$ $16\frac{2}{3}$	$\frac{2}{5} \times \frac{5}{6}$ $\frac{1}{3}$	$1\frac{1}{8} \times 1\frac{1}{3}$ $1\frac{1}{2}$
8.	$\frac{7}{8} \times 20$ $17\frac{1}{2}$	$1\frac{1}{5} \times \frac{5}{6}$ 1	$\frac{1}{3} \times \frac{5}{6}$ $\frac{5}{18}$	$3\frac{3}{8} \times 2\frac{2}{3}$ 9
9.	$9 \times 2\frac{2}{3}$ 24	$\frac{2}{5} \times 15$ 6	$\frac{2}{3} \times \frac{3}{8}$ $\frac{1}{4}$	$5\frac{1}{4} \times 3\frac{1}{3}$ $17\frac{1}{2}$
10.	$\frac{3}{10} \times 16$ $4\frac{4}{5}$	$5\frac{4}{5} \times 5$ 29	$\frac{9}{10} \times \frac{8}{9}$ $\frac{4}{5}$	$8\frac{3}{4} \times 1\frac{1}{5}$ $10\frac{1}{2}$
11.	$10 \times \frac{7}{16}$ $4\frac{3}{8}$	$\frac{15}{16} \times \frac{3}{10}$ $\frac{9}{32}$	$\frac{1}{2} \times \frac{2}{5}$ $\frac{1}{5}$	$1\frac{2}{3} \times 1\frac{2}{3}$ $2\frac{7}{9}$
12.	$6 \times 1\frac{1}{5}$ $7\frac{1}{5}$	$\frac{3}{4} \times 7\frac{1}{5}$ $5\frac{2}{5}$	$\frac{5}{8} \times \frac{4}{5}$ $\frac{1}{2}$	$9\frac{1}{2} \times 1\frac{1}{2}$ $14\frac{1}{4}$
13.	$\frac{3}{4} \times 26$ $19\frac{1}{2}$	$\frac{4}{5} \times 3\frac{3}{4}$ 3	$\frac{4}{5} \times \frac{2}{5}$ $\frac{8}{25}$	$4\frac{7}{12} \times 16$ $73\frac{1}{3}$
14.	$\frac{3}{10} \times 6\frac{2}{3}$ 2	$7\frac{3}{8} \times 10$ $73\frac{3}{4}$	$\frac{3}{10} \times 6$ $1\frac{4}{5}$	$13 \times 1\frac{1}{2}$ $19\frac{1}{2}$
15.	$8 \times 1\frac{5}{6}$ $14\frac{2}{3}$	$6\frac{1}{4} \times \frac{4}{5}$ 5	$9 \times \frac{11}{12}$ $8\frac{1}{4}$	$1\frac{1}{4} \times \frac{2}{3}$ $\frac{5}{6}$
16.	$\frac{7}{8} \times 1\frac{1}{3}$ $1\frac{1}{6}$	$2\frac{1}{2} \times \frac{3}{5}$ $1\frac{1}{2}$	$11 \times 2\frac{1}{2}$ $27\frac{1}{2}$	$15 \times \frac{7}{10}$ $10\frac{1}{2}$
17.	$\frac{5}{6} \times 2\frac{1}{4}$ $1\frac{7}{8}$	$4\frac{1}{8} \times 5\frac{1}{3}$ 22	$20 \times 5\frac{1}{3}$ $106\frac{2}{3}$	$5\frac{5}{6} \times 1\frac{4}{5}$ $10\frac{1}{2}$
18.	$8 \times 2\frac{3}{4}$ 22	$7\frac{1}{2} \times 1\frac{1}{6}$ $8\frac{3}{4}$	$1\frac{1}{2} \times \frac{11}{12}$ $1\frac{3}{8}$	$\frac{9}{10} \times \frac{5}{12}$ $\frac{3}{8}$

Have all pupils do ex. 1–5. Then assign half of ex. 6–18 to one group, and half to other group. Papers should be checked carefully to determine if further reteaching or reinforcement of concepts is necessary.

Present first set of improvement tests in multiplication. Review arithmetical terms.

Improving by Practice

STATE INSTITUTE OF EDUCATION
Govt of West Bengal

Multiplication Test 1a.

Time: 4½ min. after copying.

1.	485	923	431	607	568	792	
	293	187	406	359	624	510	⑥
	142,105	172,601	174,986	217,913	354,432	403,920	

Multiplication Test 1b.

Time: 4½ min. after copying.

2.	279	568	123	876	709	681	
	406	517	839	258	963	140	⑥
	113,274	293,656	103,197	226,008	682,767	95,340	

Multiplication Test 1c.

Time: 4½ min. after copying.

3.	957	268	192	463	704	345	
	738	496	702	143	651	870	⑥
	706,266	132,928	134,784	66,209	458,304	300,150	

Tests are to be given, scored, recorded in same way as addition tests (see pages 46–50).

The Language of Arithmetic

Show that you know what each word or group of words means by using it in a sentence or by giving an example of it:

4. divisor
5. average
6. addend
7. product
8. quotient
9. quarter
10. dividend
11. minuend
12. difference
13. invert
14. denominator
15. subtrahend
16. cancellation
17. lowest terms
18. mixed number
19. addition facts
20. proper fraction
21. partial product
22. hidden question
23. whole number
24. checking
25. multiplier
26. remainder
27. fraction
28. multiplicand
29. plus sign
30. numerator
31. factor
32. divisible
33. minus sign
34. commutative principle for addition
35. associative principle for multiplication

Pupils should copy examples carefully before test is begun. Hold conferences with pupils to determine causes of errors (facts, carrying, zero difficulties, and so on). Provide further specialized review based on results of ex. 4–35.

Reteach solving of problems with several steps (pages 70–71).

Problems with Several Steps

1. Problem At the railroad ticket office Mrs. Williams bought 2 round-trip tickets at $14.85 each and a one-way ticket for $9.63. How much change did she get from $40.00? $.67

Explanation This is a three-step problem. Study the work below:

First Step. You should answer the first **hidden question,** which is: "How much did the 2 round-trip tickets cost?" They cost $29.70, as shown below in A.

A	B	C
$14.85	$29.70	$40.00
2	9.63	39.33
$29.70	$39.33	$ .67

Second Step. Next answer the second **hidden question,** which is: "How much did all the tickets cost?" They cost $39.33, as shown above in B.

Third Step. Now you can answer the question asked in the problem, which is: "How much change did she get from $40.00?" By subtracting $39.33 from $40.00, as shown above in C, you find that Mrs. Wilson got $.67 in change.

2. How many **hidden questions** are there in a three-step problem? 2 in a four-step problem? 3

3. Of the 625 tickets that were sold for the school play 375 were sold to pupils and the rest to parents and friends. If the tickets for pupils cost $.25 each and the rest cost $.35 each, how much money came from the sale of tickets? $181.25

4. At the ribbon counter Mrs. Day bought 3 yd. of blue ribbon at $.35 a yard, 2 yd. of red ribbon at $.49 a yard, and $\frac{3}{4}$ yd. of green ribbon at $.60 a yard. Find the cost of all the ribbon Mrs. Day bought. $2.48

5. Last summer during her vacation Betsy knit a sweater for her little sister. She bought 3 balls of yarn at $.79 a ball, and $\frac{2}{3}$ doz. buttons at $.65 a dozen. Find the cost of the materials for the sweater. $2.81

In discussing ex. 1 with pupils, stress that they must look for and answer hidden questions first, before they can answer problem question. Give another problem as class practice exercise, then assign ex. 3–5.

Problems on pages 70–71 should be done as written assignment to be handed in and corrected. Check carefully the form of work as well as answers. Group pupils for further explanation if needed.

More Problems

There are two or three hidden questions in each problem on this page. Stress: **In working each problem first find the hidden questions and answer them. Then answer the question asked in the problem:**

1. Betty is planning to spend \$24.00 for a vacation at camp and \$4.90 for a round-trip bus ticket. She has saved \$8.70 so far and her father says he will give her \$10.00. How many hours must she work to earn the rest of the money if she earns \$.60 an hour? 17
2. Dan sold Mrs. Martin 6 lb. of oranges at 2 lb. for \$.27, 9 rolls at \$.03 each, a box of crackers for \$.21, and a cabbage for \$.15. What did Dan charge Mrs. Martin? \$1.44
3. At a sale Miss Brown bought a coat marked \$39.00 and a suit marked \$33.00, both for $\frac{1}{4}$ off. How much in all did Miss Brown pay for these two articles? \$54.00
4. At field day the Athletic Association made \$435 to purchase new outfits for the football team. The suits cost \$10.50 each, the shoulder pads cost \$6.75 a pair, the helmets cost \$5.79 each, and the shoes cost \$8.95 a pair. If 18 outfits are needed for the team and extra players, how much more money does the association need in order to purchase them? \$140.82
5. The Garden Club has 500 packages of seeds to sell at 10¢ a package for vegetable seeds and 15¢ a package for flower seeds. There are 150 packages of flower seeds and the rest are vegetable seeds. How much will the club get from the sale of all these seeds? \$57.50
6. The Garden Club paid \$29.00 for the seeds and received \$57.50 for them. It had expenses of \$3.75 in selling the seeds. If $\frac{1}{3}$ of the profit was set aside to buy garden tools, how much was used for this purpose? \$8.25

Urge better pupils to make up original 2- and 3-step problems.

Reteach division of whole number or fraction by fraction. See Guide for important prelesson activities. Give oral practice on inverting divisors and whole numbers in particular.

Dividing by a Fraction

1. Examples in division that have fractions as divisors are often worked by **inverting the divisor.** The divisor $\frac{3}{4}$ is inverted by turning it upside down like this: $\frac{4}{3}$. If $\frac{1}{4}$ is inverted, it becomes $\frac{4}{1}$, or 4. To invert the number 5, first write it as the fraction $\frac{5}{1}$; inverting $\frac{5}{1}$ you get $\frac{1}{5}$.

2. How many fourths of an inch are there in 3 in.? You can write the question like this: $3 \div \frac{1}{4} = ?$ On your ruler you can count the number of fourths, which is 12. You know that there are 4 fourths in 1 in., so there are 3×4 fourths, or 12 fourths, in 3 in. This shows that the answer for $3 \div \frac{1}{4}$ is the same as the answer to 3×4, which is 12.

Stress.

3. In ex. 2, you found that $3 \div \frac{1}{4}$ has the same answer as 3×4. This shows that you can do the division example $3 \div \frac{1}{4}$ by changing it to a multiplication example with the divisor $\frac{1}{4}$ inverted, which gives $\frac{4}{1}$, or 4. You can write the work like this: $3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 3 \times 4 = 12$.

Stress after understanding as assured.

To divide by a fraction, invert the divisor and multiply.

Illustrate more examples like ex. 2–3 before discussing ex. 4.

4. Study these examples carefully:

$$12 \div \frac{3}{8} = \frac{\overset{4}{\cancel{12}}}{1} \times \frac{8}{\underset{1}{\cancel{3}}} = \frac{32}{1} = 32 \qquad \frac{9}{10} \div \frac{3}{8} = \frac{\overset{3}{\cancel{9}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{3}}} = \frac{12}{5} = 2\frac{2}{5}$$

Divide. Cancel when possible. Check by going over the work:

5. $6 \div \frac{2}{5}$ 15 $\quad \frac{3}{8} \div \frac{1}{4}$ $1\frac{1}{2}$ $\quad \frac{3}{4} \div \frac{5}{8}$ $1\frac{1}{5}$ $\quad \frac{5}{6} \div \frac{15}{16}$ $\frac{8}{9}$ $\quad 12 \div \frac{3}{4}$ 16

6. $9 \div \frac{3}{8}$ 24 $\quad \frac{2}{3} \div \frac{1}{2}$ $1\frac{1}{3}$ $\quad \frac{5}{6} \div \frac{1}{2}$ $1\frac{2}{3}$ $\quad \frac{1}{5} \div \frac{3}{10}$ $\frac{2}{3}$ $\quad 15 \div \frac{2}{3}$ $22\frac{1}{2}$

7. $7 \div \frac{1}{3}$ 21 $\quad \frac{3}{4} \div \frac{1}{8}$ 6 $\quad \frac{1}{2} \div \frac{3}{4}$ $\frac{2}{3}$ $\quad \frac{5}{6} \div \frac{5}{12}$ 2 $\quad 10 \div \frac{5}{6}$ 12

8. $8 \div \frac{2}{5}$ 20 $\quad \frac{5}{6} \div \frac{1}{5}$ $4\frac{1}{6}$ $\quad \frac{5}{8} \div \frac{2}{3}$ $\frac{15}{16}$ $\quad \frac{3}{8} \div \frac{5}{16}$ $1\frac{1}{5}$ $\quad 15 \div \frac{5}{8}$ 24

After prelesson activities, follow development as given. Supplement explanation by using subtractive aspect of division (have pupils find out how many times $\frac{1}{4}$ can be subtracted from 3, $\frac{1}{6}$ from 2, and so on).

Pupils should be above to explain why answers are sensible; for example, in $4 \div \frac{4}{5}$, a number less than 1 is the divisor. Answer must be greater than 4; therefore 5 is sensible.

Dividing by a Fraction

Divide. Cancel when possible. Check your work:

1. $3 \div \frac{1}{6}$ 18 $\frac{1}{6} \div \frac{3}{8}$ $\frac{4}{9}$ $6 \div \frac{4}{5}$ $7\frac{1}{2}$ $\frac{1}{6} \div \frac{5}{12}$ $\frac{2}{5}$ $6 \div \frac{9}{10}$ $6\frac{2}{3}$

2. $6 \div \frac{3}{8}$ 16 $\frac{2}{3} \div \frac{5}{6}$ $\frac{4}{5}$ $\frac{1}{8} \div \frac{5}{6}$ $\frac{3}{20}$ $\frac{3}{4} \div \frac{15}{16}$ $\frac{4}{5}$ $\frac{5}{6} \div \frac{7}{10}$ $1\frac{4}{21}$

3. $8 \div \frac{1}{2}$ 16 $6 \div \frac{1}{8}$ 48 $\frac{2}{3} \div \frac{1}{4}$ $2\frac{2}{3}$ $\frac{2}{3} \div \frac{5}{12}$ $1\frac{3}{5}$ $\frac{3}{4} \div \frac{9}{16}$ $1\frac{1}{3}$

4. $7 \div \frac{3}{4}$ $9\frac{1}{3}$ $\frac{2}{3} \div \frac{3}{4}$ $\frac{8}{9}$ $\frac{5}{8} \div \frac{1}{2}$ $1\frac{1}{4}$ $\frac{5}{8} \div \frac{15}{16}$ $\frac{2}{3}$ $8 \div \frac{5}{12}$ $19\frac{1}{5}$

5. $\frac{3}{5} \div \frac{2}{3}$ $\frac{9}{10}$ $9 \div \frac{3}{4}$ 12 $4 \div \frac{2}{3}$ 6 $9 \div \frac{9}{16}$ 16 $\frac{3}{5} \div \frac{9}{10}$ $\frac{2}{3}$

More Practice. See ㉒ on page 333.

6. The girls in Miss Wood's sewing class have 21 yd. of cloth for aprons. They will make the aprons to sell at the school fair. If they use an apron pattern that requires $\frac{7}{8}$ yd. for each apron, how many aprons can they make from 21 yd. of cloth? 24

7. If the girls in ex. 6 use $\frac{3}{4}$ yd. for each apron, how many aprons can they make? 28

8. Suppose Miss Wood buys 7 yd. more of the same material. How many more aprons that require $\frac{7}{8}$ yd. each can be made from this extra material? 8

While you work with pupils who need help, have others make up original problems, using numbers in ex. 1–5.

Reteach division of mixed number by fraction.

Dividing by a Fraction

1. Problem Mr. Ford gives lessons on the drum for $4\frac{1}{2}$ hr. each day. How many lessons a day does Mr. Ford give if each lesson is $\frac{3}{4}$ hr. long.? 6

Explanation Change $4\frac{1}{2}$ to the improper fraction $\frac{9}{2}$, invert the divisor, and multiply. The quotient is 6. Mr. Ford gives 6 lessons each day. Check the work by going over it.

$$4\frac{1}{2} \div \frac{3}{4} = \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}} = 6$$

2. Study this example carefully:

$$2\frac{5}{8} \div \frac{7}{12} = \frac{\overset{3}{\cancel{21}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{\underset{1}{\cancel{7}}} = \frac{9}{2} = 4\frac{1}{2}$$

3. Our school day runs from 8:30 A.M. until 2:30 P.M. There is a lunch period of $\frac{3}{4}$ hr. How many class periods are there in the rest of the time if a period lasts $\frac{3}{4}$ hr.? 7

Divide. Check by going over the work:

4. $1\frac{1}{8} \div \frac{3}{8}$ 3 $6\frac{1}{8} \div \frac{7}{16}$ 14 $\frac{9}{16} \div \frac{3}{10}$ $1\frac{7}{8}$ $15 \div \frac{5}{8}$ 24

5. $4\frac{1}{6} \div \frac{5}{8}$ $6\frac{2}{3}$ $3\frac{3}{5} \div \frac{9}{10}$ 4 $\frac{3}{10} \div \frac{9}{10}$ $\frac{1}{3}$ $16 \div \frac{1}{4}$ 64

6. $1\frac{2}{3} \div \frac{2}{3}$ $2\frac{1}{2}$ $5\frac{3}{4} \div \frac{1}{12}$ 69 $\frac{9}{10} \div \frac{3}{16}$ $4\frac{4}{5}$ $3\frac{3}{4} \div \frac{5}{8}$ 6

7. $1\frac{7}{8} \div \frac{5}{6}$ $2\frac{1}{4}$ $2\frac{3}{8} \div \frac{3}{16}$ $12\frac{2}{3}$ $\frac{15}{16} \div \frac{9}{16}$ $1\frac{2}{3}$ $3\frac{1}{3} \div \frac{5}{6}$ 4

8. $1\frac{3}{4} \div \frac{1}{3}$ $5\frac{1}{4}$ $7\frac{1}{3} \div \frac{5}{12}$ $17\frac{3}{5}$ $\frac{5}{12} \div \frac{5}{16}$ $1\frac{1}{3}$ $2\frac{1}{4} \div \frac{1}{6}$ $13\frac{1}{2}$

9. $1\frac{1}{2} \div \frac{3}{4}$ 2 $4\frac{1}{2} \div \frac{15}{16}$ $4\frac{4}{5}$ $\frac{3}{16} \div \frac{15}{16}$ $\frac{1}{5}$ $27 \div \frac{9}{10}$ 30

10. $1\frac{4}{5} \div \frac{2}{3}$ $2\frac{7}{10}$ $4\frac{3}{8} \div \frac{7}{12}$ $7\frac{1}{2}$ $\frac{15}{16} \div \frac{5}{12}$ $2\frac{1}{4}$ $25 \div \frac{15}{16}$ $26\frac{2}{3}$

11. $3\frac{1}{2} \div \frac{3}{5}$ $5\frac{5}{6}$ $3\frac{3}{10} \div \frac{11}{12}$ $3\frac{3}{5}$ $\frac{11}{12} \div \frac{11}{16}$ $1\frac{1}{3}$ $21 \div \frac{7}{10}$ 30

12. $2\frac{1}{4} \div \frac{3}{8}$ 6 $2\frac{1}{12} \div \frac{5}{16}$ $6\frac{2}{3}$ $\frac{7}{16} \div \frac{7}{16}$ 1 $1\frac{1}{2} \div \frac{9}{10}$ $1\frac{2}{3}$

13. $4\frac{1}{5} \div \frac{2}{3}$ $6\frac{3}{10}$ $2\frac{1}{16} \div \frac{11}{12}$ $2\frac{1}{4}$ $\frac{5}{16} \div \frac{1}{12}$ $3\frac{3}{4}$ $14 \div \frac{7}{12}$ 24

More Practice. See ㉓ on page 333.

Review that in multiplying mixed numbers, it was necessary to change them to improper fractions. Emphasize that quotient must be changed back to mixed number and fractional amount changed to lowest terms.

Reteach division of fraction or mixed number by whole number.

Dividing Fractions by Whole Numbers

1. **Problem** Bill's mother had $\frac{3}{4}$ of a pie. She gave it to Bill to share with 5 friends. What part of a pie did each boy get? $\frac{1}{8}$

Explanation When you divide any number by 6, you take $\frac{1}{6}$ of the number. So $\frac{3}{4} \div 6$ is the same as $\frac{1}{6}$ of $\frac{3}{4}$, or $\frac{1}{6} \times \frac{3}{4}$. Then $\frac{3}{4} \div 6 = \frac{3}{4} \times \frac{1}{6}$, or $\frac{1}{8}$. Each piece was $\frac{1}{8}$ of the pie.

You can do the example $\frac{3}{4} \div 6$ by inverting the divisor and multiplying, as shown at the right. You get the answer $\frac{1}{8}$, as before. You see that the rule for inverting the divisor can also be used when the divisor is a whole number.

$$\frac{3}{4} \div 6 = \frac{3}{4} \div \frac{6}{1} = \frac{\overset{1}{\cancel{3}}}{4} \times \frac{1}{\underset{2}{\cancel{6}}} = \frac{1}{8}$$

2. **Problem** Fred walked $10\frac{1}{2}$ mi. to Long Lake in 3 hr. How many miles per hour did he average on the trip? $3\frac{1}{2}$

Explanation You must divide $10\frac{1}{2}$ by 3. First change $10\frac{1}{2}$ to $\frac{21}{2}$ and write 3 as $\frac{3}{1}$. Then invert the divisor and multiply. Fred averaged $3\frac{1}{2}$ mi. per hour.

$$10\frac{1}{2} \div 3 = \frac{21}{2} \div \frac{3}{1} = \frac{\overset{7}{\cancel{21}}}{2} \times \frac{1}{\underset{1}{\cancel{3}}} = \frac{7}{2} = 3\frac{1}{2}$$

Emphasize. →

To divide a fraction or a mixed number by a whole number, invert the divisor and multiply.

Divide. Check by going over your work:

3. $\frac{3}{4} \div 9$ $\frac{1}{12}$ $\frac{5}{8} \div 10$ $\frac{1}{16}$ $\frac{3}{16} \div 6$ $\frac{1}{32}$ $8\frac{3}{4} \div 14$ $\frac{5}{8}$ $1\frac{3}{4} \div 14$ $\frac{1}{8}$

4. $\frac{7}{8} \div 2$ $\frac{7}{16}$ $\frac{3}{4} \div 15$ $\frac{1}{20}$ $\frac{9}{10} \div 3$ $\frac{3}{10}$ $6\frac{3}{4} \div 12$ $\frac{9}{16}$ $3\frac{1}{8} \div 10$ $\frac{5}{16}$

5. $\frac{2}{3} \div 4$ $\frac{1}{6}$ $\frac{4}{5} \div 12$ $\frac{1}{15}$ $\frac{3}{10} \div 9$ $\frac{1}{30}$ $5\frac{5}{6} \div 24$ $\frac{35}{144}$ $7\frac{1}{2} \div 12$ $\frac{5}{8}$

6. $\frac{4}{5} \div 8$ $\frac{1}{10}$ $\frac{5}{8} \div 20$ $\frac{1}{32}$ $\frac{15}{16} \div 6$ $\frac{5}{32}$ $7\frac{1}{3} \div 11$ $\frac{2}{3}$ $5\frac{3}{5} \div 21$ $\frac{4}{15}$

More Practice. See ㉔ on page 333.

First use diagrams to illustrate some simple examples ($\frac{1}{2} \div 2$, $\frac{2}{3} \div 2$) to illustrate that dividing by whole number is the same as multiplying by reciprocal of whole number. You may wish to teach pupils meaning and use of term reciprocal.

Dividing by a Mixed Number

1. Problem Mr. Wood has $6\frac{3}{4}$ yd. of red canvas. How many porch chairs can he recover with this canvas if each chair takes $1\frac{1}{8}$ yd. of canvas? 6

Explanation You need to divide $6\frac{3}{4}$ by $1\frac{1}{8}$. First change $6\frac{3}{4}$ to $\frac{27}{4}$ and $1\frac{1}{8}$ to $\frac{9}{8}$. Next invert the divisor and multiply, which gives 6. So Mr. Wood can recover 6 chairs.

$$6\tfrac{3}{4} \div 1\tfrac{1}{8} = \frac{27}{4} \div \frac{9}{8} = \frac{\overset{3}{\cancel{27}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{9}}} = 6$$

Emphasize.

To divide by a mixed number, change the mixed number to an improper fraction; then invert the divisor and multiply.

Do ex. 2 as class activity to be sure all have good understanding of process.

Divide. Check your work by going over it:

2. $3\frac{3}{4} \div 1\frac{1}{8}$ $3\frac{1}{3}$ $\frac{3}{4} \div 1\frac{1}{8}$ $\frac{2}{3}$ $9 \div 1\frac{1}{2}$ 6 $12\frac{1}{2} \div 1\frac{1}{4}$ 10

3. $6\frac{1}{8} \div 5\frac{1}{4}$ $1\frac{1}{6}$ $\frac{7}{8} \div 1\frac{1}{4}$ $\frac{7}{10}$ $7 \div 9\frac{1}{3}$ $\frac{3}{4}$ $10\frac{2}{3} \div 2\frac{2}{3}$ 4

4. $4\frac{1}{2} \div 3\frac{3}{4}$ $1\frac{1}{5}$ $\frac{5}{8} \div 4\frac{1}{6}$ $\frac{3}{20}$ $4 \div 2\frac{1}{2}$ $1\frac{3}{5}$ $10\frac{1}{2} \div 1\frac{1}{6}$ 9

5. $4\frac{1}{2} \div 2\frac{1}{4}$ 2 $\frac{4}{5} \div 3\frac{1}{5}$ $\frac{1}{4}$ $\frac{5}{6} \div 5\frac{1}{3}$ $\frac{5}{32}$ $11\frac{1}{4} \div 1\frac{4}{5}$ $6\frac{1}{4}$

6. $3\frac{2}{3} \div 7\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{2} \div 2\frac{2}{3}$ $\frac{3}{16}$ $\frac{3}{8} \div 1\frac{7}{8}$ $\frac{1}{5}$ $12\frac{1}{2} \div 1\frac{7}{8}$ $6\frac{2}{3}$

7. $1\frac{7}{8} \div 1\frac{1}{4}$ $1\frac{1}{2}$ $\frac{7}{8} \div 1\frac{3}{4}$ $\frac{1}{2}$ $\frac{5}{6} \div 4\frac{1}{6}$ $\frac{1}{5}$ $10\frac{1}{2} \div 1\frac{3}{4}$ 6

More Practice. See ㉕ on page 333.

Emphasize that mixed numbers must be changed to improper fractions and that whole numbers must be written as fractions.

Give practice in division of fractions.

Making Stuffed Animals

1. The girls in the sewing class saw some stuffed animals in a toy store so they decided to make small stuffed animals to send to the Children's Home. They had 9 yd. of brown cloth which they used to make ponies. It took $\frac{3}{4}$ yd. of cloth for each pony. How many ponies could they make from 9 yd.? 12

2. They had $11\frac{1}{4}$ yd. of pink and white cloth which they used to make elephants. It took $1\frac{1}{4}$ yd. for each elephant. How many elephants did they make? 9

3. If Betty made 2 giraffes from a piece of brown and white cloth that was $2\frac{1}{2}$ yd. long, how many yards did she use for each giraffe? $1\frac{1}{4}$

Divide. **Check the work by going over it:**

4. $15 \div \frac{5}{6}$ 18 $\frac{1}{4} \div \frac{3}{8}$ $\frac{2}{3}$ $\frac{7}{8} \div 1\frac{5}{16}$ $\frac{2}{3}$ $16 \div 2\frac{2}{3}$ 6

5. $2\frac{1}{4} \div 6$ $\frac{3}{8}$ $\frac{2}{3} \div 4$ $\frac{1}{6}$ $6 \div 1\frac{1}{3}$ $4\frac{1}{2}$ $1\frac{7}{8} \div 8\frac{3}{4}$ $\frac{3}{14}$

6. $9\frac{1}{3} \div \frac{5}{6}$ $11\frac{1}{5}$ $\frac{4}{5} \div \frac{3}{10}$ $2\frac{2}{3}$ $\frac{3}{4} \div 2\frac{1}{2}$ $\frac{3}{10}$ $12 \div 1\frac{1}{2}$ 8

7. $2\frac{1}{2} \div \frac{2}{5}$ $6\frac{1}{4}$ $4 \div \frac{3}{5}$ $6\frac{2}{3}$ $\frac{4}{5} \div 12$ $\frac{1}{15}$ $6\frac{2}{3} \div 2\frac{2}{3}$ $2\frac{1}{2}$

8. $5\frac{1}{4} \div 6$ $\frac{7}{8}$ $\frac{9}{16} \div \frac{3}{8}$ $1\frac{1}{2}$ $\frac{7}{8} \div 8\frac{3}{4}$ $\frac{1}{10}$ $42 \div 2\frac{1}{10}$ 20

9. $1\frac{1}{2} \div \frac{9}{16}$ $2\frac{2}{3}$ $\frac{4}{5} \div 6$ $\frac{2}{15}$ $\frac{3}{4} \div 12$ $\frac{1}{16}$ $2\frac{1}{4} \div 2\frac{7}{10}$ $\frac{5}{6}$

These examples summarize all types of division of fractions. Papers should be checked carefully to see that all pupils are able to work each type. Reteach if necessary, emphasizing meanings.

Chapter Review

1. Using 3, 4, and 9, or any two of them, write each of the following: a mixed number, a proper fraction in lowest terms, a proper fraction not in lowest terms, an improper fraction that can be changed to a whole number, an improper fraction that can be changed to a mixed number.

2. Write a proper fraction and name its terms.

3. Change to lowest terms: $\frac{14}{16}$, $\frac{7}{8}$ $\frac{24}{32}$, $\frac{3}{4}$ $\frac{15}{48}$, $\frac{5}{16}$ $\frac{75}{100}$, $\frac{3}{4}$ $\frac{16}{80}$, $\frac{1}{5}$ $\frac{45}{90}$. $\frac{1}{2}$

4. Change to whole or mixed numbers: $\frac{17}{3}$, $5\frac{2}{3}$ $\frac{24}{6}$, 4 $\frac{25}{8}$, $3\frac{1}{8}$ $\frac{40}{16}$, $2\frac{1}{2}$ $\frac{75}{20}$, $3\frac{3}{4}$ $\frac{100}{40}$, $2\frac{1}{2}$ $\frac{100}{25}$, 4 $\frac{95}{10}$, $9\frac{1}{2}$ $\frac{19}{4}$. $4\frac{3}{4}$

5. Write 4 different fractions that each equal $\frac{2}{3}$.

6. Write these fractions in order of size, beginning with the largest: $\frac{7}{16}$, $\frac{3}{4}$, $\frac{3}{2}$, $\frac{11}{12}$, $\frac{2}{3}$, $\frac{5}{8}$. $\frac{3}{2}$, $\frac{11}{12}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{16}$

7. How much larger is $\frac{7}{8}$ than $\frac{5}{16}$? $\frac{9}{16}$

Find the answers. Watch the signs:

8. $2\frac{2}{3} + 1\frac{1}{6}$ $3\frac{5}{6}$ — $\frac{2}{3} \times \frac{5}{8}$ $\frac{5}{12}$ — $9\frac{2}{3} - 4\frac{1}{2}$ $5\frac{1}{6}$ — $5\frac{1}{4} \div \frac{7}{16}$ 12

9. $4\frac{1}{3} + 2\frac{3}{4}$ $7\frac{1}{12}$ — $\frac{4}{5} \div \frac{2}{3}$ $1\frac{1}{5}$ — $1\frac{4}{5} \times \frac{9}{10}$ $1\frac{31}{50}$ — $1\frac{3}{4} - \frac{7}{10}$ $1\frac{1}{20}$

10. $12 - 6\frac{3}{4}$ $5\frac{1}{4}$ — $\frac{3}{8} + \frac{5}{6}$ $1\frac{5}{24}$ — $2\frac{2}{3} \div 10$ $\frac{4}{15}$ — $\frac{15}{16} \times \frac{4}{5}$ $\frac{3}{4}$

11. $16 \times 1\frac{7}{8}$ 30 — $\frac{3}{4} \times 5$ $3\frac{3}{4}$ — $15 \div 1\frac{2}{3}$ 9 — $\frac{15}{16} \div 10$ $\frac{3}{32}$

12. $4\frac{1}{2} \div 3\frac{3}{4}$ $1\frac{1}{5}$ — $6 \div \frac{2}{3}$ 9 — $7\frac{1}{2} \times 1\frac{3}{5}$ 12 — $1\frac{1}{3} \times 5\frac{1}{4}$ 7

13. $3\frac{7}{8} + 2\frac{3}{4}$ $6\frac{5}{8}$ — $1 - \frac{5}{8}$ $\frac{3}{8}$ — $20 \div \frac{15}{16}$ $21\frac{1}{3}$ — $6\frac{2}{3} \times 45$ 300

14. $4\frac{3}{8} - 2\frac{5}{8}$ $1\frac{3}{4}$ — $\frac{3}{4} \div \frac{1}{8}$ 6 — $\frac{3}{10} \times 6\frac{1}{4}$ $1\frac{7}{8}$ — $2\frac{1}{4} - 1\frac{3}{4}$ $\frac{1}{2}$

15. $25 \div 3\frac{1}{3}$ $7\frac{1}{2}$ — $6 \times \frac{3}{8}$ $2\frac{1}{4}$ — $1\frac{2}{5} - \frac{9}{10}$ $\frac{1}{2}$ — $6\frac{7}{16} + \frac{9}{16}$ 7

16. $3\frac{1}{2} \times 12$ 42 — $\frac{5}{6} + \frac{3}{4}$ $1\frac{7}{12}$ — $\frac{9}{16} \div 4\frac{1}{2}$ $\frac{1}{8}$ — $14\frac{5}{6} - 2\frac{1}{3}$ $12\frac{1}{2}$

17. $6\frac{3}{4} - 4\frac{1}{2}$ $2\frac{1}{4}$ — $\frac{3}{4} - \frac{2}{3}$ $\frac{1}{12}$ — $4\frac{1}{3} + 2\frac{1}{2}$ $6\frac{5}{6}$ — $4\frac{1}{16} \div 1\frac{5}{8}$ $2\frac{1}{2}$

18. $2\frac{3}{5} + 1\frac{1}{2}$ $4\frac{1}{10}$ — $\frac{1}{8} \div \frac{5}{6}$ $\frac{3}{20}$ — $7\frac{1}{5} - 3\frac{1}{2}$ $3\frac{7}{10}$ — $2\frac{9}{10} + 1\frac{3}{4}$ $4\frac{13}{20}$

19. $1\frac{7}{8} + 1\frac{7}{8}$ $3\frac{3}{4}$ — $\frac{1}{2} \times \frac{3}{4}$ $\frac{3}{8}$ — $2\frac{3}{8} + 4\frac{1}{2}$ $6\frac{7}{8}$ — $7\frac{3}{16} - 4\frac{5}{8}$ $2\frac{9}{16}$

Check papers carefully and note kinds of errors. Return them to pupils so they may find and correct errors (under your supervision). Let others work on original problems, projects, and so on.

A Problem Test

1. Oranges are sold at 10¢ a pound, or at 59¢ for an 8-pound bag. If Mrs. Williams can use 8 lb. of oranges, how much will she save by purchasing an 8-pound bag? 21¢

2. Sam weighs $93\frac{1}{2}$ lb., Bill weighs $91\frac{3}{4}$ lb., and Joe weighs $89\frac{1}{4}$ lb. Find the average weight of these boys. $91\frac{1}{2}$ lb.

3. At 3 pencils for 20¢, find the cost of 9 pencils. 60¢ How many pencils can be bought for $1.00? 15

4. Three girls packed $8\frac{1}{4}$ lb. of candy in boxes holding $\frac{3}{4}$ lb. each. They sold it at $.75 a box and shared the money equally after paying $1.20 for expenses. Find each girl's share. $2.35

5. Mr. Hall drove 82 mi. in $2\frac{1}{2}$ hr. Find his average speed in miles per hour. $32\frac{4}{5}$

6. Grace is knitting a scarf that will be $1\frac{1}{2}$ yd. long when finished. The scarf now measures 2 ft. 8 in. How much longer must the scarf be? 1 ft. 10 in.

7. A farmer had 72 qt. of strawberries to sell at 55¢ a quart. He sold most of the berries at this price. To get rid of the last 15 qt. he sold them at 40¢ each. How much did all the berries bring? $37.35

8. Dick Reed found that his watch had gained 13 min. from 7 A.M. Wednesday until 9 A.M. Thursday. How many minutes had it gained per hour? $\frac{1}{2}$

9. Miss West earns $340 a month for 12 mo. Miss Banks earns $75 a week for 52 wk. Which one earns more per year and how much more? Miss West earns $180 more.

10. Yesterday Mr. Baker drove his car 265 mi. If he used 14 gal. of gasoline on this trip, how many miles did he average per gallon of gasoline? $18\frac{13}{14}$, or about 19

SCORE	0–5	6–7	8–9	10
	You need help	Fair	Good	Excellent

Remind pupils to record scores on own charts (page 41). Group pupils who had errors to help them discover causes (computational, wrong process, lack of understanding of problem situations, and so on).

Present diagnostic test of skills retaught
in Chapter 2 with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Add. Check the work by going over it:

						Practice Pages
1.	$\begin{array}{r} 6\frac{7}{8} \\ \underline{2\frac{5}{8}} \\ 9\frac{1}{2} \end{array}$	$\begin{array}{r} 9\frac{5}{6} \\ \underline{4\frac{2}{3}} \\ 14\frac{1}{2} \end{array}$	$\begin{array}{r} 1\frac{1}{2} \\ \underline{1\frac{1}{3}} \\ 2\frac{5}{6} \end{array}$	$\begin{array}{r} 2\frac{7}{8} \\ \underline{8\frac{5}{16}} \\ 11\frac{3}{16} \end{array}$	$\begin{array}{r} 3\frac{7}{10} \\ \underline{4\frac{3}{5}} \\ 8\frac{3}{10} \end{array}$	54, 55
2.	$\begin{array}{r} 2\frac{3}{4} \\ 1\frac{1}{6} \\ \underline{2\frac{1}{2}} \\ 6\frac{5}{12} \end{array}$	$\begin{array}{r} 3\frac{1}{3} \\ 4\frac{1}{4} \\ \underline{7\frac{1}{6}} \\ 14\frac{3}{4} \end{array}$	$\begin{array}{r} 5\frac{7}{8} \\ 1\frac{5}{6} \\ \underline{2\frac{1}{4}} \\ 9\frac{23}{24} \end{array}$	$\begin{array}{r} 8\frac{3}{4} \\ 4\frac{1}{5} \\ \underline{2\frac{3}{10}} \\ 15\frac{1}{4} \end{array}$	$\begin{array}{r} 6\frac{5}{6} \\ 5\frac{1}{2} \\ \underline{2\frac{3}{5}} \\ 14\frac{14}{15} \end{array}$	61, 62

Subtract. Check the work by going over it:

3.	$\begin{array}{r} 3\frac{5}{6} \\ \underline{1\frac{1}{6}} \\ 2\frac{2}{3} \end{array}$	$\begin{array}{r} 9\frac{4}{5} \\ \underline{5\frac{1}{2}} \\ 4\frac{3}{10} \end{array}$	$\begin{array}{r} 7\frac{1}{2} \\ \underline{2\frac{3}{16}} \\ 5\frac{5}{16} \end{array}$	$\begin{array}{r} 8\frac{11}{12} \\ \underline{5\frac{1}{6}} \\ 3\frac{3}{4} \end{array}$	$\begin{array}{r} 6\frac{3}{4} \\ \underline{3\frac{5}{8}} \\ 3\frac{1}{8} \end{array}$	56
4.	$\begin{array}{r} 7 \\ \underline{2\frac{5}{8}} \\ 4\frac{3}{8} \end{array}$	$\begin{array}{r} 2\frac{1}{2} \\ \underline{1\frac{7}{10}} \\ \frac{4}{5} \end{array}$	$\begin{array}{r} 9\frac{1}{2} \\ \underline{3\frac{5}{6}} \\ 5\frac{2}{3} \end{array}$	$\begin{array}{r} 4\frac{3}{4} \\ \underline{2\frac{11}{12}} \\ 1\frac{5}{6} \end{array}$	$\begin{array}{r} 3\frac{9}{16} \\ \underline{\frac{7}{8}} \\ 2\frac{11}{16} \end{array}$	58, 59

Multiply and check the work:

5.	$\frac{3}{4} \times 17$ $12\frac{3}{4}$	$5\frac{1}{3} \times 12$ 64	$9 \times \frac{5}{12}$ $3\frac{3}{4}$	63, 64
6.	$\frac{5}{6} \times \frac{5}{6}$ $\frac{25}{36}$	$\frac{15}{16} \times \frac{2}{3}$ $\frac{5}{8}$	$\frac{9}{10} \times \frac{5}{6}$ $\frac{3}{4}$	65, 66
7.	$\frac{5}{6} \times 1\frac{1}{5}$ 1	$\frac{7}{8} \times 5\frac{1}{3}$ $4\frac{2}{3}$	$3\frac{1}{3} \times \frac{5}{6}$ $2\frac{7}{9}$	67
8.	$7\frac{1}{2} \times \frac{4}{5}$ 6	$4\frac{2}{5} \times 7\frac{1}{2}$ 33	$2\frac{1}{4} \times 4\frac{2}{3}$ $10\frac{1}{2}$	67

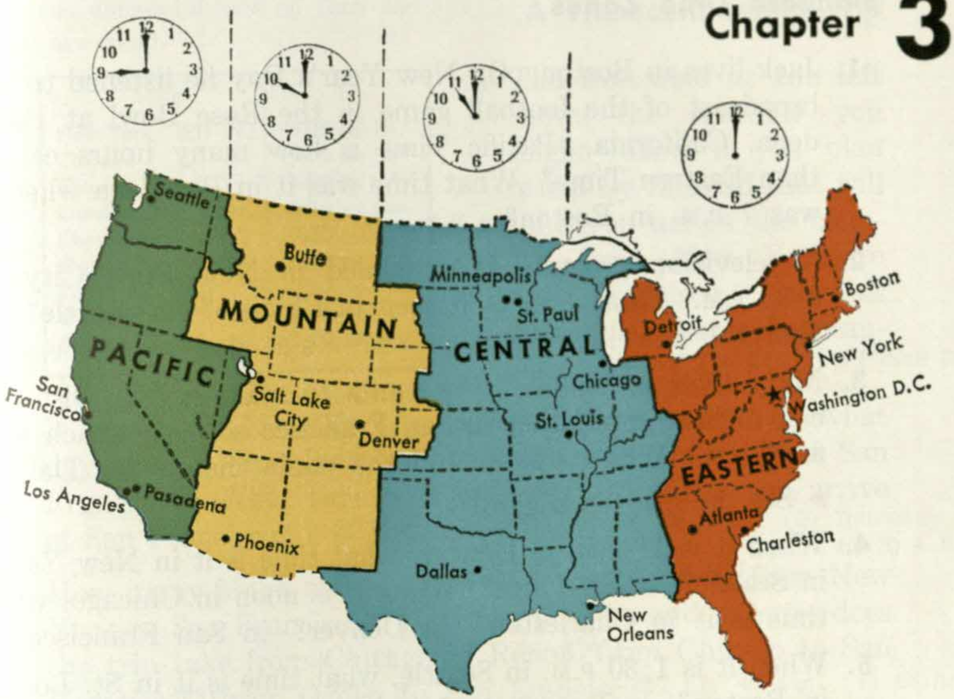
Divide and check the work:

9.	$9 \div \frac{1}{3}$ 27	$\frac{15}{16} \div \frac{5}{6}$ $1\frac{1}{8}$	$\frac{3}{4} \div \frac{5}{6}$ $\frac{9}{10}$	72, 73
10.	$1\frac{1}{2} \div \frac{2}{3}$ $2\frac{1}{4}$	$2\frac{2}{3} \div \frac{5}{12}$ $6\frac{2}{5}$	$7\frac{1}{2} \div \frac{3}{10}$ 25	74
11.	$\frac{5}{8} \div 10$ $\frac{1}{16}$	$\frac{3}{4} \div 1\frac{1}{8}$ $\frac{2}{3}$	$12 \div 1\frac{1}{3}$ 9	75, 76
12.	$6\frac{2}{3} \div 15$ $\frac{4}{9}$	$4\frac{1}{2} \div 1\frac{1}{2}$ 3	$1\frac{5}{16} \div 1\frac{1}{2}$ $\frac{7}{8}$	75, 76

All work on common fractions is included in this test. After checking papers, help pupils determine causes of difficulties. Reteach or re-explain as needed before assigning remedial work.

Explain and give practice on the time zones of the United States (pages 81-82).

Chapter 3



Standard Time Zones

1. The United States, not including Alaska and Hawaii, is divided into the 4 standard time zones shown on the map above. Cities in the eastern part of the United States, such as Boston and Atlanta, all use Eastern Time (ET). Chicago and other cities in the Central States use Central Time (CT), which is 1 hr. earlier than Eastern Time. When it is 7 P.M. in Chicago, what time is it in Boston? 8 P.M.
2. Denver and Salt Lake City are in the zone that uses Mountain Time (MT), which is 1 hr. earlier than Central Time, and 2 hr. earlier than Eastern Time. When it is 3 P.M. in Denver, what time is it in Chicago? in Boston?
   4 P.M. 5 P.M.
3. Los Angeles, San Francisco, and Seattle are in the zone that uses Pacific Time (PT), which is 1 hr. earlier than Mountain Time. When it is 9 A.M. in San Francisco, what time is it in Denver? in Chicago? in Boston? in Seattle?
   10 A.M. 11 A.M. 12 noon 9 A.M.

Discuss map with pupils and do ex. 1-3 orally. Make up similar questions about other cities on the map.

Standard Time Zones

1. Jack lives in Boston. On New Year's Day he listened to the broadcast of the football game in the Rose Bowl at Pasadena, California. Pacific Time is how many hours earlier than Eastern Time? 3 What time was it in Pasadena when it was 7 P.M. in Boston? 4 P.M.

2. A television program is scheduled in New York City at 9:30 P.M. What time is it seen in Chicago? 8:30 P.M. in Seattle? 6:30 P.M. in Denver? 7:30 P.M.

3. Mary lives in Dallas. She wants to make a long distance call to her grandfather in San Francisco so as to reach him at 8 P.M. What time should she make the call in Dallas? 10 P.M.

   ▶ See the map on page 81.

4. When it is 11 A.M. in Denver, what time is it in New York? 1 P.M. in Seattle? 10 A.M. in St. Louis? 12 noon When it is noon in Chicago, what time is it in Charleston? 1 P.M. in Denver? 11 A.M. in San Francisco? 10 A.M.

5. When it is 1:30 P.M. in Seattle, what time is it in St. Louis? 3:30 P.M. in Boston? 4:30 P.M. in Butte? 2:30 P.M. in Los Angeles? 1:30 P.M.

6. Time in London is 5 hours later than Eastern Time. When it is 9 A.M. in New York, what time is it in London? 2 P.M.

7. At what time could a radio broadcast in London at 6 P.M. be heard in New York? 1 P.M. in St. Louis? 12 noon in San Francisco? 10 A.M.

8. Time in Honolulu and in Anchorage, Alaska, is 2 hours earlier than Pacific Time. When it is 7 A.M. in Honolulu and Anchorage, what time is it in Seattle? 9 A.M. in Chicago? 11 A.M. in New York? 12 noon

9. During the summer many cities and towns use Daylight-Saving Time. This kind of time is one hour ahead of Standard Time. When it is 8 A.M. Daylight-Saving Time, it is 7 A.M. Standard Time in any time zone. What are the advantages of Daylight-Saving Time? the disadvantages? Answers will vary.

10. When it is 12:30 P.M. Daylight-Saving Time, what time is it by Standard Time? 11:30 A.M.

11. A train is scheduled to arrive at 10:30 A.M. Standard Time. What time should it arrive by Daylight-Saving Time? 11:30 A.M.

Let pupils use map on page 81 in discussing some of these examples. Have more advanced pupils investigate and report (illustrate on maps and globe) on time zones around the world. Supplement ex. 1-11 with extra questions as needed.

Teach pupils how to interpret and use timetables which use standard time zones. Have pupils bring in timetables of transcontinental railroads or airlines. Discuss abbreviations on them and how they are read.

A Transcontinental Trip

Station			Time
New York	(ET)	Lv.	6:00 PM Fri.
Chicago	(CT)	Ar.	9:00 AM Sat.
Chicago	"	Lv.	6:00 PM "
Omaha	"	Ar.	2:20 AM Sun.
Cheyenne	(MT)	"	9:20 AM "
Ogden	"	"	6:40 PM "
Reno	(PT)	"	3:30 AM Mon.
Oakland	"	"	10:50 AM "
S. Francisco	"	Ar.	11:20 AM Mon.

1. The timetable at the left shows the schedule you might follow if you plan a trip by railroad that will take you across the country from New York to San Francisco. On this timetable, what is the meaning of ET? CT? MT? PT? See Guide.

Have pupils make similar timetables for trip across United States.

2. If you leave New York on a Monday afternoon, on what day and at what hour will you arrive at Chicago?(1) at San Francisco?(2) What time is it in New York when you arrive at San Francisco? 2:20 P.M. (1) Tuesday at 9:00 A.M.; (2) Thursday at 11:20 A.M.

3. How many hours and minutes does the trip take from New York to San Francisco?(1) How many hours and minutes does the trip take from Chicago to Reno?(2) from Chicago to San Francisco?(3) from Ogden to San Francisco? 17 hr. 40 min. (1) 68 hr. 20 min; (2) 35 hr. 30 min.; (3) 43 hr. 20 min.

4. On the trip from New York to San Francisco, will you turn the hands of your watch forward or backward, and how much, as you enter each new time zone? Backward, 1 hr. How many times will you have to set your watch on this trip? 3

5. When you return to New York from San Francisco, how will you set your watch as you enter each new time zone? Ahead 1 hr.

6. In ex. 5, suppose your watch is set at Pacific Time when you leave San Francisco and you do not set your watch again until you reach New York. When you reach New York, will your watch be fast or slow and how much? 3 hr. slow

7. An airplane flew nonstop from California to New York, leaving California at 8 A.M. and arriving at New York at 3:35 P.M. How many hours of flying did the trip take? 4 hr. 35 min.

8. A train leaves Chicago at 7 P.M. on Wednesday and arrives at Los Angeles $39\frac{1}{2}$ hr. later. On what day and at what hour does it arrive at Los Angeles? Friday at 8:30 A.M.

Emphasize fact that total elapsed time in some cases is more less (depending on direction) than difference between arrival and departure times by 1, 2, or 3 hours, depending on zones involved.

Review and reteach meaning of decimals (pages 84–85). Emphasize that decimal fractions are another way of writing common fractions whose denominators are 10 or powers of 10.

Meaning of Decimals

1. The common fractions $\frac{3}{10}$, $\frac{49}{100}$, and $\frac{721}{1000}$, which have denominators of 10, 100, and 1000, may be written as decimals. The fraction $\frac{3}{10}$ may be written as the decimal .3; in this case only the numerator 3 of the fraction is written and a decimal point is placed before it. When the denominator of a fraction is 10, the decimal has only 1 place. Stress.

Stress.

In the same way, $\frac{49}{100}$ may be written as the decimal .49. When the denominator is 100, the decimal has 2 places. Stress. The fraction $\frac{731}{1000}$ may be written .731. Point out that decimal means decimal fraction.

2. When a fraction is written as a decimal, the decimal must have as many places to the right of the decimal point as there are zeros in the denominator of the fraction. For example, to write $\frac{7}{100}$ as a decimal, you must place a zero before 7 to give it two places; then $\frac{7}{100}$ becomes .07. Likewise, $\frac{7}{1000}$ becomes .007 when written as a decimal. Emphasize concept in first sentence in ex. 2 through more examples.

3. To read a decimal, read it as a whole number and then give it the name of the place of its last figure. For example, .9 is read 9 **tenths** because 9 is in tenths place; .53 is read 53 **hundredths** because 3 is in hundredths place; and .006 is read 6 **thousandths** because 6 is in thousandths place.

4. The number 34.708, which consists of a whole number and a decimal, is called a **mixed decimal.** It means $34\frac{708}{1000}$. Sometimes 34.708 is read **three-four-point-seven-o-eight.** Zero is read like the letter o. Stress.

Stress.

5. The decimal .5 may also be written 0.5. When there is no whole number, a zero is often written to the left of the decimal point to call attention to the point.

Have pupils write numbers in ex. 6–8 as common fractions and mixed numbers.

Read these numbers:

4 tenths 176 thousandths 1 thousandth 5 and 385 thousandths

6. .4 .79 .176 .06 .001 1.46 5.385 2.009

79 hundredths 6 hundredths 1 and 46 hundredths 2 and 9 thousandths

7. .8 0.4 0.19 .74 1.86 29.1 7.004 3.204

See Guide for ex. 7–8.

8. 1 0.6 0.05 .69 6.25 30.6 1.025 5.875

Read some fractions, such as "thirteen thousandths," and so on, and have pupils write them as common and decimal fractions.

More about Decimals

1. You know the names of the first 3 decimal places. The number below has 6 decimal places. Learn the names of all 6 decimal places, starting at the decimal point and reading toward the right. For the names of the places in whole numbers, start at the decimal point and read toward the left.

Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
5,	2	8	1,	6	0	3,	4	9	7	.	4	2	5	9	1	8

The number above is read 5 billion, 281 million, 603 thousand, 497 **and** 425 thousand, 918 millionths. The decimal Stress. point is read **and.** Any decimal is read as a whole number and then given the name of the place of its last figure; for example, .076 is read 76 **thousandths** because 6 is in **thousandths** place; .000307 is read 307 **millionths.**

Give further practice like that in ex. 2-4.

Read each decimal below as explained in ex. 1: See Guide.

2. .3725 .18875 .879043 2793.46718

3. .0932 .00614 .405916 719.322761

4. .0008 .00045 .600172 400.260004

Have pupils read numbers in ex. 5-6 first, be sure they say "tenths," "hundredths," and so on.

Write each number as a decimal or a mixed decimal:

5. $\frac{6354}{10000}$ .6354 $\frac{571824}{1000000}$ .571824 $\frac{3}{1000}$ .003 $\frac{718}{100000}$ .00718 $\frac{586}{100}$ 5.86 $\frac{92847}{10000}$ 9.2847

6. $\frac{216}{10000}$ .0216 $\frac{39476}{1000000}$ .039476 $\frac{27}{1000}$ .027 $\frac{48175}{100000}$ .48175 $\frac{379}{100}$ 3.79 $\frac{84309}{10000}$ 8.4309

Write each of the following as a decimal or a mixed decimal:

7. Two and seven hundred fourteen millionths. 2.000714

8. Five thousand six hundred twelve ten-thousandths. .5612

9. Eighteen and four hundred twenty-five thousandths. 18.425

Have pupils make own place value chart, as in ex. 1, to use to read and write other decimal fractions and mixed decimals. Give further practice as needed.

Review and reteach place value. Carefully develop use of exponents to tell how many times 10 is a factor. After meaning is clear, extend to other factors.

Place Value

1. Study the numbers below:

A. In 3000.07 the 7 stands for .07 or 7 hundredths
B. In 3000.7 the 7 stands for .7 or 7 tenths
C. In 3007 the 7 stands for 7 or 7 ones
D. In 3070 the 7 stands for 70 or 7 tens
E. In 3700 the 7 stands for 700 or 7 hundreds

In **B** the value of the 7 is 10 times as great as in **A**; in **C** the value of the 7 is 10 times as great as in **B**. You see that the value of 7 becomes 10 times as great if you move it one place to the left.

2. Our system of writing numbers has *place value* because the value of each digit depends upon the place it occupies.

If any digit of a number is moved one place to the left, its value is multiplied by ten.

3. The values of the first five places in whole numbers are:

ten thousand	thousand	hundred	ten	one
10,000	1000	100	10	1
$10 \times 10 \times 10 \times 10$	$10 \times 10 \times 10$	10×10	10	1

These values are often written by using an **exponent**, which is a small number placed as shown below:

$$10 \times 10 \times 10 \times 10 = 10^4 \qquad 10 \times 10 = 10^2$$
$$10 \times 10 \times 10 = 10^3 \qquad 10 = 10^1$$

The exponent tells how many factors of 10 are in the product. 10^4 means a product of four 10's and is read "ten to the fourth power." Ten to the first power is usually written without the exponent 1.

4. Give the meaning of these numbers: 10^5(1) 5^2(2) 7^3(3) 4^4(4)
(1) 10 x 10 x 10 x 10 x 10; (2) 5 x 5; (3) 7 x 7 x 7; (4) 4 x 4 x 4 x 4

5. The value of 7324 can be expressed with exponents this way:

$$7324 = 7 \text{ thousands} + 3 \text{ hundreds} + 2 \text{ tens} + 4 \text{ ones}$$
$$7324 = (7 \times 10 \times 10 \times 10) + (3 \times 10 \times 10) + (2 \times 10) + (4)$$
$$7324 = (7 \times 10^3) + (3 \times 10^2) + (2 \times 10^1) + (4)$$

6. Write the values of these numbers using exponents: See Guide.

485 6946 9247 23,843

In ex. 1 point out that if a digit is moved one place to the right, it has only 1/10 of its previous value. Compare place values when digits are more than one place apart. Compare different place values using exponents.

Present first set of improvement tests in division.

Improving by Practice

Division Test 1a.

Time: 5 min. after copying.

1. $\overset{528}{76\overline{)40128}}$ $\overset{931\ R8}{62\overline{)57730}}$ $\overset{224}{39\overline{)8736}}$

2. $\overset{583\ R11}{43\overline{)25080}}$ $\overset{446}{55\overline{)24530}}$ $\overset{879}{84\overline{)73836}}$ ⑥

Division Test 1b.

Time: 5 min. after copying.

3. $\overset{959}{58\overline{)55622}}$ $\overset{456\ R9}{99\overline{)45153}}$ $\overset{227}{23\overline{)5221}}$

4. $\overset{757}{78\overline{)59046}}$ $\overset{731\ R16}{64\overline{)46800}}$ $\overset{487}{32\overline{)15584}}$ ⑥

Division Test 1c.

Time: 5 min. after copying.

5. $\overset{993\ R28}{47\overline{)46699}}$ $\overset{230}{62\overline{)14260}}$ $\overset{662}{86\overline{)56932}}$

6. $\overset{387\ R14}{34\overline{)13172}}$ $\overset{889}{75\overline{)66675}}$ $\overset{785}{98\overline{)76930}}$ ⑥

Remind pupils to record results in Score Books (page 50).

Mixed Practice

Present mixed review of computational skills. Determine causes of errors through individual conferences.

Find the answers:

7. \$7.28 + \$9.72 \$17.00 — 339,336 ÷ 54 6284 — $9 - 2\frac{3}{4}$ $6\frac{1}{4}$
8. 329 × 4207 1,384,103 — $\frac{3}{4} + \frac{5}{8} + \frac{11}{16}$ $2\frac{1}{16}$ — $\frac{7}{8} \times 20$ $17\frac{1}{2}$
9. 36,356 ÷ 61 596 — 6290 × 8337 52,439,730 — $1\frac{1}{2} \times 1\frac{1}{4}$ $1\frac{7}{8}$
10. 9000 − 2994 6006 — \$36.75 − \$9.98 \$26.77 — $15 \div 3\frac{1}{3}$ $4\frac{1}{2}$
11. 15,259 ÷ 76 200 R59 — 165,015 ÷ 171 965 — $5\frac{3}{5} - 2\frac{1}{10}$ $3\frac{1}{2}$
12. 19,590 ÷ 247 79 R77 — $\frac{7}{12} + \frac{5}{6} + \frac{1}{4}$ $1\frac{2}{3}$ — $3\frac{3}{4} \div 1\frac{7}{8}$ 2
13. 735 × 9238 6,789,930 — \$26.33 + \$3.67 \$30.00 — $7\frac{1}{12} - 3\frac{3}{4}$ $3\frac{1}{3}$
14. 8007 − 2699 5308 — 408,865 ÷ 797 513R4 — $2\frac{2}{3} + 1\frac{3}{4}$ $4\frac{5}{12}$
15. 11,814 ÷ 29 407 R11 — $\frac{3}{4} + \frac{5}{8} + \frac{5}{6}$ $2\frac{5}{24}$ — $4\frac{1}{6} \div \frac{5}{12}$ 10

To the Teacher. In ex. 1–6, pupils should write R before remainders, if any.

Note that division examples should be copied first, before test is begun. Method of giving, scoring, and recording tests is same as for others (see pages 46-50, and 344). Group pupils for reteaching.

To summarize the understanding of the Roman Numeral System have each pupil make his own chart for rapid conversion to Roman numerals.

The Roman Numeral System

1. The symbols used in the Roman Numeral System are I, V, X, L, C, D, M, and a bar. A bar written over a Roman numeral multiplies it by 1000. Thus $\overline{V} = 5000$. What is $\overline{X}$? The Roman numerals from 1 to 9 are shown in column 4; those for tens from 10 to 90 in column 3; those for hundreds from 100 to 900 in column 2; and those for thousands from 1000 to 9000 in column 1. The numerals in columns 3 and 4 make all the Roman numerals from 1 to 99. Write in Roman numerals:

1	2	3	4
M	C	X	I
MM	CC	XX	II
MMM	CCC	XXX	III
$\overline{IV}$	CD	XL	IV
$\overline{V}$	D	L	V
$\overline{VI}$	DC	LX	VI
$\overline{VII}$	DCC	LXX	VII
$\overline{VIII}$	DCCC	LXXX	VIII
$\overline{IX}$	CM	XC	IX

32	74	92	89	36	55	30
XXXII	LXXIV	XCII	LXXXIX	XXXVI	LV	XXX

2. The primary symbols are those in the first row: I, X, C, M. The secondary symbols are in the fifth row: V, L, D. The symbol I is written before V and X and subtracted from them. Thus IV = 4 and IX = 9. In like manner X is written before L and C, and C is written before D and M. These are the only places where this subtractive principle is used. What is XC? CD?
90 400

3. The Roman numerals in these four columns make all the Roman numerals from 1 to 9999. To write 3947 in Roman numerals, first select the symbol for thousands, MMM; then the symbol for hundreds, CM; then the symbol for tens, XL; and finally the symbol for ones, VII. These symbols are then written from left to right:

3947 = MMM + CM + XL + VII = MMMCMXLVII

Notice that when you write 3947 in Roman numerals you write, as in our system, thousands first, then hundreds, tens, and ones from left to right. The Roman system, however, does not have place value. Write in Roman numerals:

704	490	1382	43,498	345,678
DCCIV	CDXC	MCCCLXXXII	$\overline{XLIII}$CDXCVIII	$\overline{CCCXLV}$DCLXXVIII

It is important to emphasize that Roman numerals do not have place value such as in our decimal system. This is the first opportunity to analyze a system radically different from the decimal system. Other systems do exist.

Reteach addition of decimals (with and without same name) pages 89–90.

Adding Decimals

Review fact that only fractions with like denominators can be added.

1. The depth of snow is measured in **tenths** of an inch. In three storms one week, these amounts of snow fell: 3.4 in., 2.8 in., 4.0 in. How many inches of snow fell that week? 10.2

▶ When you add decimals, be sure to keep the decimal points in a line under one another. The decimal point in the answer must be kept in line also.

3.4
2.8
4.0

2. **Problem** Bob bought a bat for $2, a baseball for $1.57, a glove for $6, and shoes for $5.75. How much did he spend?

Explanation You can write the amounts in this problem as at A and add them, which gives $15.32. It may be easier, however, for you to add these amounts if you write a decimal point and two zeros after $2 and $6 as at B. When you have an addition example in decimals where the numbers do not all have the same number of decimal places, you must keep the decimal points under one another.

A	B
$2.	$2.00
1.57	1.57
6.	6.00
5.75	5.75
$15.32	$15.32

When you add decimals that represent measurements they should all be to tenths, or all to hundredths, and so on. Emphasize.

Copy in columns, add, and check:

3. 6.3 + 9.2 + 8.4 + 0.7 + 4.5 + 3.3 + 2.8   35.2

4. 9.25 + 14.28 + 6.37 + 1.75 + 4.50 + 10.16   46.31

Remind pupils to copy carefully and keep columns straight in ex. 3–4.

Emphasize that in addition of whole numbers and decimals, decimal point should be written to right of each whole number.

Problems and Practice

1. Mr. Lewis wants to drive to Chicago tomorrow by way of Readville and Columbus. He knows it is 18.5 mi. from his house to Readville, 39.4 mi. from Readville to Columbus, and 78.2 mi. from Columbus to Chicago. How many miles will the trip to Chicago be? 136.1

2. If the speedometer on Mr. Lewis's car registers 8024.3 mi. just before he leaves for Chicago, what should it register when he arrives in Chicago? See ex. 1. 8160.4

3. Joe's cow gave 38.6 lb. of milk on Friday, 42.6 lb. on Saturday, and 39.8 lb. on Sunday. Find the total number of pounds of milk the cow gave in three days. 121

Have each pupil make up problem involving addition of decimals, for partners to solve.

Add. Check the work by going over it:

4.	2.76	9.40	11.6	31.8	6.333	22.476
	.45	3.20	9.4	27.1	2.500	50.107
	6.67	.88	18.2	20.5	1.395	15.245
	5.74	5.14	7.3	49.0	7.250	19.018
	15.62	18.62	46.5	128.4	17.478	106.846
5.	19.5	5.34	7.98	26.1	5.500	31.128
	31.8	.26	1.67	15.4	9.450	10.411
	12.9	1.53	4.40	7.6	2.667	6.025
	4.3	.50	.85	2.7	.375	11.600
	10.1	.75	1.60	12.8	.625	8.750
	8.6	9.15	2.25	10.0	6.000	15.625
	87.2	17.53	18.75	74.6	24.617	83.539

Copy in columns, add and check:

6. 2.6 + 5.2 + 3.6 + 8.4 + 7.5 + 1.7 + 4.3 33.3

7. 1.3 + 2.8 + 5.6 + 4.2 + 1.5 + 6.9 + 0.4 + 3.5 26.2

8. 2.125 + 4.375 + 6.333 + 4.875 + .625 + 1.125 19.458

9. \$7 + \$2.12 + \$6.80 + \$9.50 + \$11.75 + \$2 + \$7.37 \$46.54

10. .375 + 4.5 + 10.25 + 8.75 + .625 + 3.5 + 6.125 34.125

11. 4.17 + 3.89 + 12.44 + 7.13 + 5.72 + .77 34.12

More Practice. See ㉖ on page 334. Use to increase skill. Have pupils do ex. 1–3 independently. Then discuss solutions with class. Ex. 4–11 should also be done as independent work. Let different pupils explain solutions at board.

Subtracting Decimals

1. Problem On a timetable Jim found that the distance from Chicago to Fort Johnson is 232.9 mi., while the distance from Chicago to Center City, which is farther along on the same route, is 451.1 mi. How far is it from Fort Johnson to Center City? 218.2 mi.

Explanation To find the distance between Fort Johnson and Center City, you subtract as shown at the right. Put the decimal point in the answer under the decimal points above. To check the work add 218.2 and 232.9.

451.1
232.9
218.2

Emphasize importance of checking carefully.

2. Maria gave the clerk $10 when she paid for a dress that cost $5.98. How much change did she get? $4.02

▶ You can write the numbers as shown at A, but the work is easier if you place two zeros after $10., as shown at B.

A	B
$10.	$10.00
5.98	5.98
	$4.02

Subtract and check the work:

3.	11.6 9.8 1.8	1.25 .38 .87	12.25 10.78 1.47	28.667 18.875 9.792	625.438 109.875 515.563
4.	6.53 5.88 .65	3.50 2.75 .75	9.442 8.716 .726	306.50 198.75 107.75	100.375 89.625 10.750
5.	31.5 16.5 15.0	45.8 36.3 9.5	5.556 2.818 2.738	46.324 17.022 29.302	320.180 119.500 200.680
6.	5.33 2.67 2.66	21.0 15.6 5.4	112.5 38.7 73.8	31.000 25.375 5.625	$4525.50 2618.49 $ 1907.01
7.	15.8 11.6 4.2	6.80 3.85 2.95	17.20 13.75 3.45	8.0000 5.4375 2.5625	$2000.00 1297.35 $ 702.65
8.	9.81 4.18 5.63	17.6 15.8 1.8	1.000 .875 .125	416.38 227.85 188.53	$9800.00 4209.31 $ 5590.69

More Practice. See ㉗ on page 334.

Ex. 1–2 should be discussed carefully with pupils, stressing how and why they may change 1 one to 10 tenths and 1 tenth to 10 hundredths. If there are no apparent difficulties, have pupils complete ex. 3–8, and explain some examples.

Mr. Wood Keeps Cows

1. Mr. Wood has 10 cows and keeps a daily record of the amount of milk each cow gives. Last week Dot gave these amounts: 28.3 lb., 25.6 lb., 26.5 lb., 29.3 lb., 30.2 lb., 29.1 lb., 27.9 lb. How many pounds of milk did Dot give last week? 196.9

2. Daisy gave these numbers of pounds of milk last week: 32.5 lb., 33.0 lb., 31.9 lb., 32.6 lb., 30.8 lb., 34.4 lb., 32.9 lb. How many pounds of milk did Daisy give? 228.1

3. How many more pounds of milk did Daisy give last week than Dot? See ex. 1 and 2. 31.2

4. Last week the 10 cows together gave these amounts of milk daily: 312.6 lb., 306.2 lb., 309.3 lb., 313.1 lb., 308.7 lb., 307.5 lb., 311.2 lb. Find the total number of pounds for the week and the average number of pounds per day. 309.8 2168.6

5. It takes 8.6 lb. of milk to make 1 gallon of milk. About how many gallons of milk did the 10 cows together average per day? See ex. 4. 36 R2, or about 36

▶ You can divide 309.8 by 8.6 by dividing 3098 by 86.

6. An empty quart milk bottle weighs 1.12 lb. A quart of milk weighs 2.15 lb. Find the weight of the bottle filled with milk. 3.27 lb.

Scales on dairy farms do actually show weight to tenths of a pound. After discussing solutions to ex. 1-6, have pupils make up original problems involving addition and subtraction of decimals.

Changing Decimals to Fractions

1. **Problem** Ann read in the newspaper that .75 in. of rain fell during yesterday's storm. Ann says that this is the same as $\frac{3}{4}$ in. of rain. Is Ann right? Yes

Explanation You can show that $.75 = \frac{3}{4}$ by writing .75 as $\frac{75}{100}$ and changing $\frac{75}{100}$ to lowest terms. If you divide both numerator and denominator of $\frac{75}{100}$ by 25, you get $\frac{3}{4}$. You should remember that $.75 = \frac{3}{4}$ because you will often use this fact.

2. Change .25 and .50 each to fractions. $\frac{1}{4}$, $\frac{1}{2}$

3. Change .325 to a fraction. $\frac{13}{40}$

▶ Write .325 as $\frac{325}{1000}$ and change the fraction to lowest terms. The result is $\frac{13}{40}$. So $.325 = \frac{13}{40}$.

4. **Problem** Change $.66\frac{2}{3}$ to a fraction. $\frac{2}{3}$

Explanation First write $.66\frac{2}{3}$ as $\dfrac{66\frac{2}{3}}{100}$. You then multiply the numerator of $\dfrac{66\frac{2}{3}}{100}$ by a number that will give a whole number in the new numerator. If you multiply $66\frac{2}{3}$ by 3, you get 200. Then you must multiply 100 by 3, which gives 300. So $\dfrac{66\frac{2}{3}}{100} = \dfrac{200}{300} = \dfrac{2}{3}$.

Change these decimals to fractions: Stress changing to lowest terms.

5. .45 $\frac{9}{20}$ .80 $\frac{4}{5}$ $.12\frac{1}{2}$ $\frac{1}{8}$ .875 $\frac{7}{8}$ .625 $\frac{5}{8}$ .375 $\frac{3}{8}$
6. .32 $\frac{8}{25}$ .42 $\frac{21}{50}$ $.16\frac{2}{3}$ $\frac{1}{6}$ .275 $\frac{11}{40}$ .150 $\frac{3}{20}$ $.33\frac{1}{3}$ $\frac{1}{3}$
7. .76 $\frac{19}{25}$ .95 $\frac{19}{20}$ $.83\frac{1}{3}$ $\frac{5}{6}$ .425 $\frac{17}{40}$ .225 $\frac{9}{40}$ $.87\frac{1}{2}$ $\frac{7}{8}$
8. .40 $\frac{2}{5}$ .55 $\frac{11}{20}$ $.37\frac{1}{2}$ $\frac{3}{8}$ .125 $\frac{1}{8}$ .450 $\frac{9}{20}$ $.62\frac{1}{2}$ $\frac{5}{8}$

9. Remember these decimals and their fractional values: Following should be memorized after understanding is assured.

$.25 = \frac{1}{4}$ $.12\frac{1}{2} = \frac{1}{8}$ $.87\frac{1}{2} = \frac{7}{8}$ $.16\frac{2}{3} = \frac{1}{6}$
$.50 = \frac{1}{2}$ $.37\frac{1}{2} = \frac{3}{8}$ $.33\frac{1}{3} = \frac{1}{3}$ $.83\frac{1}{3} = \frac{5}{6}$
$.75 = \frac{3}{4}$ $.62\frac{1}{2} = \frac{5}{8}$ $.66\frac{2}{3} = \frac{2}{3}$ $1.00 = 1$

First review that number of places to right of decimal point tells denominator of the fraction and that the number to right of decimal point makes up the numerator. Then follow development as given.

Have actual objects for counting and grouping. Group by tens first and then start over and group by fives. Considerable practice in grouping aids understanding.

Number Bases and Place Value

1. Our number system is called the **decimal system** because it uses groups of ten. You have learned that in our number system the value of a digit depends upon its position or place. You also know that the value of each place is ten times the value of the place on its right. In the number 3124.89, the 4 represents 4 ones, the 2 represents 2 tens, and the 8 represents 8 tenths. What do 1,(1) 3,(2) and 9(3) represent?
(1) 1 hundred; (2) 3 thousands; (3) 9 hundredths

2. When you write numbers in the decimal system you use **only ten** distinct symbols. These symbols are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. With these symbols you can write all numbers. In counting, ten follows nine. For ten you have no digit. So you make a group of ten ones and write 10, which means 1 group of ten ones and no ones. 24 means 2 groups of ten ones and 4 ones. 376 means 3 hundreds, 7 tens, and 6 ones. Notice that one hundred is a group of ten tens. Give the meaning of: See ans. in Guide

348 1648 873.68 20,000 870.089

3. You can build a number system using only the symbols 0, 1, 2, 3, and 4. For one you can write 1, for two write 2, for three write 3, and for four write 4. For five you have no digit to use. So you make a group of five ones and write 10. In this new system 10 means 1 group of five ones and no ones, 13 means 1 group of five ones and 3 ones, and 42 means 4 groups of five ones and 2 ones. The value of each place is five times the value of the place on its right. So the 3 in 342 means 3 twenty-fives, since five times five is twenty-five. ▶ The decimal system, in which you use groups of ten, is called the **base ten** system. This new system, in which you use groups of five, is called the **base five** system.

4. Write one to twenty-six in base ten and in base five:
12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25
Base ten: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . ., 26
Base five: 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, . . ., 101
22, 23, 24, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 100

After writing all the numerals from 1 to 26 in ex. 4, observe which are even and count by twos, by threes, by fives, and by tens, in both bases.

Ex. 5 is long, so have class work together and make checks as work proceeds. Have pupils observe relation between the two sets of numerals.

5. Write five to two hundred by fives in base ten and in base five.

See ans. in Guide. ▶ Base ten: 5, 10, 15, 20, 25, 30, 35, . . ., 200
Base five: 10, 20, 30, 40, 100, 110, 120, . . ., 1300

6. You have used a new number system with a base of five. You can build number systems with other bases such as eight or two. In order to tell the base of the number system a **subscript** is used when writing a number. In 13_5, the 5 is the subscript and it tells that the base is five. In 25_7, the base is seven. Notice that the subscript is written in base ten. If no subscript is used, the base is ten.

You know that 13_5 means 1 five and 3 ones. Study these:

23_5 = 2 fives and 3 ones
25_7 = 2 sevens and 5 ones
423_5 = 4 twenty-fives, 2 fives, and 3 ones
312_4 = 3 sixteens, 1 four, and 2 ones
1220_3 = 1 twenty-seven, 2 nines, 2 threes, and no ones

Give the value of each digit: See ans. in Guide.

7. 31_5 14_5 42_5 121_5 240_5

8. 342_5 2134_5 1320_5 2859 8300

9. 21_7 153_6 627_8 11_2 1010_2

10. To change a number written in base five to base ten you express the base five place values in base ten:

23_5 = 2 fives and 3 ones
$23_5 = (2 \times 5) + (3 \times 1) = 10 + 3 = 13$

Change to base ten:

11. 31_5 16 14_5 9 42_5 22 121_5 36 240_5 70

12. 342_5 97 2134_5 294 1320_5 210 2044_5 274 3003_5 378

13. 125_6 53 21_7 15 153_6 69 11_2 3 213_4 39

▶ $125_6 = (1 \times 6 \times 6) + (2 \times 6) + (5 \times 1) = 36 + 12 + 5 = 53$

14. 1111_2 15 1010_2 10 2134_6 490 2122_3 71 3221_4 233

▶ $1111_2 = (1 \times 2 \times 2 \times 2) + (1 \times 2 \times 2) + (1 \times 2) + (1 \times 1) = 15$

In examples where new base causes difficulty do not hesitate to use objects again. Count and write the digits as objects are grouped according to new base.

Reteach multiplication of decimal by whole number.

Multiplying Decimals

1. Problem Swan Lake is 11.7 mi. from Peter's house. How far does Peter ride on his bicycle if he goes to Swan Lake and back by the same road? 23.4 mi.

Explanation When you multiply a decimal by a whole number, the product must have as many decimal places as the decimal itself. There is one decimal place in 11.7, so the product must have one decimal place. Then the decimal point is placed at the left of 4. The answer is 23.4.

11.7
2
23.4

To show that 23.4 is correct you can add 11.7 to 11.7 to get the total distance. The sum is 23.4, which equals the product above.

Stress understanding, not memorization of following. →

To multiply a decimal by a whole number, multiply as with whole numbers. Then, beginning at the right, point off as many decimal places in the product as there are in the multiplicand.

2. Study these examples carefully: Have pupils verify decimal-point position through estimation.

.21	.7	3.2	.06	.0017	.008
4	8	9	7	4	3
.84	5.6	28.8	.42	.0068	.024

Have pupils estimate answers before multiplying in ex. 3-6.

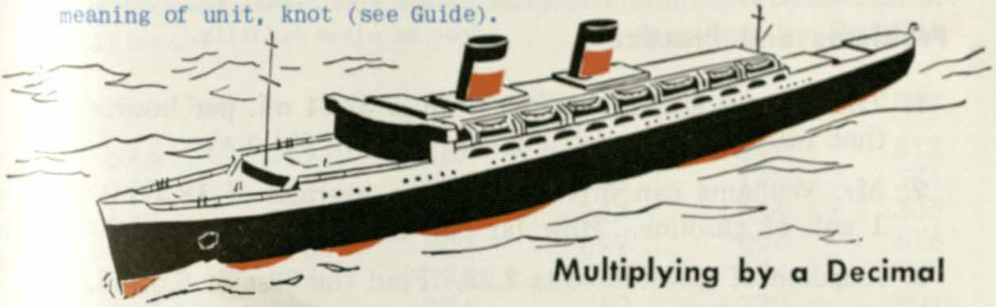
Multiply. Check the work by going over it:

3.	.212	11.9	42.6	8.37	.009	.025
	9	4	2	8	7	5
	1.908	47.6	85.2	66.96	.063	.125
4.	.026	6.38	.012	.004	.349	7.53
	3	6	8	5	9	4
	.078	38.28	.096	.020	3.141	30.12
5.	.063	.005	.147	16.5	2.54	60.7
	15	19	31	28	67	79
	.945	.095	4.557	462.0	170.18	4795.3
6.	.035	9.03	.006	70.8	.024	46.5
	31	72	64	35	74	49
	1.085	650.16	.384	2478.0	1.776	2278.5

More Practice. See ㉘ on page 335.

Lead pupils to discover and verify rule (boxed above) by having pupils use addition and estimation, that is, answer for 2 x 11.7 must be larger than 22 (2 x 11) and smaller than 24 (2 x 12). Therefore decimal point should be at right of 23.

Reteach multiplication of decimal by decimal (pages 97-98). Review meaning of unit, knot (see Guide).



Multiplying by a Decimal

1. Problem The speed of a ship is measured in **knots.** A speed of 1 knot is equal to a speed of 1.15 mi. per hour. If a large steamship has an average speed of 29.3 knots, find its speed in miles per hour. 33.695

Explanation Multiply 1.15 by 29.3 as you multiply whole numbers. The product is 33695. To find where to put the decimal point, make a rough estimate of the answer by multiplying the 1 of 1.15 by the 29 of 29.3. This gives 29. Since the answer should be a little more than 29, the decimal point should be placed after 33, which gives 33.695. The speed is 33.695 mi. per hour.

$$\begin{array}{r} 1.15 \\ \underline{29.3} \\ 345 \\ 10\,35 \\ \underline{23\,0} \\ 33.695 \end{array}$$

2. The decimal point in the product can also be placed by rule. Since there are 2 decimal places in 1.15 and 1 decimal place in 29.3, there are 3 decimal places in 1.15 and 29.3 together. The product should then have 3 decimal places in it. Beginning at the right, point off 3 decimal places in 33695, which gives 33.695 as the answer.

Stress understanding, not memorization. →

To multiply decimals, multiply as with whole numbers. Then, beginning at the right, point off as many decimal places in the product as there are decimal places in the multiplier and the multiplicand together.

Estimate the answers. (1) Then multiply the numbers: (2)

(1); (2) 100; 103.32 | 99; 104.16 | 280; 309.52 | 80; 86.645

3. 4.1 × 25.2 | 9.3 × 11.2 | 7.3 × 42.4 | 2.15 × 40.3

48; 50.530 | 56; 52.780 | 240; 257.92 | 54; 56.2005

4. 6.2 × 8.15 | 6.5 × 8.12 | 3.2 × 80.6 | 6.21 × 9.05

Follow development and explanation in ex. 1. Before presenting rule in ex. 2, do some more examples, using estimation to locate position of decimal point. In ex. 3-4 have pupils give estimated answers first before multiplying.

Assign ex. 1–4 as independent work to see that all can use decimal skills in problem solving. Then discuss solutions with class. Ex. 5 should be done as class activity.

Problems and Practice

1. The average speed of a fast train is 52.24 mi. per hour. At that rate how far will it go in 7.5 hr.? 391.8 mi.

2. Mr. Williams can drive his car an average of 18.4 mi. on 1 gal. of gasoline. How far can he drive on 6.5 gal.? 119.6 mi.

3. A gallon of gasoline costs $.28. Find the cost of 6.5 gal. of gasoline. $1.82

4. A yard of wool cloth costs $2.50. Find the cost of 4.5 yd. of this cloth. $11.25

Second and fourth examples are wrong.

5. Study these examples. Is the decimal point in each product in the correct place? If you find any mistakes, write the answers as they should be: Be sure pupils can explain decimal-point position without using rule.

4.29	1.625	12.24	431.8
.37	4.5	2.6	1.2
3003	8 125	7 344	863 6
1 287	65 00	24 48	4318
1.5873	73.125	31.824	5181.6
	7.3125		518.16

Multiply. Check by going over the work:

6.	.04 × 9.7	.388	.15 × 3.9	.585	.19 × 2.3	.437	.67 × 4.26	2.8542
7.	.07 × 3.8	.266	6.9 × .85	5.865	8.7 × 9.2	80.04	9.8 × .123	1.2054
8.	4.6 × .53	2.438	2.1 × 8.9	18.69	.54 × .86	.4644	5.8 × 60.5	350.90
9.	.27 × .61	.1647	5.4 × .92	4.968	1.6 × 6.7	10.72	.23 × 4.52	1.0396
10.	.58 × 6.4	3.712	.16 × 4.2	.672	.23 × 3.8	.874	5.4 × .674	3.6396
11.	9.2 × 7.4	68.08	9.3 × 9.6	89.28	6.2 × 6.9	42.78	8.5 × 1.78	15.130
12.	3.1 × 9.7	30.07	2.5 × 3.8	9.50	.14 × .83	.1162	.87 × 73.8	64.206
13.	.83 × 8.3	6.889	.47 × 5.4	2.538	.68 × 4.6	3.128	.32 × .705	.22560
14.	1.9 × .56	1.064	.85 × .75	.6375	3.5 × 7.4	25.90	6.9 × 18.9	130.41
15.	.36 × .71	.2556	.79 × 7.4	5.846	.57 × 1.3	.741	7.8 × 2.59	20.202

More Practice. See ㉙ on page 335.

In ex. 6–15 pupils should be able to explain decimal-point position by estimation, using common fractions, and by place-value knowledge: product of tenths and hundredths is thousandths (3 decimal places).

Present second set of improvement tests in addition. Review arithmetic vocabulary.

Improving by Practice

Addition Test 2a.

Time: 4 min.

1.	574	962	487	213	541	813
	199	191	636	680	173	581
	680	827	391	327	806	305
	125	510	509	699	894	616
	227	374	390	822	654	270
	547	806	874	569	230	901
	316	182	729	157	947	871 ⑥
	2668	3852	4016	3467	4245	4357

Addition Test 2b.

Time: 4 min.

2.	917	392	195	400	353	820
	884	574	466	554	606	189
	773	740	310	765	457	758
	606	512	198	200	183	931
	180	404	759	151	832	273
	964	823	306	295	242	706
	835	321	543	728	496	467 ⑥
	5159	3766	2777	3093	3169	4144

Addition Test 2c.

Time: 4 min.

3.	302	883	168	129	837	706
	690	254	129	955	194	585
	703	278	204	880	956	303
	554	555	597	744	427	974
	144	461	975	962	872	860
	106	309	183	338	791	172
	845	316	832	319	316	265 ⑥
	3344	3056	3088	4327	4393	3865

The Language of Arithmetic

Show that you know what each word or group of words means:

4. tenths

5. estimate

6. point off

7. round off

8. mixed decimal

9. factor

10. place value

11. decimal point

12. hundredths

13. millionths

14. time zone

15. round number

Note that only answers to ex. 1–3 should be written on folded paper (see pages 46–50, 344). Do ex. 4–15 as oral activity, with pupils using chalkboard to write examples of terms. Provide further specialized review as needed.

Reteach zero cases in multiplication of decimals.

Watching the Decimal Point

1. **Problem** Find the product of .16 and .03. .0048

Explanation In this example, it is easy to multiply 16 by 3 but it is difficult to place the decimal point correctly. You know that 3 × 16 = 48. If you follow the rule on page 95, you know there should be 4 decimal places in the product, counting from the right. Since there are only two figures in 48, two zeros are written before 48 so that there will be 4 decimal places. The answer is .0048.

$$\begin{array}{r} .16 \\ .03 \\ \hline .0048 \end{array}$$

2. To show that .0048 is the correct product, you can write the two decimals as fractions and then multiply the fractions. The answer is $\frac{48}{10000}$, which equals .0048 when written as a decimal. This shows that the work in ex. 1 is correct.

$$\frac{3}{100} \times \frac{16}{100} = \frac{48}{10000}$$

Stress understanding.

When the number of decimal places to be pointed off is greater than the number of figures in the product, put as many zeros to the left of the product as are needed to point off correctly.

Have pupils check some answers in ex. 3–5 by using common fractions.

Multiply these numbers:

3. .5 × .03 .015 .6 × .012 .0072 .05 × .113 .00565 .01 × 2.13 .0213

4. .7 × .12 .084 .9 × .003 .0027 1.2 × .004 .0048 0.4 × .004 .0016

5. .3 × .02 .006 5 × .015 .075 .04 × .006 .00024 6.3 × .003 .0189

In ex. 6 to 9, three answers are given for each example but only one answer is correct. Tell which is the right answer: Do orally and have pupils use estimation.

6. Does .9 × 3.27 equal .02943, or 2.943, or 29.43? 2.943

7. Does .2 × .47 equal 9.4, or .94, or .094? .094

8. Does .19 × .21 equal 39.9, or .399, or .0399? .0399

9. Does .4 × .018 equal .72, or .072, or .0072? .0072

More Practice. See ㉚ on page 335. Use to reinforce understanding.

Discuss ex. 1–2 thoroughly. Have some more examples done at chalkboard, in both decimal and common fraction form. Pupils should realize that product of 2 pure decimals must be less than either factor, since each factor is less than 1.

Reteach division of decimal by whole number when there is no remainder.

Dividing Decimals by Whole Numbers

1. Study these exercises carefully:

A.	B.	C.	D.
$\overset{\$\ .59}{3\overline{)\$1.77}}$	$\overset{.126}{6\overline{).756}}$	$\overset{5.76}{4\overline{)23.04}}$	$\overset{.385}{9\overline{)3.465}}$

In A, when you divide \$1.77 by 3, you are dividing a decimal by a whole number. You place the decimal point in Stress the quotient **directly above** the decimal point in \$1.77. You divide the numbers in B, C, and D just as you divide dollars and cents. In this work it is important to place the first quotient figure in the proper position. In B, the 1 in the quotient is placed above 7 because 7 was divided by 6 to get the first quotient figure. In C, the 5 in the quotient is placed above the last figure of 23 because 23 was divided by 4 to get the 5. Emphasize place value also, that is, that 7 "tenths," 23 "ones," 34 "tenths" were divided.

2. In one week Mr. Gray's prize cow gave these daily amounts of milk: 48.3 lb., 46.5 lb., 50.1 lb., 47.8 lb., 48.9 lb., 51.6 lb., 50.5 lb. Find the average number of pounds the cow gave per day. 49.1

Divide and check the work:

3.	$\overset{3.9}{4\overline{)15.6}}$	$\overset{23.8}{6\overline{)142.8}}$	$\overset{9.46}{7\overline{)66.22}}$	$\overset{1.146}{6\overline{)6.876}}$	$\overset{4.3}{55\overline{)236.5}}$
4.	$\overset{.92}{8\overline{)7.36}}$	$\overset{.934}{2\overline{)1.868}}$	$\overset{88.8}{3\overline{)266.4}}$	$\overset{.3278}{3\overline{).9834}}$	$\overset{1.25}{21\overline{)26.25}}$
5.	$\overset{7.6}{2\overline{)15.2}}$	$\overset{.437}{5\overline{)2.185}}$	$\overset{.358}{9\overline{)3.222}}$	$\overset{.219}{7\overline{)1.533}}$	$\overset{42.6}{18\overline{)766.8}}$
6.	$\overset{.17}{6\overline{)1.02}}$	$\overset{.147}{8\overline{)1.176}}$	$\overset{.94}{11\overline{)10.34}}$	$\overset{42.6}{19\overline{)809.4}}$	$\overset{7.3}{84\overline{)613.2}}$
7.	$\overset{4.9}{5\overline{)24.5}}$	$\overset{28.9}{9\overline{)260.1}}$	$\overset{8.3}{16\overline{)132.8}}$	$\overset{11.8}{26\overline{)306.8}}$	$\overset{.29}{77\overline{)22.33}}$
8.	$\overset{8.4}{3\overline{)25.2}}$	$\overset{8.61}{4\overline{)34.44}}$	$\overset{.71}{22\overline{)15.62}}$	$\overset{5.13}{12\overline{)61.56}}$	$\overset{40.8}{16\overline{)652.8}}$
9.	$\overset{.67}{9\overline{)6.03}}$	$\overset{.835}{7\overline{)5.845}}$	$\overset{5.9}{31\overline{)182.9}}$	$\overset{.232}{43\overline{)9.976}}$	$\overset{.72}{58\overline{)41.76}}$

More Practice. See ㉛ on page 335.

In discussing ex. 1 be sure pupils understand any regrouping involved (ones for tenths, tenths for hundredths, and so on).

Review first that decimal point comes between whole number and decimal fraction. Refer to money ($10 or $10.00) to show that writing decimal point and annexing zeros does not change value of number.

Avoiding Remainders

1. **Problem** An airplane travels 863 mi. in 4 hr. Find its average speed in miles per hour. 215.75 mi.

Explanation If you divide 863 by 4, as shown at A, the quotient is $215\frac{3}{4}$. You can avoid the fraction $\frac{3}{4}$ by writing 863 as 863.00, as shown at B. The speed of the airplane is 215.75 mi. per hour.

A $\begin{array}{r} 215\frac{3}{4} \\ 4\overline{)863} \end{array}$ B $\begin{array}{r} 215.75 \\ 4\overline{)863.00} \end{array}$

Emphasize after understanding is assured.

To avoid remainders in division, annex zeros after the decimal point of the dividend and continue to divide.

Divide. Write enough zeros after the decimal point to avoid a remainder. Check the work:

2. $\begin{array}{r} 10.25 \\ 4\overline{)41.00} \end{array}$ $\begin{array}{r} 11.75 \\ 8\overline{)94.00} \end{array}$ $\begin{array}{r} 2.88 \\ 25\overline{)72.00} \end{array}$ $\begin{array}{r} 3.75 \\ 12\overline{)45.00} \end{array}$ $\begin{array}{r} 4.6 \\ 35\overline{)161.0} \end{array}$

3. $\begin{array}{r} 9.5 \\ 6\overline{)57.0} \end{array}$ $\begin{array}{r} 16.8 \\ 5\overline{)84.0} \end{array}$ $\begin{array}{r} 11.25 \\ 12\overline{)135.00} \end{array}$ $\begin{array}{r} 2.96 \\ 25\overline{)74.00} \end{array}$ $\begin{array}{r} 37.5 \\ 18\overline{)675.0} \end{array}$

4. **Problem** A 10-pound bag of potatoes costs \$.51. How much does 1 lb. of potatoes cost? $\$.05\frac{1}{10}$, or \$.051

Explanation C shows that 1 lb. of potatoes costs $\$.05\frac{1}{10}$, which means $5\frac{1}{10}$¢. D shows that \$.051 is another way to write $\$.05\frac{1}{10}$ or $5\frac{1}{10}$¢. The 1 in \$.051 is in **thousandths** place and means $\frac{1}{10}$ of a cent. This shows that when you write money, a figure in thousandths place represents tenths of a cent. Emphasize

Stress.

C $\begin{array}{r} \$.05\frac{1}{10} \\ 10\overline{)\$.51} \\ \underline{50} \\ 1 \end{array}$ D $\begin{array}{r} \$.051 \\ 10\overline{)\$.510} \\ \underline{50} \\ 10 \\ \underline{10} \end{array}$

5. Study these numbers containing **tenths** of a cent:

\$.259 may be written as $\$.25\frac{9}{10}$
\$.875 may be written as $\$.87\frac{5}{10}$ or $\$.87\frac{1}{2}$

6. Write each in two other ways: \$.075, \$4.255, \$.087, \$.164, \$11.125, \$.732, $8\frac{1}{2}$¢. $7\frac{1}{2}$¢, $\$.07\frac{5}{10}$, $\$.07\frac{1}{2}$; $\$4.25\frac{5}{10}$, $\$4.25\frac{1}{2}$; $8\frac{7}{10}$¢ $\$.08\frac{7}{10}$; 16 $\$.16\frac{4}{10}$, $\$.16\frac{2}{5}$; $\$11.12\frac{5}{10}$, $\$11.12\frac{1}{2}$; $73\frac{1}{5}$¢, $\$.73\frac{2}{10}$, $\$.73\frac{1}{5}$; $\$.08\frac{5}{10}$, $\$.08\frac{1}{2}$, \$.085

Ex. 4-5 should be discussed thoroughly. Point out that $\frac{1}{10}$ of a cent is represented by "1" in thousandths place ($\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000}$).

Write the answer to each problem in two different ways:

1. Mr. Jackson receives \$19 a day for working 8 hr. How much is he paid per hour? $\$2.37\frac{1}{2}$; \$2.375
2. Mrs. Smith paid \$4.39 for a remnant of cloth 5 yd. long. What did the cloth cost her per yard? $\$.87\frac{4}{5}$; \$.878
3. John paid \$.69 for 4 packages of crackers. What did each package cost? Is $\$.17\frac{1}{4}$ the same as \$.1725? Yes
   $\$.17\frac{1}{4}$; \$.1725

   ▶ When a decimal ends in $\frac{1}{4}$, you can write 25 in place of $\frac{1}{4}$.
4. In ex. 3, suppose that John had paid \$.75 for the 4 packages of crackers. Find the cost of each package. Is $\$.18\frac{3}{4}$ the same as \$.1875? Yes.
   $\$.18\frac{3}{4}$; \$.1875

   ▶ When a decimal ends in $\frac{3}{4}$, you can write 75 in place of it.

Write the following entirely as decimals:

5. $\overset{\$1.0625}{\$1.06\frac{1}{4}}$ $\overset{\$1.1775}{\$1.17\frac{3}{4}}$ $\overset{\$.065}{\$.06\frac{1}{2}}$ $\overset{\$.123}{\$.12\frac{3}{10}}$ $\overset{\$.1575}{\$.15\frac{3}{4}}$ $\overset{\$.132}{\$.13\frac{1}{5}}$

6. $\overset{\$3.185}{\$3.18\frac{1}{2}}$ $\overset{\$2.1025}{\$2.10\frac{1}{4}}$ $\overset{\$.1675}{\$.16\frac{3}{4}}$ $\overset{\$.757}{\$.75\frac{7}{10}}$ $\overset{\$.366}{\$.36\frac{3}{5}}$ $\overset{\$.125}{\$.12\frac{1}{2}}$

Divide. Write enough zeros after the decimal point to avoid a remainder. Check the work:

7. $\overset{7.4}{5\overline{)37.0}}$ $\overset{31.75}{4\overline{)127.00}}$ $\overset{99.25}{12\overline{)1191.00}}$ $\overset{53.875}{56\overline{)3017.000}}$ $\overset{39.75}{52\overline{)2067.00}}$

8. $\overset{4.5}{8\overline{)36.0}}$ $\overset{47.5}{6\overline{)345.0}}$ $\overset{31.125}{48\overline{)1494.000}}$ $\overset{88.8}{45\overline{)3996.0}}$ $\overset{116.4}{25\overline{)2910.0}}$

9. $\overset{19.5}{2\overline{)39.0}}$ $\overset{28.4}{5\overline{)142.0}}$ $\overset{107.2}{25\overline{)2680.0}}$ $\overset{15.25}{76\overline{)1159.00}}$ $\overset{3.0725}{28\overline{)86.0300}}$

10. $\overset{15.5}{6\overline{)93.0}}$ $\overset{13.125}{8\overline{)105.000}}$ $\overset{1.2825}{12\overline{)15.3900}}$ $\overset{61.75}{44\overline{)2717.00}}$ $\overset{31.6}{75\overline{)2370.0}}$

11. $\overset{11.5}{8\overline{)92.0}}$ $\overset{92.5}{6\overline{)555.0}}$ $\overset{94.8}{35\overline{)3318.0}}$ $\overset{58.5}{64\overline{)3744.0}}$ $\overset{5.1125}{24\overline{)122.7000}}$

12. $\overset{.65}{4\overline{)2.60}}$ $\overset{7.55}{2\overline{)15.10}}$ $\overset{8.26}{25\overline{)206.50}}$ $\overset{2.25}{22\overline{)49.50}}$ $\overset{7.66}{15\overline{)114.90}}$

13. $\overset{.46}{5\overline{)2.30}}$ $\overset{3.325}{4\overline{)13.300}}$ $\overset{9.525}{24\overline{)228.600}}$ $\overset{.65}{46\overline{)29.90}}$ $\overset{6.24}{35\overline{)218.40}}$

More Practice. See ㉜ on page 336. Use for remedial work.

Do ex. 1–4 with pupils, letting different ones explain solutions at board. It may be necessary to review page 93 for ex. 4–6. Assign ex. 7–13 as independent work. Group pupils who had errors to help determine causes.

Reteach rounding off decimals (pages 104-105).

Rounding Off Results

$$7\overline{)17.000}^{\;2.428}$$

1. Sometimes in division you will be told to find the answer to the **nearest tenth** or **nearest hundredth.** In the example at the right, if you carry the answer to two places, you get 2.42. To find whether 2.42 is nearer to 2.4 or to 2.5, you can think of 2.4 as 2.40 and 2.5 as 2.50. You know that 2.42 is nearer to 2.40 than it is to 2.50. So 2.42 rounded off to the **nearest tenth** is 2.4. In this case the last 2 of 2.42 is dropped.

2. Suppose the answer carried to two places had been 2.48. You see that 2.48 is closer to 2.50 than it is to 2.40. This means that 2.48 rounded off to the nearest tenth is 2.5. In this case the 8 of 2.48 is dropped but the figure before 8 is made 1 larger.

3. If 2.45 is rounded off to the nearest tenth, it becomes 2.5. You see that 2.45 is halfway between 2.40 and 2.50; in such cases the larger tenth is selected.

4. Answers having 3 decimal places can be rounded off to the **nearest hundredth.** For example, 2.428 is rounded off to 2.43 because 2.428 is closer to 2.430 than to 2.420. Likewise, 2.423 is rounded off to 2.42 because it is closer to 2.420 than to 2.430. When you round off 2.425, it becomes 2.43, just as \$2.42½ is rounded off to \$2.43.

Emphasize.

To round off a decimal, use this rule: If the last figure of the decimal is less than 5, drop it; if the last figure of the decimal is 5 or more, drop it and make the figure before it 1 larger.

5. Round off each decimal correct to the nearest tenth:

4.83 4.8 1.05 1.1 2.96 3.0 71.94 71.9 2.07 2.1 .98 1.0 1.41 1.4 5.99 6.0 3.80 3.8

6. Round off each decimal correct to the nearest hundredth:

.721 .72 .276 .28 3.455 3.46 1.399 1.40 6.072 6.07 4.295 4.30 7.123 7.12

Emphasize that decimal is expressed to one more decimal place than will be retained in answer. See Guide for further exercise.

Stress that quotient is carried to one more decimal place than is wanted in answer. In ex. 11, point out that nearest cent is same as nearest hundredth of a dollar.

Rounding Off Results

Divide. Carry the quotient to two decimal places; then round off the result correct to the nearest tenth: Stress.

1. $\overset{1.6}{7\overline{)11}}$ $\overset{.1}{9\overline{)1.33}}$ $\overset{19.5}{11\overline{)215}}$ $\overset{1.1}{21\overline{)22.6}}$ $\overset{.7}{18\overline{)11.8}}$

2. $\overset{1.8}{9\overline{)16}}$ $\overset{4.5}{6\overline{)27.1}}$ $\overset{9.8}{13\overline{)127}}$ $\overset{.6}{32\overline{)18.7}}$ $\overset{.5}{76\overline{)39.8}}$

3. $\overset{5.6}{8\overline{)45}}$ $\overset{1.3}{7\overline{)8.76}}$ $\overset{8.8}{17\overline{)150}}$ $\overset{.2}{48\overline{)9.62}}$ $\overset{1.3}{12\overline{)15.2}}$

4. $\overset{17.3}{3\overline{)52}}$ $\overset{2.9}{4\overline{)11.5}}$ $\overset{11.2}{19\overline{)212}}$ $\overset{.6}{26\overline{)15.2}}$ $\overset{7.5}{35\overline{)262}}$

Divide. Carry the quotient to three decimal places; then round off the result correct to the nearest hundredth:

5. $\overset{.98}{6\overline{)5.9}}$ $\overset{.05}{5\overline{).27}}$ $\overset{.02}{8\overline{).149}}$ $\overset{1.12}{29\overline{)32.5}}$ $\overset{.57}{31\overline{)17.8}}$

6. $\overset{.44}{7\overline{)3.1}}$ $\overset{.04}{4\overline{).15}}$ $\overset{.26}{9\overline{)2.38}}$ $\overset{1.12}{16\overline{)17.9}}$ $\overset{.58}{82\overline{)47.5}}$

7. $\overset{.96}{9\overline{)8.6}}$ $\overset{.17}{7\overline{)1.2}}$ $\overset{1.19}{6\overline{)7.12}}$ $\overset{.18}{27\overline{)4.87}}$ $\overset{1.42}{65\overline{)92.6}}$

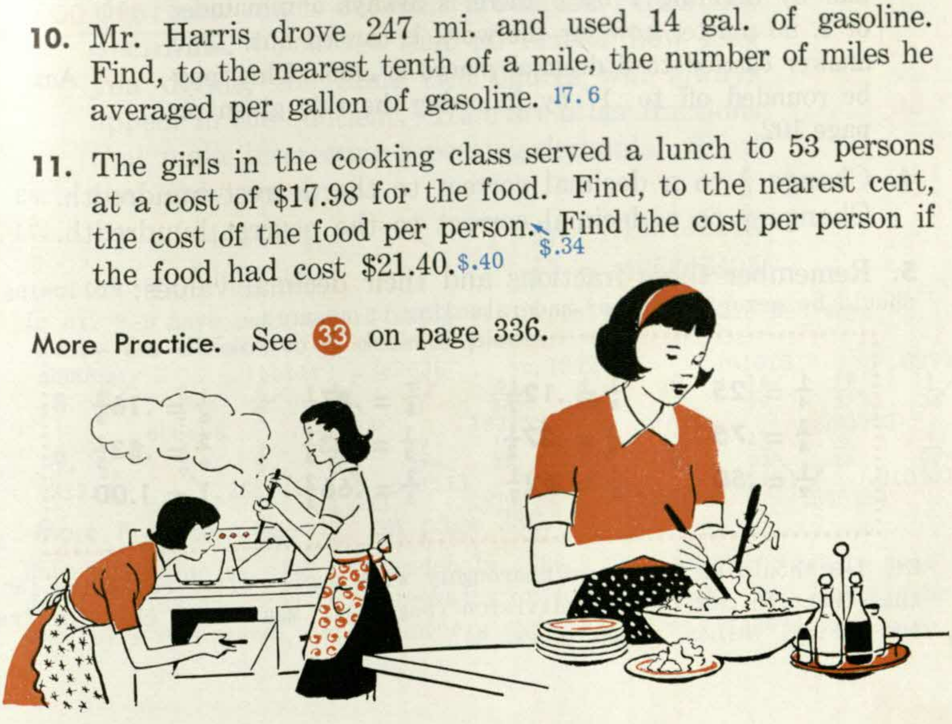
8. $\overset{.10}{8\overline{).79}}$ $\overset{.06}{3\overline{).19}}$ $\overset{7.51}{7\overline{)52.6}}$ $\overset{.20}{15\overline{)3.04}}$ $\overset{.08}{38\overline{)2.91}}$

9. An airplane traveled 1100 mi. in 6 hr. Find, to the nearest hundredth of a mile per hour, its average speed. 183.33

10. Mr. Harris drove 247 mi. and used 14 gal. of gasoline. Find, to the nearest tenth of a mile, the number of miles he averaged per gallon of gasoline. 17.6

11. The girls in the cooking class served a lunch to 53 persons at a cost of \$17.98 for the food. Find, to the nearest cent, the cost of the food per person. \$.34 Find the cost per person if the food had cost \$21.40. \$.40

More Practice. See ㉝ on page 336.



Reteach changing fractions to decimals and extend work to include repeating decimals and comparison of certain common fractions with decimals (pages 106–107).

Changing Fractions to Decimals

1. **Problem** Change $\frac{5}{8}$ to a decimal. $.62\frac{1}{2}$, or .625

Explanation You know that $\frac{5}{8}$ means $5 \div 8$. To divide 5 by 8, write a decimal point after 5 and annex zeros. The quotient is $.62\frac{1}{2}$ if you annex 2 zeros, or .625 if you annex 3 zeros. This shows that $\frac{5}{8} = .62\frac{1}{2} = .625$. Be sure pupils understand regrouping involved (5 ones changed to 50 tenths, and so on).

$$\begin{array}{r} .62\frac{1}{2} \\ 8\overline{)5.00} \end{array} \qquad \begin{array}{r} .625 \\ 8\overline{)5.000} \end{array}$$

2. **Problem** Change $\frac{2}{3}$ to a decimal. $.6\frac{2}{3}$, or $.66\frac{2}{3}$, or $.666\frac{2}{3}$

Explanation To change $\frac{2}{3}$ to a decimal, you annex zeros to 2 and then divide by 3. If you annex 1 zero, as at A, you get $\frac{2}{3} = .6\frac{2}{3}$; if you annex 2 zeros, as at B, you get $\frac{2}{3} = .66\frac{2}{3}$; if you annex 3 zeros, as at C, you get $\frac{2}{3} = .666\frac{2}{3}$. No matter how many zeros you annex to 2, you always get a remainder. You see that $\frac{2}{3} = .6\frac{2}{3}$ or $.66\frac{2}{3}$ or $.666\frac{2}{3}$.

A

$$\begin{array}{r} .6\frac{2}{3} \\ 3\overline{)2.0} \end{array}$$

B

$$\begin{array}{r} .66\frac{2}{3} \\ 3\overline{)2.00} \end{array}$$

C

$$\begin{array}{r} .666\frac{2}{3} \\ 3\overline{)2.000} \end{array}$$

3. **Problem** Change $\frac{1}{6}$ to a decimal correct to the nearest hundredth.

Explanation You see that when $\frac{1}{6}$ is changed to a decimal by dividing 1 by 6 there is always a remainder of 4, no matter how far the work is carried out. The answer carried to 3 decimal places is .166. This may be rounded off to .17 by following the rule given on page 102.

$$\begin{array}{r} .166 \\ 6\overline{)1.000} \end{array}$$

.17 **Ans.**

4. Change $\frac{5}{6}$ to a decimal correct to the nearest hundredth. .83
Change $\frac{5}{7}$ to a decimal correct to the nearest hundredth. .71

5. Remember these fractions and their decimal values: Following should be memorized after understanding is assured.

$\frac{1}{4} = .25$
$\frac{3}{4} = .75$
$\frac{1}{2} = .50$
$\frac{1}{8} = .12\frac{1}{2}$
$\frac{3}{8} = .37\frac{1}{2}$
$\frac{5}{8} = .62\frac{1}{2}$
$\frac{7}{8} = .87\frac{1}{2}$
$\frac{1}{3} = .33\frac{1}{3}$
$\frac{2}{3} = .66\frac{2}{3}$
$\frac{1}{6} = .16\frac{2}{3}$
$\frac{5}{6} = .83\frac{1}{3}$
$1 = 1.00$

Ex. 1–3 should be discussed thoroughly with pupils after reviewing fact that fraction can indicate division (page 51). Make class chart of fractions and decimal values, shown in ex. 5.

Point out usefulness of changing fraction to decimal for comparison purposes.

Changing Fractions to Decimals

Change these fractions to decimals:

1. $\frac{3}{5}$ 6 $\frac{7}{8}$ 875 $\frac{1}{4}$ .25 $\frac{3}{8}$ 375 $\frac{13}{20}$ .65 $\frac{4}{5}$ .8 $\frac{19}{25}$ .76 $\frac{9}{20}$ .45 $\frac{3}{10}$ .3 $\frac{11}{25}$ .44 $\frac{17}{20}$ .85

Change to decimals correct to the nearest tenth:

2. $\frac{2}{7}$ .3 $\frac{1}{3}$ .3 $\frac{2}{9}$ .2 $\frac{5}{6}$ .8 $\frac{4}{9}$ .4 $\frac{6}{7}$ .9 $\frac{7}{12}$ .6 $\frac{8}{11}$ .7 $\frac{11}{13}$ .8 $\frac{5}{18}$ .3 $\frac{10}{23}$ .4

Change to decimals correct to the nearest hundredth:

3. $\frac{1}{8}$ 13 $\frac{1}{7}$ 14 $\frac{8}{9}$ .89 $\frac{3}{7}$ .43 $\frac{1}{6}$ 17 $\frac{5}{9}$ .56 $\frac{13}{15}$ .87 $\frac{14}{39}$ .36 $\frac{23}{42}$ .55 $\frac{14}{19}$ 74 $\frac{19}{26}$ .73

Tell which number is larger. First change the common fraction to a decimal; then compare the decimals:

4. $\frac{3}{4}$ or .76 .76 $\frac{3}{8}$ or .369 $\frac{3}{8}$ $\frac{13}{25}$ or .55 .55 $\frac{6}{25}$ or .26 .26

5. $\frac{4}{5}$ or .78 $\frac{4}{5}$ $\frac{5}{8}$ or .614 $\frac{5}{8}$ $\frac{17}{20}$ or .84 $\frac{17}{20}$ $\frac{9}{10}$ or .87 $\frac{9}{10}$

6. $\frac{1}{4}$ or .23 $\frac{1}{4}$ $\frac{7}{8}$ or .882 .882 $\frac{11}{50}$ or .26 .26 $\frac{18}{25}$ or .71 $\frac{18}{25}$

7. Sometimes when a fraction like $\frac{9}{11}$ is changed to a decimal, the same figure, or group of figures, is repeated in the decimal as shown at the right. Such decimals are called **repeating decimals.** You see that no matter how long you divide, the same two figures will always appear in the quotient. Here are other fractions that make interesting repeating decimals. Show that they are true:

Stress.

$$\begin{array}{r} .8181 \\ 11\overline{)9.0000} \\ 8\,8 \\ \hline 20 \\ 11 \\ \hline 90 \\ 88 \\ \hline 20 \\ 11 \\ \hline 9 \end{array}$$

$\frac{4}{9} = .44444444$ $\frac{4}{27} = .148148148$

$\frac{7}{11} = .63636363$ $\frac{15}{37} = .405405405$

In ex. 8–9 have pupils tell which are repeating decimals and why.

Carry the division to 6 decimal places:

8. $\frac{8}{9}$ .888888 $\frac{2}{3}$ .666666 $\frac{1}{9}$ .111111 $\frac{4}{7}$ .571428 $\frac{4}{11}$ .363636 $\frac{6}{37}$ .162162 $\frac{4}{33}$ .121212 $\frac{10}{37}$ .270270 $\frac{16}{99}$ .161616 $\frac{19}{27}$ .703703 $\frac{1}{36}$ .027777

9. $\frac{1}{3}$ .333333 $\frac{5}{9}$ .555555 $\frac{2}{7}$ .285714 $\frac{5}{6}$ .833333 $\frac{19}{37}$ .513513 $\frac{10}{27}$ .370370 $\frac{11}{15}$ .733333 $\frac{13}{18}$ .722222 $\frac{17}{33}$ .515151 $\frac{6}{11}$ .545454 $\frac{1}{99}$ .010101

More Practice. See ㉞ on page 336.

Have pupils do ex. 1–6 independently. Check them as they work to note any difficulties and need for reteaching or remedial work. Ex. 7 should be carefully explained. Let volunteers show that $\frac{4}{9}$ = .44444444, and so on.

Provide written review of previous computation.

Mixed Practice

Add and check:

1.	871	353	418	320	306	311	554
	524	277	106	561	872	98	582
	169	526	216	404	56	201	232
	523	66	819	300	256	87	96
	196	534	212	146	68	347	468
	82	350	48	63	231	460	122
	314	99	313	107	753	132	735
	2679	2205	2132	1901	2542	1636	2789
2.	3.25	22.5	.540	5.42	.195	1.26	78.5
	1.86	44.0	.875	1.64	.009	3.25	9.8
	.92	8.3	.375	3.55	.842	3.50	8.6
	6.03	74.8	1.790	10.61	1.046	8.01	96.9
3.	$15\frac{1}{2}$	$86\frac{1}{6}$	$57\frac{11}{12}$	$84\frac{9}{10}$	$16\frac{3}{4}$	$21\frac{3}{5}$	$36\frac{1}{2}$
	$14\frac{3}{4}$	$20\frac{2}{3}$	$31\frac{1}{3}$	$15\frac{7}{10}$	$32\frac{1}{6}$	$21\frac{1}{2}$	$33\frac{7}{10}$
	$30\frac{1}{4}$	$106\frac{5}{6}$	$89\frac{1}{4}$	$100\frac{3}{5}$	$48\frac{11}{12}$	$43\frac{1}{10}$	$70\frac{1}{5}$

Divide and check:

4. $\begin{array}{r} .124 \\ 8\overline{).992} \end{array}$ $\begin{array}{r} 27.5 \\ 3\overline{)82.5} \end{array}$ $\begin{array}{r} 1.24 \\ 12\overline{)14.88} \end{array}$ $\begin{array}{r} 86 \\ 273\overline{)23478} \end{array}$ $\begin{array}{r} 3006 \\ 196\overline{)589176} \end{array}$

5. $\begin{array}{r} .82 \\ 7\overline{)5.74} \end{array}$ $\begin{array}{r} .145 \\ 5\overline{).725} \end{array}$ $\begin{array}{r} 23.7 \\ 15\overline{)355.5} \end{array}$ $\begin{array}{r} 49 \\ 423\overline{)20727} \end{array}$ $\begin{array}{r} 994 \\ 318\overline{)316092} \end{array}$

Find the answers:

6. $\frac{2}{3} \times \frac{3}{8}$ $\frac{1}{4}$ $4\frac{1}{2} \div \frac{1}{2}$ 9 $7\frac{1}{3} \times 18$ 132 $7\frac{1}{2} + 3\frac{5}{8}$ $11\frac{1}{8}$

7. $\frac{7}{8} + \frac{3}{4}$ $1\frac{5}{8}$ $1\frac{3}{4} + \frac{3}{4}$ $2\frac{1}{2}$ $1\frac{2}{3} \times 1\frac{1}{2}$ $2\frac{1}{2}$ $2\frac{7}{8} + 3\frac{1}{2}$ $6\frac{3}{8}$

8. $\frac{4}{5} \times \frac{1}{5}$ $\frac{4}{25}$ $6\frac{2}{3} \div 8$ $\frac{5}{6}$ $2\frac{1}{4} \div 1\frac{1}{3}$ $1\frac{11}{16}$ $2\frac{5}{6} - 1\frac{3}{4}$ $1\frac{1}{12}$

9. $\frac{1}{4} + \frac{5}{6}$ $1\frac{1}{12}$ $3\frac{2}{3} \times 3$ 11 $1\frac{3}{5} \div 1\frac{1}{3}$ $1\frac{1}{5}$ $3\frac{1}{4} + 2\frac{2}{3}$ $5\frac{11}{12}$

10. $\frac{3}{8} \times \frac{1}{3}$ $\frac{1}{8}$ $3\frac{1}{2} \times 9$ $31\frac{1}{2}$ $4\frac{1}{2} \div 1\frac{1}{8}$ 4 $6\frac{3}{8} + 4\frac{1}{2}$ $10\frac{7}{8}$

11. $\frac{1}{2} \times \frac{1}{3}$ $\frac{1}{6}$ $3\frac{3}{8} \div \frac{3}{4}$ $4\frac{1}{2}$ $4\frac{5}{6} + 1\frac{1}{3}$ $6\frac{1}{6}$ $5\frac{1}{3} \div 1\frac{3}{5}$ $3\frac{1}{3}$

12. $6 \div \frac{2}{3}$ 9 $6\frac{1}{4} \div \frac{5}{6}$ $7\frac{1}{2}$ $1\frac{1}{8} \times 3\frac{1}{3}$ $3\frac{3}{4}$ $1\frac{3}{4} + 1\frac{3}{4}$ $3\frac{1}{2}$

13. $9 \div \frac{1}{2}$ 18 $7\frac{1}{2} \times 8$ 60 $3\frac{1}{3} \div 1\frac{1}{2}$ $2\frac{2}{9}$ $1\frac{3}{4} \div 1\frac{1}{6}$ $1\frac{1}{2}$

14. $6 \div \frac{5}{8}$ $9\frac{3}{5}$ $2\frac{2}{3} \times 9$ 24 $7\frac{1}{4} - 2\frac{5}{8}$ $4\frac{5}{8}$ $6\frac{3}{5} + 2\frac{1}{2}$ $9\frac{1}{10}$

Pupils need not do all exercises. Be sure that all pupils do at least one of each different kind of example. Group pupils who evidence need for remedial work or reteaching.

Present set of improvement tests in subtraction.

Improving by Practice

Subtraction Test 2a.

Time: 3½ min.

1.	$91.32 20.33 $70.99	$84.88 5.99 $78.89	$90.10 29.08 $61.02	$65.42 16.47 $48.95	$83.05 35.96 $47.09	
2.	$94.89 54.79 $40.10	$87.48 11.86 $75.62	$89.06 26.49 $62.57	$60.90 9.96 $50.94	$76.23 29.49 $46.74	
3.	$73.66 59.68 $13.98	$94.10 38.30 $55.80	$50.89 33.68 $17.21	$73.86 1.53 $72.33	$49.27 19.38 $29.89	⑮

Subtraction Test 2b.

Time: 3½ min.

4.	$97.94 67.52 $30.42	$71.71 40.02 $31.69	$57.28 19.60 $37.68	$80.41 29.97 $50.44	$42.30 29.71 $12.59	
5.	$61.47 44.69 $16.78	$54.21 48.79 $ 5.42	$97.32 94.69 $ 2.63	$51.32 30.46 $20.86	$58.05 9.68 $48.37	
6.	$34.66 21.46 $13.20	$53.02 2.20 $50.82	$91.11 13.58 $77.53	$81.40 25.34 $56.06	$72.53 49.99 $22.54	⑮

Subtraction Test 2c.

Time: 3½ min.

7.	$82.80 38.78 $44.02	$73.61 14.92 $58.69	$40.48 28.69 $11.79	$41.69 3.01 $38.68	$86.14 57.46 $28.68	
8.	$68.52 40.98 $27.54	$79.59 37.43 $42.16	$36.45 8.79 $27.66	$41.73 30.96 $10.77	$83.02 16.34 $66.68	
9.	$79.26 9.57 $69.69	$69.76 18.42 $51.34	$64.06 49.53 $14.53	$40.43 13.38 $27.05	$30.34 16.99 $13.35	⑮

To the Teacher. Have each pupil compare the graph of his scores on Subtraction Tests 2a, 2b, and 2c above with the graph he made of his scores on Subtraction Tests 1a, 1b, and 1c. In this way he can see whether he is improving in subtraction.

Have all pupils determine own scores and record them on graphs. Through individual conferences with pupils, try to determine causes of errors. Have others work on original problems, projects, and so on.

Before discussing ex. 1, put some similar examples on board and lead pupils to discover short way for themselves. Then discuss ex. 1 and do ex. 2 as class activity.

Multiplying and Dividing by 10, 100

1. Study these multiplication examples:

A. $\begin{array}{r}\$11.25\\ 10\\ \hline \$112.50\end{array}$ B. $\begin{array}{r}\$11.25\\ 100\\ \hline \$1125.00\end{array}$ C. $\begin{array}{r}\$11.25\\ 1000\\ \hline \$11250.00\end{array}$

In A, when you multiply by 10, you can get the answer by moving the decimal point in $11.25 one place to the right. In B, when you multiply by 100, you can get $1125.00 by moving the decimal point in $11.25 two places to the right. In C, you can get $11,250.00 by moving the decimal point three places to the right, but in this case you have to write a zero after $11.25 so that you can move the decimal point three places.

To multiply a number by 10, 100, or 1000, move the decimal point as many places to the right as there are zeros in the multiplier.

2. Multiply each number by 10, then by 100, and then by 1000: $1.28, $13.29, 6.4, .9, 8.7, .02, $3.175, 27.2045.

See ans. in Guide.

3. Study these division examples:

D. $12.50 ÷ 10 = $1.25 E. $3.50 ÷ 100 = $.035

Since you multiply by 10 by moving the decimal point one place to the right, you divide by 10 by moving the decimal point one place to the left, as shown at D. To divide $3.50 by 100, as shown at E, move the decimal point in $3.50 two places to the left; but since there is only one figure to the left of the decimal point, you must place a zero in front of 3 so that you can move the decimal point two places. Emphasize.

To divide by 10, 100, or 1000, move the decimal point as many places to the left as there are zeros in the divisor.

4. Divide each number by 10, (1) then by 100, (2) and then by 1000: (3)

	48	321	6.4	$134	$23.00	$1237	$1427.35
(1)	4.8	32.1	.64	$13.40	$2.30	$123.70	$142.735
(2)	.48	3.21	.064	$1.34	$.23	$12.3	$14.2735
(3)	.048	.321	.0064	$.134	$.023	$1.237	$1.42735

Before discussing ex. 3, have pupils discover rule through illustrative examples. Do ex. 4 as class activity. Pupils should learn both rules after understanding is assured. Give more oral practice as in ex. 2 and 4.

Review division by decimal when quotients are whole numbers. First review principle: If both terms of fraction are multiplied by same number, value of fraction is unchanged.

Dividing by a Decimal

A. $\begin{array}{r} 4 \\ 6\overline{)24} \end{array}$ B. $\begin{array}{r} 4 \\ 60\overline{)240} \end{array}$ C. $\begin{array}{r} 4 \\ 600\overline{)2400} \end{array}$

1. In B, both the divisor and the dividend are 10 times as large as in A, but B has the same quotient as A. In C, both the divisor and the dividend are 100 times as large as in A, yet the quotient 4 remains the same.

Emphasize after discussion.

Multiplying both the divisor and the dividend by 10, by 100, or by 1000 does not change the quotient.

This is the same as multiplying both terms of a fraction by 10, by 100, or by 1000. For you can think of the above division examples as the fractions $\frac{24}{6}$, $\frac{240}{60}$, and $\frac{2400}{600}$. You know that these three fractions are equal.

2. If the divisor is a decimal, you can change it to a whole number by the above rule. Study D and E below.

D. $4.8\overline{)28.8}$ E. $\begin{array}{r} 6 \\ 48\overline{)288} \\ 288 \\ \hline \end{array}$

If you multiply both numbers in D by 10, you get E. In E, the divisor is a whole number, so you can work the example as usual. You get 6 for the answer to E; 6 is also the answer to D because the quotient does not change when both divisor and dividend are multiplied by 10.

Have pupils tell by what power of 10 they multiplied both divisor and dividend.

Divide. First change each divisor to a whole number:

3. $\begin{array}{r} 56 \\ .9\overline{)50.4} \end{array}$ $\begin{array}{r} 49 \\ .6\overline{)29.4} \end{array}$ $\begin{array}{r} 13 \\ 6.2\overline{)80.6} \end{array}$ $\begin{array}{r} 18 \\ \$.48\overline{)\$8.64} \end{array}$ $\begin{array}{r} 14 \\ .34\overline{)4.76} \end{array}$

4. $\begin{array}{r} 36 \\ .6\overline{)21.6} \end{array}$ $\begin{array}{r} 39 \\ .7\overline{)27.3} \end{array}$ $\begin{array}{r} 12 \\ 7.3\overline{)87.6} \end{array}$ $\begin{array}{r} 13 \\ \$.66\overline{)\$8.58} \end{array}$ $\begin{array}{r} 41 \\ .22\overline{)9.02} \end{array}$

5. $\begin{array}{r} 58 \\ .3\overline{)17.4} \end{array}$ $\begin{array}{r} 97 \\ .8\overline{)77.6} \end{array}$ $\begin{array}{r} 12 \\ 8.2\overline{)98.4} \end{array}$ $\begin{array}{r} 16 \\ \$.58\overline{)\$9.28} \end{array}$ $\begin{array}{r} 37 \\ 2.5\overline{)92.5} \end{array}$

6. Joe saves $.75 a week. How many weeks will it take him to save $6.00? 8

More Practice. See ㉟ on page 337. Use to increase skill.

After prelesson review, discuss ex. 1 with class. In ex. 2 try to get pupils to tell you how to divide before discussing explanation. Do ex. 3 as class activity. Then assign ex. 4-7, if there are no apparent difficulties.

Reteach division by decimal when quotients contain decimals. If use of caret is avoided in your school, rewrite examples as in B and proceed with division.

Dividing by a Decimal

1. **Problem** In a heavy rain 2.23 in. of rain fell in 3.5 hr. How many inches of rain fell per hour? Find the answer to the nearest hundredth of an inch. .64

A

$$\begin{array}{r} .637 \\ 3.5_{\wedge}\overline{)2.2_{\wedge}300} \\ 2\;10 \\ \hline 1\;30 \\ 1\;05 \\ \hline 250 \\ 245 \\ \hline 5 \end{array}$$

B

$$35\overline{)22.3}$$

Explanation Make the divisor 3.5 a whole number by multiplying it by 10; you can do this by moving the decimal point in 3.5 one place to the right, which gives 35. Also multiply the dividend 2.23 by 10 by moving the point in 2.23 one place to the right, which gives 22.3.

Instead of rewriting the example, as shown at B, you can use a caret (‸), as at A, to indicate the new position of each decimal point. Then do the work at A. Place the decimal point in the quotient Emphasize. **directly above the caret** in 2.2‸3. The rainfall per hour was .637 in., or .64 in. to the nearest hundredth of an inch.

By using carets you save rewriting the example. Remember, if you use carets, you place the decimal point in the quotient directly above the caret in the dividend. Stress.

To check the work, multiply .637 by the original divisor 3.5 and add the remainder 5, remembering that 5 is in **ten-thousandths** place and means .0005. Stress. The result should equal 2.23.

In dividing by a decimal, place a caret (‸) as many places to the right of the decimal point in both divisor and dividend as there are decimal places in the divisor.

Stress after understanding of process is assured.

Carry to 3 decimal places and round off to the nearest hundredth:

2. $\begin{array}{r}3.61\\ .7\overline{)2.53}\end{array}$ $\begin{array}{r}2.40\\ .9\overline{)2.164}\end{array}$ $\begin{array}{r}52.89\\ 2.7\overline{)142.8}\end{array}$ $\begin{array}{r}13.26\\ .93\overline{)12.333}\end{array}$

3. $\begin{array}{r}.88\\ .8\overline{).701}\end{array}$ $\begin{array}{r}5.36\\ .6\overline{)3.217}\end{array}$ $\begin{array}{r}165.94\\ .16\overline{)26.55}\end{array}$ $\begin{array}{r}1.60\\ 6.7\overline{)10.718}\end{array}$

4. $\begin{array}{r}25.11\\ .9\overline{)22.6}\end{array}$ $\begin{array}{r}15.77\\ .7\overline{)11.04}\end{array}$ $\begin{array}{r}15.80\\ .43\overline{)6.793}\end{array}$ $\begin{array}{r}3.82\\ 8.5\overline{)32.442}\end{array}$

5. $\begin{array}{r}.74\\ .4\overline{).295}\end{array}$ $\begin{array}{r}2.95\\ .6\overline{)1.768}\end{array}$ $\begin{array}{r}7.68\\ 3.2\overline{)24.59}\end{array}$ $\begin{array}{r}20.97\\ .73\overline{)15.306}\end{array}$

6. $\begin{array}{r}.65\\ .3\overline{).194}\end{array}$ $\begin{array}{r}29.76\\ .8\overline{)23.81}\end{array}$ $\begin{array}{r}7.32\\ 2.3\overline{)16.84}\end{array}$ $\begin{array}{r}57.93\\ .42\overline{)24.331}\end{array}$

Discuss ex. 1 with pupils. Caret should not be used until pupils have good understanding of process. In ex. 2–6 have pupils explain how they changed divisors and why they were able to do so.

Present problems and exercises on work taught on pages 111-112.

An Automobile Trip

1. Mr. Carver's speedometer shows that he traveled 124.4 mi. on a trip of 3 hr. 18 min. (3.3 hr.). Find his average speed, to the nearest tenth of a mile, per hour. 37.7

2. Mr. Carver had a tank full of gasoline when he left home. After driving 124.4 mi. he could put only 6.5 gal. of gasoline in the tank. What was his average mileage per gallon of gasoline? Find the answer to the nearest tenth of a mile. 19.1

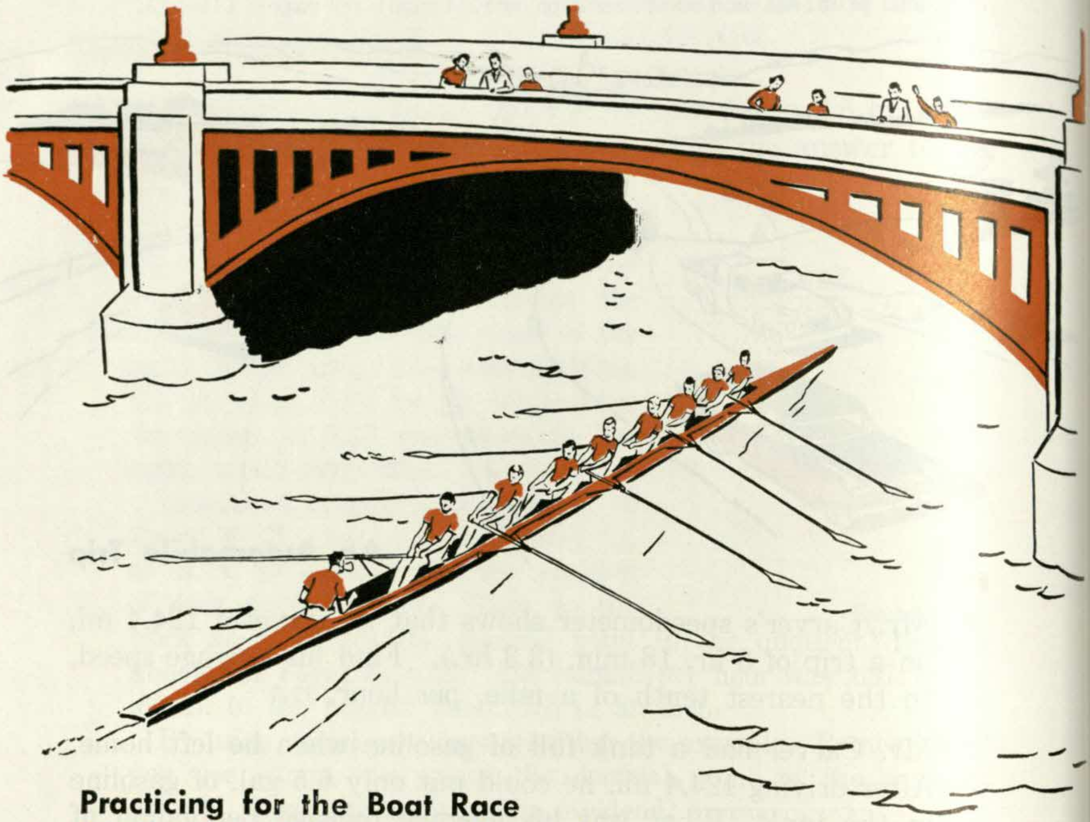
3. On a long trip that took 11 hr. 12 min. (11.2 hr.) of actual driving, Mr. Carver went 435.8 mi. How many miles per hour, to the nearest tenth of a mile, did he average? 38.9

Divide. Carry the quotient to 2 decimal places; then round off the result to the nearest tenth:

4.	$\begin{array}{r}231.8\\.17\overline{)39.41}\end{array}$	$\begin{array}{r}6.9\\2.8\overline{)19.38}\end{array}$	$\begin{array}{r}2.1\\.67\overline{)1.428}\end{array}$	$\begin{array}{r}54.2\\.74\overline{)40.08}\end{array}$
5.	$\begin{array}{r}42.8\\.38\overline{)16.25}\end{array}$	$\begin{array}{r}274.1\\.19\overline{)52.08}\end{array}$	$\begin{array}{r}3.0\\2.5\overline{)7.431}\end{array}$	$\begin{array}{r}1.0\\9.3\overline{)9.417}\end{array}$
6.	$\begin{array}{r}.6\\4.9\overline{)2.814}\end{array}$	$\begin{array}{r}.3\\6.5\overline{)1.922}\end{array}$	$\begin{array}{r}.6\\3.4\overline{)1.928}\end{array}$	$\begin{array}{r}181.7\\.18\overline{)32.71}\end{array}$
7.	$\begin{array}{r}3.4\\2.1\overline{)7.222}\end{array}$	$\begin{array}{r}6.3\\4.2\overline{)26.54}\end{array}$	$\begin{array}{r}17.5\\.56\overline{)9.815}\end{array}$	$\begin{array}{r}1.4\\6.7\overline{)9.114}\end{array}$

More Practice. See ㊱ on page 337. Use for reteaching.

Have pupils do ex. 1-3 first. Discuss solutions with class and try to clear up any difficulties before assigning ex. 4-7. Group pupils who had errors to determine causes and need for reteaching.



Practicing for the Boat Race

Extend work on distance-rate-time problems (pages 114–115).

1. In practicing for the boat race, the crew of Westfield College covered the 4-mile course in 20 min. How many miles per minute did they average?

▶ The number of miles rowed per minute is called the average **rate** or **speed** in miles per minute.

2. In ex. 1, how many miles did the Westfield crew row in 5 min.? in 10 min.? in 15 min.?
1.0 2.0 3.0

3. If the Westfield crew could row for 1 hr. at the same rate that they rowed in ex. 1, how many miles would they go in that hour? 12 Do you think that a college crew could row for a full hour at that rate? Look up in the *World Almanac* some rowing records. Have pupils report findings to class.

4. A college crew covered a 3-mile course in 15.5 min. Find their speed to the nearest hundredth of a mile per min. .19

In ex. 1 be sure pupils understand meaning of rate or speed, and the units in which it is expressed. Do ex. 1–4 with pupils and check to see if problem situations or division process causes difficulty.

Distance, Rate, and Time

1. Jack can ride his bicycle at the rate of $6\frac{1}{2}$ mi. per hour. How far can he go in 2 hr.? in $2\frac{1}{2}$ hr.? in 3 hr.?
13 mi. $16\frac{1}{4}$ mi. $19\frac{1}{2}$ mi.

2. Mr. Wright drove his car 95 mi. in $2\frac{1}{2}$ hr. Find the average rate at which he drove his car on this trip. 38 mph

3. If an airplane travels at a speed of 480 mi. per hour, how many hours will it take the plane to make a trip of 960 mi.? 2

4. On a nonstop trip an airplane traveled 945 mi. in $3\frac{1}{4}$ hr. Find its average speed to the nearest tenth of a mile per hour. First change $3\frac{1}{4}$ hr. to 3.25 hr. 290.8 mph

5. In a race of 500 mi. a racing car made a time of $4\frac{1}{4}$ hr. Find, to the nearest hundredth of a mile per hour, the average speed of the car. 117.65 mph

6. Miss Lewis finds that her average speed of driving on a long trip is about 35 mi. per hour. At this rate how many hours should she allow for a trip of 280 mi.? Include an extra hour for a stop for lunch. 9

7. An airplane flew nonstop a distance of 8193 mi. in 30 hr. Find, to the nearest tenth of a mile per hour, the average speed of this airplane. 273.1 mph

8. A fast train averages 55 mi. per hour. At that rate how many miles will it travel in 3 hr. 36 min.? 198 At that rate how many hours and minutes will the train take to travel 374 mi.? 6 hr. 48 min.

9. A train leaves St. Louis at 2:10 P.M. and arrives in Memphis at 10:55 P.M. The distance is 329.9 mi. Find, to the nearest tenth of a mile per hour, the speed of the train. 37.7 mph

10. If an airplane has a speed of 355 mi. per hour, what is its speed in miles per minute? How many feet does this airplane travel per second? $5\frac{11}{12}$ mi. $520\frac{2}{3}$ ft.

11. If d represents the distance, r the rate, and t the time:

$d = r \times t$ $r = d \div t$ $t = d \div r$

Tell in words what these formulas mean.

Discuss the formulas in ex. 11 and be certain pupils understand operations indicated. Discuss how formulas can be used to solve problems in ex. 1–10. Have pupils solve other problems using the formulas.

Reteach and give practice on zero difficulties in quotients where there is a decimal divisor and/or dividend (pages 116-117).

Zero in the Quotient

1. Study these examples carefully:

A. $\begin{array}{r} .045 \\ 6\overline{).270} \end{array}$ B. $\begin{array}{r} 43\,40. \\ .05_\wedge\overline{)217.00_\wedge} \end{array}$ C. $\begin{array}{r} 3\,600. \\ .006\overline{)21.600_\wedge} \end{array}$

In A above, the 4 in the quotient is placed above 7 since 27 was divided by 6 to get 4. The decimal point in the quotient is placed directly above the decimal point in .270. This leaves a space in the quotient, between the decimal point and 4, which should be filled with a zero. The quotient is .045.

In B, multiply .05 by 100 to make the divisor a whole number. To keep the quotient unchanged, also multiply 217 by 100. Use carets in each case to show the new positions of the decimal points. Place the decimal point in the quotient above the caret in 217.00‸. The first three figures of the quotient are 434. This leaves a space between 4 and the decimal point which should be filled by a zero. The quotient is 4340.

In C, the quotient has two spaces between 36 and the decimal point; these spaces should be filled by two zeros.

Be sure pupils know what power of 10 was used in C.

2. The bus fare on the Bee Line is $.12. How many fares were collected if the total amount received one day was $941.40? 7845

Have pupils explain zeros in quotients and placement of quotient figures in ex. 3-8.

Divide. Check by multiplying:

3.	$\begin{array}{r} .08 \\ 8\overline{).64} \end{array}$	$\begin{array}{r} 500 \\ .05\overline{)25} \end{array}$	$\begin{array}{r} .042 \\ 4\overline{).168} \end{array}$	$\begin{array}{r} 1900 \\ .19\overline{)361} \end{array}$	$\begin{array}{r} 400 \\ .014\overline{)5.6} \end{array}$
4.	$\begin{array}{r} .09 \\ 4\overline{).36} \end{array}$	$\begin{array}{r} 800 \\ .03\overline{)24} \end{array}$	$\begin{array}{r} .007 \\ 8\overline{).056} \end{array}$	$\begin{array}{r} 4800 \\ .06\overline{)288} \end{array}$	$\begin{array}{r} 3000 \\ .025\overline{)75} \end{array}$
5.	$\begin{array}{r} 120 \\ .6\overline{)72} \end{array}$	$\begin{array}{r} 60 \\ .07\overline{)4.2} \end{array}$	$\begin{array}{r} 320 \\ .7\overline{)224} \end{array}$	$\begin{array}{r} .09 \\ 1.5\overline{).135} \end{array}$	$\begin{array}{r} 12{,}000 \\ .007\overline{)84} \end{array}$
6.	$\begin{array}{r} 70 \\ .9\overline{)63} \end{array}$	$\begin{array}{r} 90 \\ .09\overline{)8.1} \end{array}$	$\begin{array}{r} .039 \\ 6\overline{).234} \end{array}$	$\begin{array}{r} 2400 \\ .07\overline{)168} \end{array}$	$\begin{array}{r} 1100 \\ .009\overline{)9.9} \end{array}$
7.	$\begin{array}{r} .06 \\ 3\overline{).18} \end{array}$	$\begin{array}{r} 1800 \\ .02\overline{)36} \end{array}$	$\begin{array}{r} 720 \\ .9\overline{)648} \end{array}$	$\begin{array}{r} .09 \\ 2.8\overline{).252} \end{array}$	$\begin{array}{r} 400 \\ .018\overline{)7.2} \end{array}$
8.	$\begin{array}{r} .07 \\ 7\overline{).49} \end{array}$	$\begin{array}{r} 900 \\ .08\overline{)72} \end{array}$	$\begin{array}{r} 250 \\ .5\overline{)125} \end{array}$	$\begin{array}{r} 1200 \\ .12\overline{)144} \end{array}$	$\begin{array}{r} 4000 \\ .023\overline{)92} \end{array}$

More Practice. See ㊲ on page 337.

When discussing ex. 1 be sure pupils understand why zero is placed in quotient (A. to show that tenths were divided; B. ones were divided; C. tens and ones were divided). Emphasize place value in ex. 3-8.

1. Sam paid \$6.60 for 240 newspapers that he sold last week. How much did he have to pay for each newspaper? $2\frac{3}{4}$¢
2. Sam sells the papers for \$.05 each. How many papers does he have to sell to take in \$16? 320
3. Joe buys a package of typewriter paper for \$2. There are 500 sheets in the package. What is the cost of this paper per sheet? $\frac{2}{5}$¢ If he uses 30 sheets of this paper to write a story, how much do the 30 sheets cost him? 12¢

Divide. Check by multiplying: Emphasize.

4. $\overset{.8}{7\overline{)5.6}}$ $\overset{.08}{7\overline{).56}}$ $\overset{80}{.7\overline{)56}}$ $\overset{80}{.07\overline{)5.6}}$ $\overset{800}{.07\overline{)56}}$

5. $\overset{.05}{9\overline{).45}}$ $\overset{5}{.9\overline{)4.5}}$ $\overset{50}{.9\overline{)45}}$ $\overset{500}{.09\overline{)45}}$ $\overset{.5}{.09\overline{).045}}$

6. $\overset{9}{.3\overline{)2.7}}$ $\overset{90}{.3\overline{)27}}$ $\overset{.9}{.3\overline{).27}}$ $\overset{900}{.03\overline{)27}}$ $\overset{90}{.03\overline{)2.7}}$

7. $\overset{40}{.8\overline{)32}}$ $\overset{.4}{8\overline{)3.2}}$ $\overset{.4}{.8\overline{).32}}$ $\overset{400}{.08\overline{)32}}$ $\overset{4}{.08\overline{).32}}$

8. $\overset{.6}{6\overline{)3.6}}$ $\overset{.6}{.6\overline{).36}}$ $\overset{60}{.6\overline{)36}}$ $\overset{60}{.06\overline{)3.6}}$ $\overset{600}{.06\overline{)36}}$

9. $\overset{.7}{4\overline{)2.8}}$ $\overset{7}{.4\overline{)2.8}}$ $\overset{70}{.4\overline{)28}}$ $\overset{700}{.04\overline{)28}}$ $\overset{.7}{.04\overline{).028}}$

In the quotients below, the decimal points are left out. In some quotients, the 0's are also left out. In each quotient, put the decimal point in the right place, supplying 0's if necessary:

10. $\overset{860.}{.7\overline{)6020.}}$ $\overset{1380.}{1.2\overline{)16560.}}$ $\overset{7\,300.}{.025\overline{)182.500}}$ $\overset{42\,500.}{.037\overline{)1572.500}}$

11. $\overset{.037}{8\overline{).296}}$ $\overset{.053}{4.6\overline{).2438}}$ $\overset{.091}{102\overline{)9.282}}$ $\overset{1\,070.}{.118\overline{)126.260.}}$

12. $\overset{940.}{.6\overline{)5640.}}$ $\overset{6200.}{.28\overline{)173600.}}$ $\overset{208,000.}{.007\overline{)1456\,000.}}$ $\overset{214,900.}{.009\overline{)1934.100.}}$

13. $\overset{.075}{9\overline{).675}}$ $\overset{490.}{6.3\overline{)30870.}}$ $\overset{90.}{3.05\overline{)274.50}}$ $\overset{.0373}{23.6\overline{).88028}}$

Use ex. 2 to point out that when divisor is decimal (less than 1) quotient will be larger than dividend. (Since \$.05 is contained in \$1 twenty times, how many times is it contained in \$2, \$5, \$10, \$16?) Give other illustrations.

Present review of the four processes with decimals, retaught in Chapter 3.

Chapter Review

Read these numbers.(1) Then write each one as a common fraction or a mixed number:(2)

(1),(2) 2 and 5 tenths, $2\frac{1}{2}$ 4 and 375 thousandths $4\frac{3}{8}$ 9 tenths, $\frac{9}{10}$

1. 2.5 7.25 4.375 11.7 0.9 0.15

7 and 25 hundredths, $7\frac{1}{4}$ 11 and 7 tenths, $11\frac{7}{10}$ 15 hundredths, $\frac{3}{20}$

Change these fractions to decimals:

2. $\frac{3}{5}$ .6 $\frac{9}{20}$ .45 $\frac{7}{8}$ .875 $\frac{3}{10}$ .3 $\frac{11}{16}$ .6875 $\frac{19}{100}$ .19 $\frac{33}{1000}$ .033 $\frac{7}{25}$ .28

Change to decimals correct to the nearest hundredth:

3. $\frac{7}{19}$ .37 $\frac{6}{11}$ .55 $\frac{2}{13}$ .15 $\frac{2}{9}$ .22 $\frac{13}{15}$ .87 $\frac{17}{21}$ .81 $\frac{5}{7}$ .71 $\frac{12}{19}$ .63

Write in Roman numerals:

4. 947 CMXLVII 1832 MDCCCXXXII 2465 MMCDLXV 35,846 $\overline{XXXV}DCCCXLVI$

Add or subtract as indicated. Check the work:

5. 1.28 + 0.36 + 7.02 + 9.45 18.11 16.375 − 2.125 14.250

6. .375 + 5.513 + 1.758 + 20.517 28.163 \$200 − \$4.95 \$195.05

Multiply. Check by going over the work:

7. $\begin{array}{r}.016\\ 9\\ \hline .144\end{array}$ $\begin{array}{r}.002\\ 15\\ \hline .030\end{array}$ $\begin{array}{r}.047\\ .43\\ \hline .02021\end{array}$ $\begin{array}{r}11.6\\ 7.5\\ \hline 87.00\end{array}$ $\begin{array}{r}.012\\ .08\\ \hline .00096\end{array}$ $\begin{array}{r}3.17\\ 40\\ \hline 126.80\end{array}$

Divide until there is no remainder. Annex zeros if necessary:

8. $\overset{7.6}{5\overline{)38.0}}$ $\overset{2.1375}{8\overline{)17.1000}}$ $\overset{38.75}{24\overline{)930.00}}$ $\overset{.9}{.06\overline{).054}}$ $\overset{.043}{13\overline{).559}}$

9. $\overset{9.5}{.2\overline{)1.90}}$ $\overset{73.5}{.6\overline{)44.10}}$ $\overset{32.8}{25\overline{)820.0}}$ $\overset{1.7625}{16\overline{)28.2000}}$ $\overset{12.75}{4.4\overline{)56.100}}$

Divide. In ex. 10, find quotients correct to the nearest tenth. Stress. In ex. 11 and 12, find quotients correct to the nearest hundredth: Stress.

10. $\overset{11.6}{7\overline{)81}}$ $\overset{1.3}{.6\overline{).77}}$ $\overset{54.1}{.9\overline{)48.7}}$ $\overset{9.1}{42\overline{)381}}$ $\overset{1.6}{.09\overline{).147}}$

11. $\overset{.07}{9\overline{).59}}$ $\overset{.55}{.8\overline{).437}}$ $\overset{.10}{.7\overline{).068}}$ $\overset{22.42}{1.9\overline{)42.6}}$ $\overset{8.19}{.36\overline{)2.95}}$

12. $\overset{.03}{8\overline{).21}}$ $\overset{.41}{.4\overline{).163}}$ $\overset{.41}{.9\overline{).372}}$ $\overset{8.91}{2.3\overline{)20.5}}$ $\overset{14.02}{.48\overline{)6.73}}$

Use review as preparation for diagnostic test (page 120). Clear up difficulties (reteach or review further) before assigning remedial work.

A Problem Test

1. At an average speed of 35.5 mi. per hour, how many hours will it take Mr. Johnson to drive 200 mi.? Find the answer to the nearest tenth of an hour. 5.6

2. Jane is 62 in. tall, Betty is 59 in. tall, and Sally is $57\frac{1}{2}$ in. tall. Find the average height of these girls. $59\frac{1}{2}$ in.

3. On a trip Bob bought 15 post cards to send to his friends. Find the cost of 15 cards at 3 cards for 10¢. 50¢

4. Mrs. Mason paid $2.50 for 1 doz. cans of soup and 2 packages of crackers. The cans of soup cost $1.96. Find the cost of 1 package of crackers. $.27

5. Jim earns 60¢ an hour working for Mr. Price. Last week he worked these 5 times: Monday, $2\frac{1}{2}$ hr.; Tuesday, $1\frac{3}{4}$ hr.; Thursday, $2\frac{1}{4}$ hr.; Friday, $2\frac{3}{4}$ hr.; Saturday, $5\frac{1}{4}$ hr. Find the average amount of money Jim earned per day. $1.74

6. A 26-trip railroad ticket to Fairfield costs $10.65. A single ticket for the same trip costs $.62. How much in all can be saved on 26 trips by buying a 26-trip ticket? $5.47

7. Find the cost of 27 in. of lace at $1.25 a yard and 1 yd. 9 in. of ribbon at $.50 a yard. $1.57

8. How much change should Tom get from $10.00 after buying 3 radio tubes at $1.45 each and wire for $2.39? $3.26 Tell what coins and bills he might receive in change. 1 penny, 1 quarter, and 3 one-dollar bills

9. A cake recipe calls for $2\frac{1}{2}$ cups of flour and $\frac{1}{2}$ cup of butter. How much flour and how much butter should be used if only $\frac{1}{2}$ the recipe is made? $1\frac{1}{4}$ cups flour, $\frac{1}{4}$ cup butter

10. Robert had 4 bu. of apples to sell. He sold 10 pk. of them at $.75 a peck and the rest at $.60 a peck. How much in all did he get for the apples? $11.10

Have pupils record scores and compare with others.

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Instruct pupils to read problems carefully. Check papers and analyze errors. Have pupils explain their solutions to help determine causes of errors (computational, lack of understanding of problem situation, and so on).

Present diagnostic test of skills retaught in Chapter 3, with

How Much Have You Learned? practice-page references.

If you miss more than one example in a row, turn to the Practice Pages for that row.

Change to decimals:

						Practice Pages
1.	$\frac{7}{8}$ .875	$\frac{4}{5}$ .8	$\frac{9}{16}$ .5625	$\frac{14}{25}$ .56	$\frac{17}{20}$ .85	106, 107

Change to decimals correct to the nearest hundredth:

2.	$\frac{1}{13}$ .08	$\frac{6}{11}$ .55	$\frac{32}{35}$ .91	$\frac{11}{24}$ .46	$\frac{5}{19}$ .26	107

Add up. Check by adding down:

3.	1.71 4.02 8.37 3.35 17.45	3.375 4.250 1.500 6.000 15.125	120.5 243.6 89.2 117.4 570.7	$16.38 9.20 7.00 28.16 $60.74	89, 90

Subtract. Check by adding:

4.	10.0 8.3 1.7	452.8 259.9 192.9	2.125 1.750 .375	$20.00 12.94 $ 7.06	91

Multiply and check the work:

5.	16.05 12 192.60	.749 4.5 3.3705	21.3 .003 .0639	64.8 .25 16.200	96–98, 100

Divide. If you continue to have a remainder, find the answer to the nearest tenth:

6.	3.875 8)31	.68 9)6.12	.35 76)26.6	.31 27)8.37	101–103
7.	.078 6).468	.031 7).217	.047 2.5).1175	.076 53)4.028	116
8.	.08 .7).056	730 .3)219	2900 .38)1102	1590 .08)127.2	111–113
9.	24.1 .9)21.7	75.2 6)451	1.7 4.9)8.147	3.1 .62)1.937	116

After checking papers and analyzing errors, confer with pupils who had mistakes to determine causes. Reteach where needed (stress meanings) before assigning remedial work. Pupils should have good mastery and understanding before beginning new work.

See Guide for specific aims of Chapter 4.

Chapter 4

Present problems on travel by air, including standard time zones and costs of tickets (pages 121-123).

Traveling by Airplane

Los Angeles	(PT)	Lv.	10:00
Chicago	(CT)	Ar.	**3:25**
		Lv.	**4:00**
Detroit	(ET)	Ar.	**5:50**

1. Susan and her family traveled by jet airplane from Los Angeles to Detroit. The table at the right gives the schedule of the plane on which they traveled. The hours printed in light type are morning hours; those in dark type are afternoon and evening hours. What do PT,(1) CT,(2) and ET(3) mean? See page 81. In which time zone did the plane not stop? Mountain

(1) Pacific Time; (2) Central Time; (3) Eastern Time

2. What time does the plane leave Los Angeles? 10:00 A.M. What time is it in Detroit when the plane leaves Los Angeles? 1:00 P.M.

3. When does the plane arrive at 3:25 P.M. and leave Chicago? 4:00 P.M. When the plane arrives in Chicago, what time is it in Los Angeles? 1:25 P.M. in Detroit? 4:25 P.M.

4. What time does the plane reach Detroit? 5:50 P.M. What time is it then in Los Angeles? 2:50 P.M. in Chicago? 4:50 P.M.

First review work on pages 81-82. These problems may be done orally with class to be sure all understand differences in time in the different zones.

Review meaning of per cent and language peculiar to percentage (pages 124-125). See Guide for discussion.

Meaning of Per Cent

1. At the right is a tag like the one that came on Jack's new sweater. This is read "80 per cent wool; 20 per cent cotton." The sign % means **per cent** or **hundredths.** Another way to write 80% is $\frac{80}{100}$, or .80.

Stress meaning.

Here are four ways of writing hundredths:

$\frac{83}{100}$ = .83 = 83% = 83 per cent

$\frac{7}{100}$ = .07 = 7% = 7 per cent

When you write **hundredths** by means of the per cent sign, you are using **percentage.** You often see per cents in newspapers and magazines. They are used in business, science, and industry. You cannot understand statements that use the words "per cent" or the sign "%" if you are not familiar with the meaning of per cent.

Emphasize that any one of the ways of writing per cent may be exchanged for another.

2. Since 87% = .87 and 3% = .03, you see that a per cent is another way to write a decimal. In working problems in percentage, it is often necessary to change per cents to decimals or to change decimals to per cents.

Stress.

Write these fractions as per cents:

3. $\frac{19}{100}$ 19% $\frac{37}{100}$ 37% $\frac{80}{100}$ 80% $\frac{52}{100}$ 52% $\frac{75}{100}$ 75% $\frac{9}{100}$ 9% $\frac{1}{100}$ 1%

Write these decimals as per cents:

4. .17 17% .40 40% .33 33% .25 25% .01 1% .08 8% .02 2%

Have pupils bring in examples of uses of per cents, as found in newspapers, and so on.

Read these statements that contain per cents:

5. Farmers seek 5% rise in the price of milk.

6. Prices on all tennis rackets are reduced 25%.

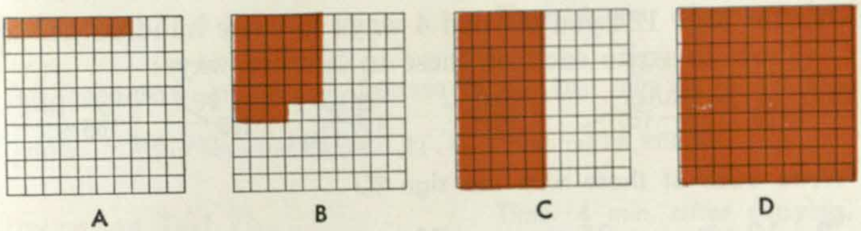
7. Automobile production has increased 10%.

8. All airline fares are subject to a 5% Government tax.

9. There is a 3% sales tax in the city where Ben lives.

10. There is a 10% tax on the price of watches.

Use a hundred board in discussing ex. 1 and point out that 80% means 80 out of 100. Stress fact that per cent is another way of writing hundredths (decimal and common fractions).



1. Each large square above is divided into 100 small squares. One small square is $\frac{1}{100}$ of the large square. What per cent of each large square is each small square? 1%

2. In square A, 7 small squares are red. What per cent of square A is red? How many small squares in A are white? 93 How can you find the number of white squares without counting them? Subtract 7 from 100 What per cent of A is white? 93%

3. Look at squares B and C and tell what per cent of each is red and what per cent of each is white.
B: 28% red, 72% white; C: 50% red, 50% white

4. In square D, you see that **all** the small squares, or 100 small squares, are red. This means that $\frac{100}{100}$ or 100% of square D is red. You see that 100% of square D means **all** of square D. Stress.

5. Draw 3 large squares like squares A, B, and C above. Divide each of your squares into 100 small squares; to do this, mark off 10 equal spaces on each side of the large square. By drawing lines you will then have 100 small squares in each large square. Shade 25% of the first square,(1) 41% of the second square,(2) and 50% of the third square.(3) What per cent of each square is not shaded? (1) 75%; (2) 59%; (3) 50%

6. Make 2 more squares like those in ex. 5. Shade all but (1) 10% of the first square and all but 25% of the second square.(2) What per cent of each square is shaded? (1) 90%; (2) 75%

7. Draw some squares and divide each into 100 small squares. Draw designs in each square by coloring some of the small squares; then tell what per cent of each square is colored and what per cent is white.

Discuss. ex. 1–4 with pupils, emphasizing meaning of per cent as "so many out of a hundred." Supply pupils with graph paper to use for ex. 5–7. Give further work in showing various per cents on diagrams.

Emphasize that any one of the 4 ways of writing per cents may be exchanged for another.

Reading and Writing Per Cents

1. On page 124 you learned 4 ways to write hundredths. Show how to write each of these in 3 other ways:

.39, 39%, .39 per cent .57, 57%, 57 per cent .09, 9%, 9 per cent .01, 1%, 1 per cent

$\frac{39}{100}$ $\frac{43}{100}$ $\frac{57}{100}$ $\frac{13}{100}$ $\frac{9}{100}$ $\frac{7}{100}$ $\frac{1}{100}$

.43, 43%, 43 per cent .13, 13%, 13 per cent .07, 7%, 7 per cent

Write each of these with the sign %:

2. .19 19% .25 25% .70 70% .08 8% .05 5% .01 1%

3. $\frac{33}{100}$ 33% $\frac{75}{100}$ 75% $\frac{22}{100}$ 22% $\frac{38}{100}$ 38% $\frac{3}{100}$ 3% $\frac{8}{100}$ 1%

Write each of these as decimals:

4. $\frac{72}{100}$ .72 $\frac{11}{100}$ .11 $\frac{60}{100}$ .60 $\frac{15}{100}$ .15 $\frac{2}{100}$ .02 $\frac{6}{100}$ .06

5. 75% .75 43% .43 68% .68 8% .08 4% .04 1% .01

6. 16 per cent .16 9 per cent .09 2 per cent .02

7. Bill read that 75% of a banana is water. Say this by using a fraction instead of a per cent. $\frac{3}{4}$

8. Sally read that 50% of the apple crop was destroyed by a storm. Say this by using a fraction instead of a per cent. $\frac{1}{2}$

9. Mr. Baker tries to save 10% of the money he earns. Say this statement, using a fraction instead of a per cent. $\frac{1}{10}$

10. Twenty-five per cent of the pupils in our class have been present in school every day this year. Say this statement, using a fraction instead of a per cent. $\frac{1}{4}$

11. In the high school 90% of the pupils voted that ice skating was their favorite winter sport. Say this statement, using a fraction instead of a per cent. $\frac{9}{10}$

12. In the election for president of the Science Club, Bill received 80% of all the votes cast. Use a fraction instead of a per cent in making this statement. $\frac{4}{5}$

13. Jack and Tom sold all the corn they raised last summer and divided the money "fifty-fifty." Say this statement, using the language of percentage. 50% each

Do this page as class activity, letting different pupils show answers at chalkboard. In ex. 1 be sure pupils use = sign ($\frac{39}{100}$ = .39 = 39% = 39 per cent). Give further practice as needed.

Instruct pupils to copy problems carefully and legibly before beginning test. After checking papers and analyzing errors, return them to pupils so they may find and correct mistakes (give help as needed).

Improving by Practice

Multiplication Test 2a.

Time: 4 min. after copying.

1.	751	378	869	102	368	472
	348	759	206	127	584	630
	261,348	286,902	179,014	12,954	214,912	297,360

⑥

Multiplication Test 2b.

Time: 4 min. after copying.

2.	374	135	546	153	809	982
	512	486	937	602	193	740
	191,488	65,610	511,602	92,106	156,137	726,680

⑥

Multiplication Test 2c.

Time: 4 min. after copying.

3.	247	295	508	143	412	736
	366	905	721	487	259	140
	90,402	266,975	366,268	69,641	106,708	103,040

⑥

Division Test 2a.

Time: 5 min. after copying.

4.	38)31312 — 824	43)34800 — 809 R13	59)15576 — 264
5.	75)60300 — 804	97)74311 — 766 R9	19)8246 — 434

⑥

Division Test 2b.

Time: 5 min. after copying.

6.	24)6330 — 263 R18	32)9792 — 306	46)39928 — 868
7.	58)51910 — 895	62)21840 — 352 R16	71)61202 — 862

⑥

Division Test 2c.

Time: 5 min. after copying.

8.	83)48400 — 583 R11	95)34580 — 364	78)11466 — 147
9.	69)48720 — 706 R6	56)29344 — 524	42)41748 — 994

⑥

To the Teacher. In ex. 4–9, pupils should write R before remainders.

Present second set of improvement tests in multiplication and in division. Have pupils determine own scores and record them.

Review meaning of multiplication when the multiplier is a common or decimal fraction.

Finding a Per Cent of a Number

1. Problem Last spring the pupils of the Park School began to earn $450 for a new movie projector, screen, and films. The Thanksgiving issue of the school paper stated that the amount earned up to that time was 62% of the whole amount. How much had they earned by Thanksgiving? $279.00

Explanation This means that 62% of $450 or $\frac{62}{100}$ of $450 had been earned. You know that $\frac{62}{100}$ of $450 = .62 × $450. Multiplying $450 by .62, you get $279.00. So $279 had been earned by Thanksgiving. Check the work by going over it.

$$\begin{array}{r} \$450 \\ .62 \\ \hline 9\;00 \\ 270\;0 \\ \hline \$279.00 \end{array}$$

2. You know that 62% of $450 means .62 × $450. What does 24% of 150 mean? How much is 24% of 150? 36

.24 x 150

Tell the number to put in each space:

3. 64% of 150 = .64 × 150 = 96

4. 30% of 320 = .30 × 320 = 96

5. 2% of 600 = .02 × 600 = 12

To find a per cent of a number, change the per cent to a decimal and multiply.

Find the answers:

6.	35% of 60 21	8% of 300 24	2% of 600 12	19% of $200 $38.00
7.	12% of 75 9	4% of 250 10	6% of 350 21	25% of $195 $48.75
8.	40% of 95 38	3% of 100 3	5% of 460 23	11% of $250 $27.50
9.	84% of 50 42	9% of 400 36	8% of 250 20	40% of $175 $70.00
10.	20% of 70 14	7% of 200 14	4% of 150 6	50% of $186 $93.00
11.	50% of 84 42	5% of 480 24	9% of 700 63	15% of $104 $15.60

More Practice. See ㊳ on page 338.

Follow development as given in ex. 1. Have pupils use common fractions also to solve some examples, to point out that using a decimal fraction is easier. Give more practice in changing per cents to decimal fractions, if needed.

Finding a Per Cent of a Number

1. On a test of 25 problems Mary worked 92% correctly. How many problems did Mary work correctly? 23

2. In a class of 40 pupils the girls made up 55% of the class. How many girls were in the class? 22

3. Bill had 95% of his answers correct on a science test of 20 questions. How many of the 20 answers did he have correct? 19 How many did he miss? 1

4. There are 2150 pupils in the Westbrook schools and 72% of them have money in the school savings bank. How many pupils have money in the school bank? 1548

5. A grocer finds that in a box of oranges about 2% of the oranges will spoil before all the oranges are sold. About how many oranges will spoil in a box of 150 oranges? 3

6. Mr. Bird bought 100 chicks in March. He was able to raise 90% of them. How many chicks did he raise? 90

Find the answers. Check the work by going over it:

7. 20% of 45 9 — 5% of $280 $14.00 — 32% of $1000 $320.00 ✓
8. 50% of 98 49 — 2% of $650 $13.00 — 45% of $2440 $1098.00
9. 25% of 48 12 — 8% of $275 $22.00 — 18% of $1550 $279.00
10. 60% of 35 21 — 6% of $950 $57.00 — 90% of $2800 $2520.00
11. 82% of 50 41 — 1% of $600 $6.00 — 26% of $6000 $1560.00
12. 24% of 75 18 — 7% of $300 $21.00 — 15% of $3680 $552.00
13. 85% of 80 68 — 4% of $725 $29.00 — 33% of $8000 $2640.00
14. 12% of 25 3 — 9% of $500 $45.00 — 50% of $2384 $1192.00
15. 40% of 90 36 — 3% of $200 $6.00 — 14% of $1750 $245.00
16. 15% of 60 9 — 8% of $450 $36.00 — 12% of $2000 $240.00
17. 30% of 20 6 — 5% of $120 $6.00 — 20% of $1635 $327.00
18. 16% of 75 12 — 7% of $900 $63.00 — 17% of $1500 $255.00

Do problems orally with class, letting different pupils explain solutions. Have pupils complete ex. 7-18. Group pupils who need further teaching. Have others make up problems using numbers in ex. 7-18.

Meaning of 100 Per Cent

1. On page 125, **all** of square D is red. So $\frac{100}{100}$ or 100% of it is red. You see that:

$$\frac{100}{100} = 1 = 100\% = 100 \text{ per cent}$$

2. 100% of 1 apple means $\frac{100}{100}$ of one apple, or $\frac{100}{100} \times 1$ apple.
Since $\frac{100}{100} \times 1 = 1 \times 1 = 1$, 100% of 1 apple means the **whole** apple.

3. 100% of 675 pupils means $\frac{100}{100} \times 675$ pupils, or 1×675 pupils. Since $1 \times 675 = 675$, you see that 100% of 675 pupils means **all** the 675 pupils.

4. 100% of the boys means **all** the boys.
100% of the tickets means **all** the tickets.
100% of the class means the **whole** class.
100% of the examples correct means **all** examples correct.

5. John read about a truck driver whose "safety record" was 100% for the past ten years. What does this mean?

6. Mrs. Walker bought a coat that was marked "100% wool." Tell the meaning of this.

7. Try to find a statement that contains the expression "100 per cent" or "100%." You may find it in your reading, or in a store, or you may hear it on the radio or television.

Have pupils bring examples to class and explain meaning.

Make these statements using the language of percentage:

8. Every member of the class belongs to a school club.

9. Jane worked every problem correctly.

10. Dick spent all of his savings for a bicycle.

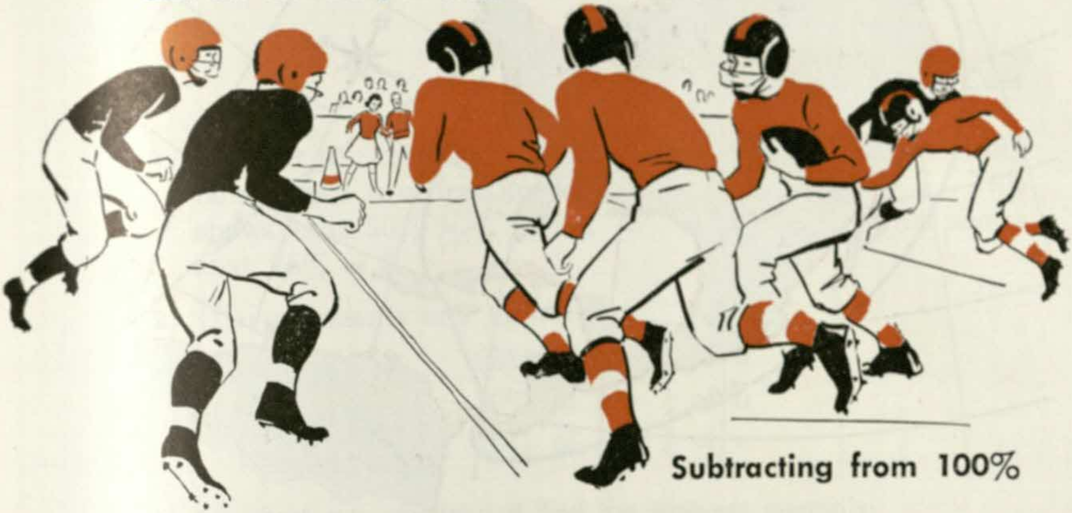
11. This year all the pupils in the fifth, sixth, and seventh grades of the George Washington School have become members of the Junior Red Cross.

12. All the employees of the Smith Company contributed to the Thanksgiving collection.

13. Mr. Hamilton built a new apartment house. Yesterday he said that all the apartments were rented.

Discuss concepts in ex. 1-6 with pupils, stressing idea that 100% means "all." Have pupils bring to class various examples of use of 100% as given in ex. 8-13.

Show how the whole, or 100%, is made up of parts that total 100%.



Subtracting from 100%

1. Our football team has won 85% of the games it has played. What per cent of the games has it lost? 15%

▶ All of the games are 100% of them. If 85% of the games have been won, the rest, or 15% of the games, have been lost. You find 15% by subtracting 85% from 100%.

2. Mary saves 10% of her allowance. What per cent of her allowance does she spend? 90%

3. Jack plays on the Newtown Junior High School baseball team. He says the team won 90% of the games it played last spring. What per cent of the games did the team lose? 10%

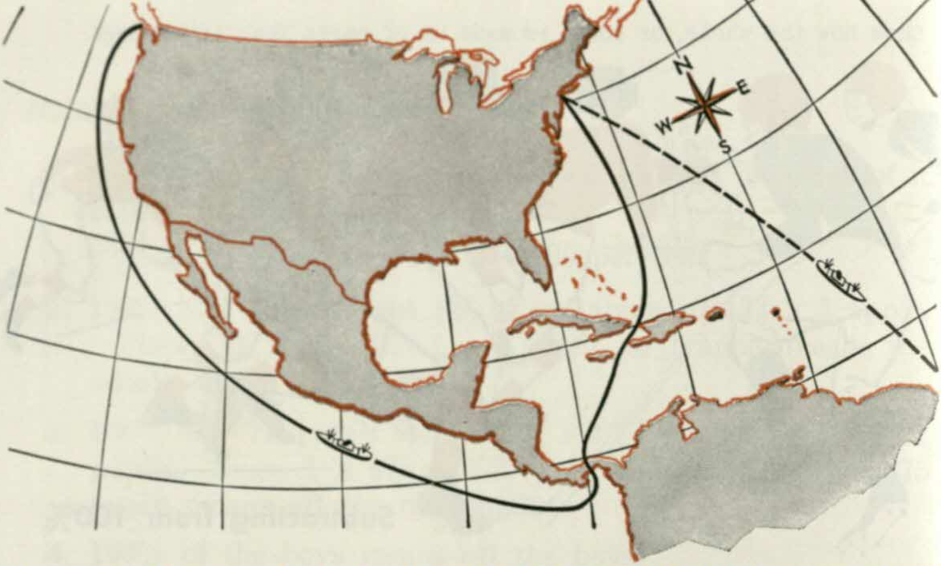
4. In December Andy bought a football at a sale and got "25% off" the regular price. What per cent of the regular price did he pay for the football? 75%

5. Betty saves 20% of the money she earns. What per cent of it does she spend? 80%

6. A clerk sold a blanket to Mrs. Brown. He said it contained 10% rayon and 30% cotton, and the rest was wool. What per cent of the blanket was wool? 60%

7. The Washington School raised some money by showing a travel movie. 50% of the money was given to the Athletic Club, 20% to the school paper, and the rest to the Girls' Club. What per cent was given to the Girls' Club? 30%

Do problems orally with the class. Emphasize why we subtract from 100% (since 100% means all or the whole). Point out also that per cents all have same denominator. Have pupils make up similar problems.



Teach fractional equivalents of frequently used per cents (pages 132–136).

Short Cuts

1. The Panama Canal is a short cut that saves time and distance. If you travel by ship from New York to Seattle by the route around South America, the distance is 13,914 mi., but if you go by way of the Panama Canal, the distance is only 6038 mi. How many miles of travel are saved by the Panama Canal route? 7876

2. There are also short cuts in finding certain per cents of a number. For example, to find 25% of \$2.00, you can multiply \$2.00 by .25, which gives \$.50 as the answer, as shown at A. A shorter way to get the same result is to take $\frac{1}{4}$ of \$2.00, as shown at B. You see that $25\% = \frac{25}{100} = \frac{1}{4}$, so $\frac{1}{4}$ of a number is the same as 25% of it.

$$\begin{array}{r} \text{A} \\ \$2.00 \\ \underline{.25} \\ 1000 \\ \underline{400} \\ \$.5000 \end{array} \qquad \begin{array}{r} \text{B} \\ \$.50 \\ 4\overline{)\$2.00} \end{array}$$

Use the short cut and find the answers mentally:

3. 25% of \$20 \$5 — 25% of \$80 \$20 — 25% of \$100 \$25

4. 25% of \$36 \$9 — 25% of \$44 \$11 — 25% of \$120 \$30

5. 25% of \$18 \$4.50 — 25% of \$50 \$12.50 — 25% of \$200 \$50

6. 25% of \$32 \$8 — 25% of \$25 \$6.25 — 25% of \$480 \$120

Follow development as given in ex. 1–2. Bring out fact that sometimes it is easier to use fractional equivalent by having pupils use decimal in some cases in ex. 3–6.

More Short Cuts

1. Another per cent that is used often is 50%. This can easily be changed to a fraction as follows: $50\% = \frac{50}{100} = \frac{1}{2}$. This shows that $50\% = \frac{1}{2}$. Also, $75\% = \frac{75}{100} = \frac{3}{4}$. This shows that $75\% = \frac{3}{4}$.

2. If **n** represents any number:

$$25\% \text{ of } \mathbf{n} = \tfrac{1}{4} \text{ of } \mathbf{n}$$
$$50\% \text{ of } \mathbf{n} = \tfrac{1}{2} \text{ of } \mathbf{n}$$
$$75\% \text{ of } \mathbf{n} = \tfrac{3}{4} \text{ of } \mathbf{n}$$

Use the short cuts above and find the answers mentally:

3. 25% of 4 1 75% of 36 27 50% of 14 7 75% of 40 30
4. 50% of 2 1 25% of 44 11 75% of 20 15 25% of 36 9
5. 75% of 8 6 50% of 26 13 25% of 32 8 50% of 18 9
6. 50% of 6 3 75% of 28 21 25% of 28 7 75% of 12 9

7. Jack bought an old bicycle from Jim and paid Jim 50% of the amount he had paid for it when new. If the bicycle had cost Jim \$46.50, find the amount Jack paid for it. \$23.25

8. At a sale Mr. Baker bought a coat marked \$50.00 and a suit marked \$38.60. There was 25% off on each garment. How much did Mr. Baker save by buying at the sale? \$22.15

Use short cuts to find the answers:

9. 75% of 92 69 25% of \$130 \$32.50 50% of \$11.50 \$5.75
10. 25% of 76 19 50% of \$215 \$107.50 75% of \$16.24 \$12.18
11. 50% of 98 49 75% of \$160 \$120 25% of \$20.36 \$5.09
12. 25% of 68 17 50% of \$311 \$155.50 75% of \$37.48 \$28.11
13. 75% of 96 72 25% of \$178 \$44.50 50% of \$25.00 \$12.50
14. 50% of 88 44 75% of \$300 \$225 25% of \$12.60 \$3.15

More Practice. See ㊴ on page 338. Use to develop facility.

Discuss ex. 1–2 carefully with pupils. Ask them why $50\% = \frac{50}{100} = \frac{1}{2}$ and $75\% = \frac{75}{100} = \frac{3}{4}$. Understanding of how to change per cents to fractional equivalents should be assured before memorization of them is stressed.

Urge pupils to solve problems mentally. Discuss solutions with class, and note if fractional equivalents, interpreting problems, and so on, cause difficulty. Plan remedial work accordingly.

Using Short Cuts

1. The price of an arithmetic workbook is \$.84. When a school orders workbooks, there is a reduction of 25% on the price. How much is the reduction on an order from a school for 45 workbooks? \$9.45

2. In a drive for new members, the Boy Scouts in Miller County are trying to raise their membership 25%. There were 680 Boy Scouts before the drive began. How many new members do they want to enroll? 170

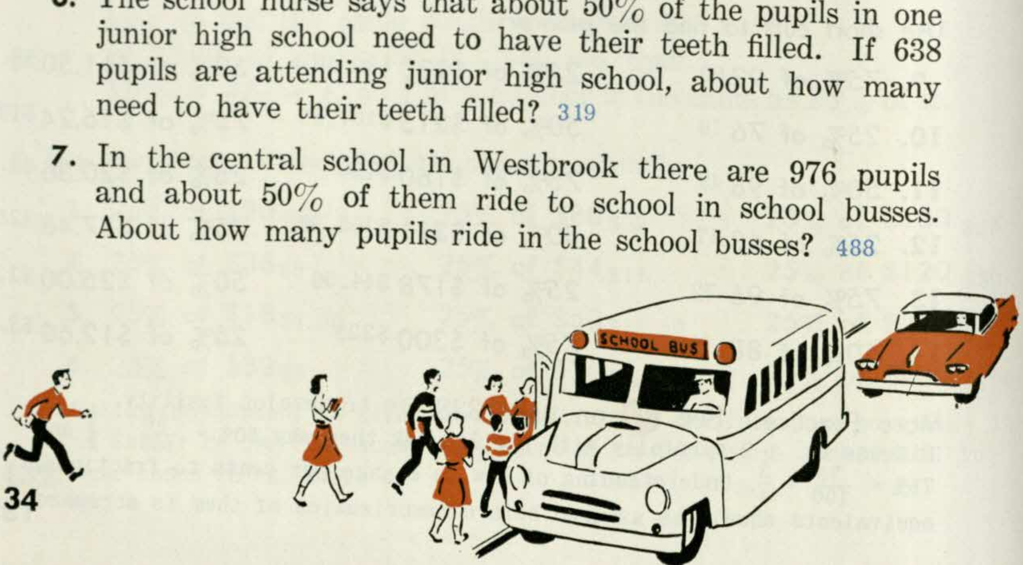
3. Jim works in a bicycle store after school and on Saturday. Because he works in the store, he can buy a bicycle for 75% of its regular price. If he buys a bicycle that sells for \$50.00, how much will he pay for it? \$37.50 If he buys one that sells for \$45.80, how much will he pay for it? \$34.35 How much will he save on the bicycle selling for \$45.80? \$11.45

4. Mr. Jackson says it takes about 50% of his income to pay for rent and food for his family. Mr. Jackson's income is \$425 a month. About how much does he spend for rent and food in a year? \$2550.00

5. When Mr. Hill bought a new car he turned in his old car and was allowed "25% off" the price of the new car. How much was he allowed for the old car if the price of the new car was \$2680? \$670

6. The school nurse says that about 50% of the pupils in one junior high school need to have their teeth filled. If 638 pupils are attending junior high school, about how many need to have their teeth filled? 319

7. In the central school in Westbrook there are 976 pupils and about 50% of them ride to school in school busses. About how many pupils ride in the school busses? 488



Other Short Cuts

State Institute of Education, Govt. of West Bengal

1. There are other per cents that can be easily represented by fractions. For example, $10\% = \frac{10}{100} = \frac{1}{10}$. So 10% of a number is the same as $\frac{1}{10}$ of it. In the same way $20\% = \frac{1}{5}$, $40\% = \frac{2}{5}$, $60\% = \frac{3}{5}$, and $80\% = \frac{4}{5}$.

2. Fractions can be changed to per cents in this way: $\frac{1}{2} = \frac{1}{2}$ of $1 = \frac{1}{2}$ of $100\% = 50\%$. So: Work these out with class.

$\frac{1}{3} = \frac{1}{3}$ of $100\% = 33\frac{1}{3}\%$
$\frac{2}{3} = \frac{2}{3}$ of $100\% = 66\frac{2}{3}\%$
$\frac{1}{6} = \frac{1}{6}$ of $100\% = 16\frac{2}{3}\%$
$\frac{5}{6} = \frac{5}{6}$ of $100\% = 83\frac{1}{3}\%$

$\frac{1}{8} = \frac{1}{8}$ of $100\% = 12\frac{1}{2}\%$
$\frac{3}{8} = \frac{3}{8}$ of $100\% = 37\frac{1}{2}\%$
$\frac{5}{8} = \frac{5}{8}$ of $100\% = 62\frac{1}{2}\%$
$\frac{7}{8} = \frac{7}{8}$ of $100\% = 87\frac{1}{2}\%$

3. Learn these relationships: Have some pupils make class chart of following, including 25%, 50%, 75%.

$10\% = \frac{1}{10}$
$20\% = \frac{1}{5}$
$40\% = \frac{2}{5}$

$60\% = \frac{3}{5}$
$33\frac{1}{3}\% = \frac{1}{3}$
$66\frac{2}{3}\% = \frac{2}{3}$

$16\frac{2}{3}\% = \frac{1}{6}$
$83\frac{1}{3}\% = \frac{5}{6}$
$12\frac{1}{2}\% = \frac{1}{8}$

$37\frac{1}{2}\% = \frac{3}{8}$
$62\frac{1}{2}\% = \frac{5}{8}$
$87\frac{1}{2}\% = \frac{7}{8}$

Use short cuts to find the answers mentally:

4. 60% of 25 15 — $66\frac{2}{3}\%$ of 18 12 — $83\frac{1}{3}\%$ of 24 20
5. 40% of 35 14 — $37\frac{1}{2}\%$ of 24 9 — $62\frac{1}{2}\%$ of 40 25
6. 25% of 48 12 — $87\frac{1}{2}\%$ of 16 14 — $16\frac{2}{3}\%$ of 12 2
7. 20% of 30 6 — $33\frac{1}{3}\%$ of 15 5 — $12\frac{1}{2}\%$ of 56 7

Find the answers:

8. 60% of 75 45 — $83\frac{1}{3}\%$ of 216 180 — $66\frac{2}{3}\%$ of 159 106
9. 20% of 90 18 — $12\frac{1}{2}\%$ of 184 23 — $87\frac{1}{2}\%$ of 344 301
10. 80% of 65 52 — $16\frac{2}{3}\%$ of 144 24 — $37\frac{1}{2}\%$ of 256 96
11. 75% of 92 69 — $62\frac{1}{2}\%$ of 200 125 — $33\frac{1}{3}\%$ of 225 75

More Practice. See ㊵ on page 338.

In ex. 1 point out that per cents are hundredths; therefore $20\% = \frac{20}{100} = \frac{1}{5}$. Do same for others. Have pupils prove short cuts in ex. 3 by changing fraction to 2-place decimal ($1 \div 10 = .10$), then write decimal as per cent.

Problems and Practice

Find the answers, using short cuts:

1. 20% of \$2.90 \$.58 $33\frac{1}{3}\%$ of \$171 \$57 $87\frac{1}{2}\%$ of \$23.36 \$20.44
2. 60% of \$4.55 \$2.73 $62\frac{1}{2}\%$ of \$416 \$260 $66\frac{2}{3}\%$ of \$13.35 \$8.90
3. 10% of \$3.20 \$.32 $12\frac{1}{2}\%$ of \$304 \$38 $62\frac{1}{2}\%$ of \$37.04 \$23.15
4. 75% of \$2.76 \$2.07 $37\frac{1}{2}\%$ of \$152 \$57 $83\frac{1}{3}\%$ of \$15.42 \$12.85
5. 40% of \$1.75 \$.70 $16\frac{2}{3}\%$ of \$258 \$43 $33\frac{1}{3}\%$ of \$19.50 36.50

Change each fraction to a per cent and tell which is greater:

6. $\frac{7}{8}$ or 90% 90% 37% or $\frac{3}{8}$ $37\frac{1}{2}\%$ $\frac{1}{3}$ or $\frac{3}{8}$ $37\frac{1}{2}\%$ $\frac{5}{8}$ or $\frac{3}{5}$ $62\frac{1}{2}\%$
7. $\frac{1}{2}$ or 45% 50% 60% or $\frac{2}{3}$ $66\frac{2}{3}\%$ $\frac{4}{5}$ or $\frac{5}{6}$ $83\frac{1}{3}\%$ $\frac{2}{5}$ or $\frac{3}{8}$ 40%
8. $\frac{1}{3}$ or 34% 34% 65% or $\frac{5}{8}$ 65% $\frac{2}{3}$ or $\frac{5}{8}$ $66\frac{2}{3}\%$ $\frac{5}{6}$ or $\frac{7}{8}$ $87\frac{1}{2}\%$

9. By paying cash, Mrs. Lee can save $12\frac{1}{2}\%$ on a set of living room furniture that costs \$330. How much can she save by paying cash? \$41.25

10. Miss Lane bought a book selling for \$1.98 at $33\frac{1}{3}\%$ off because the book was soiled. How much did Miss Lane save on the price of the book? \$.66

11. Tickets for the school play brought in \$525. Of this amount $66\frac{2}{3}\%$ was used to buy books for the school library. How much money was used in this way? \$350

12. In April, Jack bought a pair of skis marked "$33\frac{1}{3}\%$ off." They had been priced at \$19.95. How much did Jack save on the skis? \$6.65 Can you think of a reason why Jack was able to buy the skis for so much less in April?

13. At a sale Patty bought a 50-dollar coat for $87\frac{1}{2}\%$ of its marked price. How much did Patty pay for the coat? \$43.75

14. Mr. West says that when he sells a rug his profit is about $16\frac{2}{3}\%$ of the selling price. How much is his profit on a rug that sells for \$108? \$18

Urge pupils to do exercises mentally if possible. Be sure all pupils understand how to change fraction to a per cent in ex. 6-8. If understanding is assured, drill on per cents and fractional equivalents may be used.

Teach discount in form used in retail stores, and vocabulary for it.

Buying at a Sale

1. Jean lives in a large city. Her mother finds that she can buy Jean better clothes for the money she can afford to spend by waiting until "sales" are advertised where clothes are sold at reduced prices. Here are the regular prices of the winter clothes Jean bought in February:

Winter coat, $39.00	$26.00	Wool suit, $27.75	$18.50
Wool dress, $12.80	$9.60	Silk blouse, $4.00	$3.00
Cotton dress, $7.00	$5.25	Wool sweater, $9.00	$6.75

Find the amount Jean's mother paid for each article if there was "$33\frac{1}{3}\%$ off" on the coat and suit and "25% off" on the other articles. Ans. above. Also find the total cost of these clothes. $69.10

2. When articles are sold at less than the regular price, they are sold at a **discount.** The regular price of the article is the **marked price;** the amount saved is the **discount;** the per cent taken off is the **rate of discount;** and the price paid after subtracting the discount is the **net price.**

3. Bert can get a 15% discount on a radio that has a marked price of $35. How much money can he save by buying the radio at this discount? $5.25 Find the net price. $29.75

4. In the Sports Store all the goods are being sold at a discount of 20%. If Fred buys a pair of skates marked $10.00, a pair of skis marked $18.00, and a cap marked $2.00, how much will he spend in all? $24.00

Find the discount (1) and the net price (2) if you buy articles having these marked prices and discounts: Have pupils make up sensible problems using numbers in ex. 5–8.

5.	$24.50, 10% (1), (2) $2.45, $22.05	$12.00, $37\frac{1}{2}\%$ $4.50, $7.50	$11.35, 20% $2.27, $9.08
6.	$18.00, 30% $5.40, $12.60	$15.60, $12\frac{1}{2}\%$ $1.95, $13.65	$18.98, 50% $9.49, $9.49
7.	$19.00, 25% $4.75, $14.25	$15.75, $33\frac{1}{3}\%$ $5.25, $10.50	$10.50, 40% $4.20, $6.30
8.	$14.75, 20% $2.95, $11.80	$20.00, $12\frac{1}{2}\%$ $2.50, $17.50	$16.80, 10% $1.68, $15.12

More Practice. See ㊶ on page 338. Use for reteaching.

Discuss ex. 1–4 with pupils. Be sure they understand and correctly use terms in ex. 2. Have them complete ex. 5–8. Have pupils bring to class advertisements of sales on which discounts are allowed, and make up problems based on these.

Teach rounding off answers correct to nearest cent.

Rounding Off to the Nearest Cent

1. **Problem** When Mr. Carter bought a new outboard motor for his boat he saved 5% of the price by paying cash for it. This means that the price was $148.75 with a 5% discount for cash. How much did Mr. Carter save by paying cash? $7.44

Explanation Find 5% of $148.75 as shown at the right. The answer is $7.4375, which is called $7.44 **to the nearest cent,** because $7.4375 is closer to $7.44 than it is to $7.43. Mr. Carter saved $7.44 by paying cash.

$$\begin{array}{r} \$148.75 \\ \underline{.05} \\ \$7.4375 \end{array}$$

If the answer had been $7.4125, you would call it $7.41 **to the nearest cent.** Why? If the answer had been $7.4250, which is halfway between $7.42 and $7.43, you would call it $7.43 **to the nearest cent.** See page 104.

Emphasize.

> **To find an answer correct to the nearest cent, if the figure in the third decimal place is 5 or more, drop it but make the figure before it 1 larger; if the figure in the third decimal place is less than 5, drop it.**

When an answer is written correct to the nearest cent, it is **rounded off** to the nearest cent, or to the nearest hundredth of a dollar. Be sure pupils understand this. Give oral practice in rounding off before assigning ex. 2–6.

Find the answers. Round off the answers to the nearest cent:

2. 5% of $83.75 $4.19   2% of $40.19 $.80   15% of $14.49 $2.17

3. 7% of $18.65 $1.31   4% of $15.35 $.61   12% of $17.98 $2.16

4. 8% of $29.29 $2.34   1% of $12.25 $.12   25% of $11.50 $2.88

5. 2% of $63.36 $1.27   6% of $76.39 $4.58   50% of $19.89 $9.95

6. 4% of $56.62 $2.26   9% of $55.85 $5.03   14% of $15.83 $2.22

Check pupils' work carefully to be sure all have good understanding.

More Practice. See ㊷ on page 339.

Conduct quick review of rounding off decimals as given on page 104. Then discuss ex. 1 with class. Emphasize that figure in third place is important one, and figures in fourth and fifth places do not affect rounding off.

Teach how to estimate answer when finding per cent of a number.

Estimating Answers

In working percentage problems, you can often avoid foolish mistakes by first making a rough estimate of the answer. $120

1. **Problem** Find 35% of $360. Estimate the answer; then find the exact answer. $126

Explanation 35% is a little over $33\frac{1}{3}$%, which equals $\frac{1}{3}$. Then $\frac{1}{3}$ of $360, or $120, is a rough estimate of the answer. Since 35% is a little over $\frac{1}{3}$, the exact answer is a little more than $120. The work at the right gives $126 as the answer, which is probably right since it is a little over $120. Check the work.

$$\begin{array}{r} \$360 \\ .35 \\ \hline 18\,00 \\ 108\,0 \\ \hline \$126.00 \end{array}$$

This example shows that you can make a rough estimate of a per cent of a number if the given per cent is close to $33\frac{1}{3}$%, or 25%, or some other per cent that is a short cut.

2. Bill read that there are about 180,000,000 people living in the United States and that 30% live in rural areas. Estimate the number living in rural areas. Find 30% of 180,000,000 exactly. Which answer is greater and why? 60,000,000 54,000,000
Exact answer is less since 30% is less than $33\frac{1}{3}$%.

Estimate the answers. Then find the exact answers:

(1), (2) $32, $32.64 — $2, $1.92 — $30, $30.60 — $15, $14.70
3. 51% of $64 — 12% of $16 — 34% of $90 — 49% of $30

$36, $35.52 — $5, $4.75 — $45, $44.64 — $39, $39.52
4. 74% of $48 — 19% of $25 — 62% of $72 — 76% of $52

$22, $22.88 — $42, $42.21 — $12, $12.16 — $3, $3.12
5. 26% of $88 — 67% of $63 — 38% of $32 — 13% of $24

After each exercise three answers are given but only one is correct. Estimate the answer and tell which answer is nearest your estimate:

6. Does 11% of 520 = 57.2, 572.0, or 5.72? 57.2
7. Does 24% of 824 = 1977.6, 197.76, or 19.776? 197.76
8. Does 38% of 480 = 18.24, 182.40, or 1824.0? 182.40
9. Does 49% of 250 = 12.25, 1225.0, or 122.50? 122.50
10. Does 13% of 488 = 6.344, 63.44, or 634.4? 63.44

Note if some pupils need drill on short cuts.

More Practice. See ㊸ on page 339.

In ex. 1 emphasize that the given per cent must be close to some per cent that you can find by a short cut. Ask pupils what short cut they would use in estimating with following per cents: 13%, 19%, 11%, 24%, 34%, 67%, 61%, 76%, 85%, 36%.

Selling on Commission

Teach simple commission through meaningful problems.

1. Jack and Bill make birdhouses to sell. Their sister Betty sells the birdhouses for them because they are too busy to do it. The boys pay Betty 20% of the price of each birdhouse. If a small birdhouse sells for $.85, how much does Betty get from the sale? $.17 How much do the boys get? $.68

▶ To find the amount of money Betty gets, take 20% of $.85, which is $.17. Betty gets $.17 and the boys get the rest, which is the difference between $.85 and $.17, or $.68.

The money Betty gets is called her **commission.** The per cent she gets is the **rate of commission.** Betty is the **agent** because she sells the goods.

2. The boys make other kinds of birdhouses selling at $1.00, $1.25, and $1.50 each. If Betty sells 5 houses at $1.00, 3 at $1.25, and 2 at $1.50, what is the total amount of her commission? $2.35 What is the total amount the boys get? $9.40

3. Betty tried to sell Mrs. King a birdhouse but Mrs. King wanted a large one made to order. The boys said they would have to charge $2.00 for it. How much would Betty's commission be on that birdhouse? $.40 How much would the boys receive for that birdhouse? $1.60

In discussing ex. 1 be sure all pupils understand meaning of new terms and are able to use them correctly. Point out that commission is merely one way in which a person may be paid for services.

4. Last summer Tom sold magazine subscriptions on commission. He got 30 subscriptions to a magazine that sells at \$3.50 a year. His commission was 25% on each subscription. How much did Tom earn by selling that magazine? \$26.25

5. Tom also sold 52 subscriptions to another magazine selling at \$3.00 a year. His commission was 20% on each subscription. How much did Tom earn selling this magazine? \$31.20 How much did Tom earn all together? See ex. 4. \$57.45

6. Mr. Price is an automobile salesman. He is paid a commission of 15% on each car that he sells. Last week he sold one car at \$2340 and another at \$2585. How much in all did Mr. Price get on these two sales? \$738.75

7. An agent sold a house for \$17,500 and received 3% of that amount as his commission. How much did the agent receive for his work? \$525.00

8. Miss Williams works in a store at a weekly salary of \$45. Besides her salary she gets a commission of 5% of her weekly sales. How much in all does Miss Williams get the week her sales amount to \$350? \$62.50 How much does she get the week her sales are \$100? \$50.00 \$500? \$70.00

9. Ann makes candy to sell. She asks \$1.20 a pound for it. Judy sells the candy for Ann and receives a commission of 15% for her work. How much commission does Judy get when she sells 2 lb. of candy? \$.36 when she sells 5 lb.? \$.90 when she sells $6\frac{1}{2}$ lb.? \$1.17 How much does Ann get for each pound of candy that Judy sells? \$1.02

Have pupils who seem to have difficulty explain their solutions so you can determine causes.

Extend use of per cents to those ending in fractions (not short cuts).
Note that this work is difficult for below-average students.

Per Cents Ending in Fractions

1. **Problem** An agent selling cottages at Sandy Beach was paid $3\frac{1}{2}\%$ of the selling price as his commission. How much commission did he get for selling a cottage for $5950? $208.25

Explanation To find $3\frac{1}{2}\%$ of $5950, change $3\frac{1}{2}\%$ to the decimal $.03\frac{1}{2}$. Then multiply as shown at A. The agent's commission was $208.25.

On page 102 you learned that the decimal $.03\frac{1}{2}$ can also be written as .035. If you use .035 instead of $.03\frac{1}{2}$, you can do the work as shown at B.

Both A and B give the same answer, which is $208.25.

A	B
$5950	$5950
$.03\frac{1}{2}$	.035
2975	29750
178 50	178 50
$208.25	$208.250

Study these per cents written as decimals:

2. $2\frac{1}{2}\% = .025$ $5\frac{1}{2}\% = .055$ $16\frac{1}{2}\% = .16\frac{1}{2}$ or $.165$

3. $15\frac{1}{3}\% = .15\frac{1}{3}$ $4\frac{2}{3}\% = .04\frac{2}{3}$ $7\frac{1}{4}\% = .07\frac{1}{4}$ or $.0725$

4. $11\frac{2}{3}\% = .11\frac{2}{3}$ $1\frac{1}{3}\% = .01\frac{1}{3}$ $10\frac{3}{4}\% = .10\frac{3}{4}$ or $.1075$

Change these per cents completely to decimals:

5. $1\frac{1}{2}\%$ .015 $4\frac{1}{4}\%$ .0425 $10\frac{1}{2}\%$ .105 $3\frac{1}{4}\%$ .0325 $24\frac{1}{2}\%$ .245 $12\frac{1}{4}\%$ .1225

6. $2\frac{1}{2}\%$ .025 $3\frac{3}{4}\%$ .0375 $15\frac{1}{4}\%$ .1525 $5\frac{1}{2}\%$ .055 $21\frac{1}{4}\%$ .2125 $28\frac{3}{4}\%$ .2875

Change these per cents to two-place decimals ending in a fraction:

7. $6\frac{2}{3}\%$ $.06\frac{2}{3}$ $2\frac{1}{3}\%$ $.02\frac{1}{3}$ $83\frac{1}{3}\%$ $.83\frac{1}{3}$ $1\frac{1}{6}\%$ $.01\frac{1}{6}$ $33\frac{1}{3}\%$ $.33\frac{1}{3}$ $66\frac{2}{3}\%$ $.66\frac{2}{3}$

Find the answers:

8. $2\frac{1}{2}\%$ of $58 $1.45 $11\frac{1}{4}\%$ of $280 $31.50 $6\frac{1}{2}\%$ of $2350 $152.75

9. $3\frac{1}{4}\%$ of $96 $3.12 $12\frac{3}{4}\%$ of $124 $15.81 $9\frac{1}{3}\%$ of $1680 $156.80

10. $5\frac{1}{2}\%$ of $88 $4.84 $16\frac{1}{2}\%$ of $236 $38.94 $5\frac{3}{4}\%$ of $1264 $72.68

11. $1\frac{3}{4}\%$ of $48 $.84 $12\frac{1}{6}\%$ of $144 $17.52 $3\frac{2}{3}\%$ of $4038 $148.06

In discussing ex. 1 point out that in some cases ($2\frac{1}{3}\%$, $15\frac{1}{3}\%$) the method in A should be used. Go over this method carefully, emphasizing place value of partial products. The method in B may be preferred in other cases ($3\frac{1}{2}\%$, $7\frac{1}{4}\%$).

Present another set of improvement tests in addition.

Improving by Practice

Addition Test 3a.

Time: 4 min.

1.	567	412	217	272	357	473
	101	806	550	841	603	970
	420	790	712	324	640	207
	265	883	895	301	503	136
	864	342	987	319	121	415
	688	824	364	197	863	764
	199	709	202	832	739	155
	949	542	751	678	106	118
	4053	5308	4678	3764	3932	3238

⑥

Addition Test 3b.

Time: 4 min.

2.	540	801	106	722	845	747
	458	937	539	370	328	383
	428	964	690	605	358	238
	110	249	328	427	634	729
	507	500	881	893	613	812
	273	288	599	904	283	608
	459	572	662	966	507	849
	625	991	126	628	170	965
	3400	5302	3931	5515	3738	5331

⑥

Addition Test 3c.

Time: 4 min.

3.	639	804	406	174	892	301
	129	825	973	950	520	568
	420	695	161	601	883	280
	462	814	340	347	205	149
	515	704	673	967	517	161
	944	561	257	232	867	236
	577	172	788	410	484	103
	269	747	595	538	731	649
	3955	5322	4193	4219	5099	2447

⑥

To the Teacher. If individual pupils get low scores on Improvement Tests in addition, subtraction, multiplication, and division, such pupils may be assigned extra practice which will be found on pages 325–328.

Check papers carefully and note types of errors (facts, adding by endings, carrying). Help pupils find and correct errors. Have them determine own scores and record them. Clear up difficulties before assigning extra practice.

Extend use of per cents ending in fractions to examples where there is a fraction in the product.

Per Cents Ending in Fractions

1. Study these examples carefully:

$$\begin{array}{r} \$125 \\ .12\frac{3}{4} \\ \hline 93\frac{3}{4} \\ 2\,50 \\ 12\,5 \\ \hline \$15.93\frac{3}{4} \\ \\ \$15.94 \text{ Ans.} \end{array} \qquad \begin{array}{r} \$208 \\ .02\frac{1}{3} \\ \hline 69\frac{1}{3} \\ 4\,16 \\ \hline \$4.85\frac{1}{3} \\ \\ \$4.85 \text{ Ans.} \end{array} \qquad \begin{array}{r} \$150 \\ .11\frac{1}{4} \\ \hline 37\frac{1}{2} \\ 1\,50 \\ 15\,0 \\ \hline \$16.87\frac{1}{2} \\ \\ \$16.88 \text{ Ans.} \end{array}$$

By studying the examples above you see that each of these answers is a decimal which ends with a fraction. Since the answers represent dollars and cents, only two decimal places are needed to express them. Stress. The answers should be rounded off to the nearest cent: if the fraction at the end of the result is less than ½, it is dropped; but if it is equal to or more than ½, it is counted as an extra cent. Emphasize.

Find the answers to the nearest cent:

2. $3\frac{1}{4}\%$ of \$150 \$4.88 $1\frac{1}{3}\%$ of \$135 \$1.80 $15\frac{1}{3}\%$ of \$2200 \$337.33

3. $2\frac{1}{3}\%$ of \$250 \$5.83 $2\frac{3}{4}\%$ of \$263 \$7.23 $10\frac{3}{4}\%$ of \$1885 \$202.64

4. $1\frac{2}{3}\%$ of \$320 \$5.33 $4\frac{1}{4}\%$ of \$142 \$6.04 $19\frac{2}{3}\%$ of \$1250 \$245.83

5. $6\frac{1}{4}\%$ of \$125 \$7.81 $5\frac{2}{3}\%$ of \$175 \$9.92 $14\frac{1}{4}\%$ of \$1575 \$224.44

6. $4\frac{1}{3}\%$ of \$200 \$8.67 $7\frac{1}{3}\%$ of \$265 \$19.43 $12\frac{1}{3}\%$ of \$1150 \$141.83

Estimate the answer.(1) Then tell which one of the three numbers is the correct answer:(2) Remind pupils to use short cuts.

(1),(2)**7.** $19\frac{1}{3}\%$ of \$150 = \$2.90, \$29.00, or \$290.00. \$30, \$29.00

8. $24\frac{1}{4}\%$ of \$188 = \$455.90, \$4.56, or \$45.59. \$47, \$45.59

9. $36\frac{2}{3}\%$ of \$120 = \$44.00, \$4.40, or \$440.00. \$45, \$44.59

10. $66\frac{1}{2}\%$ of \$240 = \$159.60, \$15.96, or \$1596.00. \$160, \$159.60

Model examples in ex. 1 should be put on board and discussed thoroughly with class. Be sure all know how first partial products are obtained. Do ex. 2 and 3 with class. If there are no apparent difficulties, assign ex. 4–10 as independent work.

Teach meaning of per cents larger than 100% and use them in practical exercises (pages 145-146). First give oral practice on 100% of certain numbers, such as 100% of 8, 100% of 312, and so on.

Per Cents over 100%

1. In Westover the population is now 400% of what it was five years ago. This means that the population is 4 times its former size since 400% = $\frac{400}{100}$, or 4.

▶ 400% of a number equals 4 times that number.

2. What does "300% of the original price" mean? 3 times the original price

3. Tell the meaning of 800% of a number; 200% of a number; 600%; 300%; 900%; 1000%. 8 times 2 times 6 times 3 times 9 times 10 times

Have pupils use graph paper to show the meaning of some of these per cents.

4. If the population is 150% of what it was two years ago, it is $1\frac{1}{2}$ times, or 1.5 times, its former size. When you change 150% to $\frac{150}{100}$ and reduce the fraction, you get $\frac{3}{2}$, or $1\frac{1}{2}$, or 1.5. You see that any per cent larger than 100% is more than 1. When you find a per cent of a number, using a per cent larger than 100%, the result will be larger than the number. Point out also that in finding per cent of number under 100%, answer will be smaller than original number.

Find the answers, mentally if possible:

5. 200% of 35 70 800% of 11 88 1200% of 30 360
6. 300% of 21 63 650% of 10 65 1000% of 82 820
7. 550% of 60 330 400% of 25 100 1500% of 20 300

8. Jane read about a college that now has 1800% of the students it had the year it was started. There were only 75 students that first year. How many are there now? 1350 students

9. Our school library was started with 550 volumes. Now it has grown so that it is 650% of its original size. How many volumes has the library now? 3575

Tell what number belongs in each space:

10. 500% of a number equals .5. . times the number.

11. 850% of a number equals .8.5 times the number.

12. 1300% of a number equals .13 . times the number.

13. 2800% of a number equals .28. times the number.

After discussing 100% of some numbers, show by using graph paper that 125% is 100% plus 25%. This means that 125% of a number must be equal to the number plus another amount; therefore, it will be larger than the original number.

Large Per Cents

1. Problem The school registration in Johnstown last year was 1750 pupils. This year the registration is 112% as great as last year. What is the registration this year? 1960

Explanation To find 112% of 1750, write 112% as $\frac{112}{100}$, or 1.12. Then multiply 1750 by 1.12, which gives 1960, as shown at the right.

You see that when 112% is changed to a decimal, the per cent sign is dropped and the decimal point is moved two places to the left.

$$\begin{array}{r} 1750 \\ 1.12 \\ \hline 35\,00 \\ 1750\, \\ 1750\, \\ \hline 1960.00 \end{array}$$

Change to decimals:

2. 185% 1.85 327% 3.27 406% 4.06 503% 5.03 1556% 15.56

3. 324% 3.24 215% 2.15 700% 7.00 800% 8.00 1323% 13.23

Find answers to the nearest cent:

4. 124% of $173 $214.52 360% of $975 $3510 175% of $236.50 $413.88

5. 145% of $450 $652.50 225% of $246 $553.50 240% of $510.23 $1224.55

6. 310% of $625 $1937.50 130% of $550 $715 135% of $140.28 $189.38

7. 212% of $184 $390.08 550% of $319 $1754.50 415% of $253.27 $1051.07

8. Bob, Joe, and Bill made $45 last summer selling old papers. If they divided the money equally, how much did each boy get? $15 Next summer they want to make 200% of $45. How much is that? $90 What will each boy get? $30

9. If they make 200% more than $45 next summer, how much will they make? $135 What will each boy get? $45

10. Is 300% of $45 the same as 300% **more than** $45? No What is the answer in each case? $135; $180

More Practice. See ㊹ on page 339.

Ex. 1 should be discussed carefully with pupils. Be sure all understand how 112% is changed to 1.12. Emphasize that per cents are fractions whose denominators are 100. Give more practice like that in ex. 2-3 before doing ex. 4-10.

Teach some new technical expressions in percentage. Put model examples in ex. 1 on board and discuss both methods thoroughly with class.

Understanding Percentage Language

1. Problem The population of West Fairfield, which was 12,880 ten years ago, is now 15% more than that. What is the population now? 14,812

Explanation There are two ways to work this problem.

First Method. Find 15% of 12,880, which is 1932. Add this number to 12,880 to find the present population, as shown at A.

$$\text{A.}\quad \begin{array}{r} 12880 \\ .15 \\ \hline 64400 \\ 12880 \\ \hline 1932.00 \end{array} \qquad \begin{array}{r} 12880 \\ 1932 \\ \hline 14812 \end{array} \qquad\qquad \text{B.}\quad \begin{array}{r} 12880 \\ 1.15 \\ \hline 64400 \\ 12880 \\ 12880 \\ \hline 14812.00 \end{array}$$

Second Method. Since the population is now 15% **more than** it was, it is 100% + 15%, or 115% as great as it was ten years ago. Since 115% = 1.15, multiply 12,880 by 1.15, which gives 14,812, as shown at B. This is the same answer as that found at A. Point out that present population is all of old population plus 15% more.

If the population were now 15% **less than** it was ten years ago, you would subtract 1932 from 12,880 if you used Method A. If you used Method B, you would first subtract 15% from 100%, which gives 85%; then find 85% of 12,880.

Tell the number to put in each space:

2. 20% more than $30 = . . . 120 % of $30, or $ . . . 36.

3. 25% less than $36 = . . . 75 % of $36, or $ . . . 27

4. $33\frac{1}{3}$% more than $24 = . . . $133\frac{1}{3}$% % of $24, or $ . . . 32

5. 100% more than $60 = . . . 200 % of $60, or $ . . . 120

6. In our school the registration this year **has increased** 25% over last year's, when there were 1820 pupils registered. How many pupils are registered this year? 2275

7. In ex. 6, you see that **"has increased** 25%" means the same as "25% **more than."** What does the expression **"has decreased** 10%" mean? "10% less than"

Do ex. 2-7 orally. To assure understanding of terms, give more oral class work; take the number $20 and ask what it will be if it increases 25%; if it becomes 25% more than originally; if it is decreased 25%. Do the same with other numbers.

Present review of language, problems, and computation of percentage. Use review as preparation for problem test and diagnostic test.

Chapter Review

Make these statements in the language of percentage:

1. One fourth of the pupils play musical instruments. 25% of the pupils

2. At present there are twice as many people living in Charleston as there were 20 yr. ago. 200% as many people

Write each of these with the sign %:

3. $\frac{50}{100}$ 50% $\frac{73}{100}$ 73% $\frac{1}{100}$ 1% $\frac{235}{100}$ 235% $\frac{100}{100}$ 100% $\frac{9}{100}$ 9% $\frac{175}{100}$ 175%

4. .28 28% .64 64% .15 15% .98 98% .04 4% .08 8% .09 9% .06 6%

Write these per cents as decimals:

5. 42% .42 $5\frac{1}{2}\%$ .055 $9\frac{3}{4}\%$ .0975 172% 1.72 1275% 12.75 900% 9.00

Find the answers:

6. 18% of 450 81 50% of 278 139 43% of 3350 1440.5

7. $2\frac{1}{2}\%$ of 520 13 $3\frac{3}{4}\%$ of 224 8.4 $15\frac{1}{2}\%$ of 1200 186

8. 100% of 325 325 225% of 168 378 312% of 2275 7098

Find the answers:

9. In Eagle Rock only 35% of the days of the year are free from frost. What per cent of the days have frost? 65%

10. Miss King paid a 2% sales tax when she purchased a coat for $45. How much was the tax on her coat? $.90

11. Find a 50% discount on a pair of white shoes originally marked $8.50. $4.25 What is the net price of this pair of shoes? $4.25

12. Find 6% of $54.75 correct to the nearest cent. $3.29

13. Estimate 24% of $220 (1) and then find the exact answer. (2) (1) $55; (2) $52.80

14. An agent sells houses on a commission of 5% of the sale price. Find his commission on a house selling for $18,300. $915.00

15. Show that you know the meaning of 25% of $400, $100 25% more than $400, $500 and 25% less than $400. $300

Check papers carefully and note kinds of errors. Through individual conferences with pupils, try to determine causes of errors and need for reteaching or further review.

A Problem Test

1. The jet airplane fare from Philadelphia to Tulsa is $87.75. A tax of 5% is not included in this figure. Find the total fare including the tax, figured to the nearest cent. $92.14

2. Miss Williams bought a bracelet. The price was $6.85 but that did not include a Government tax of 10% and a city tax of 3% of the cost. Find the total cost of the bracelet. $7.75

3. Martha and Betty bought a camera and shared the cost. They paid $33.50 for the camera, $3.64 for a carrying case, and $1.80 for films. Find each girl's share of the cost. $19.47

4. Martha had saved $20 for her share of the camera. How many months did it take her to save $20 if she saved $2.50 each month? 8

5. Last summer Mr. Ball used 153 gal. of gasoline on a trip of 2860 mi. to Texas and back. How many miles did he average per gallon of gasoline? Find the answer to the nearest tenth of a mile. 18.7

6. As treasurer of the school band, Bill received these amounts of money in May: $2.50, $1.75, $.80, $1.00, $.15, $4.25. He paid out these amounts: $.26, $1.33, $2.06, $.19, $.88, $1.65, $.74. Balance the account and find the cash on hand. $3.34

7. Find the cost of $4\frac{1}{2}$ lb. of meat at $.45 a pound. $2.03

8. How many pieces of ribbon each $1\frac{1}{2}$ yd. long can be cut from a roll of ribbon 30 yd. long? 20

9. Find the net cost of an article if the marked price is $32.50 and the discount is 20% $26.00

10. Before a trip the speedometer on a car read 2191.6 mi. At the end of the trip it read 2470.3 mi. Find the number of miles covered on the trip. 278.7

Have pupils record scores on own graphs.

SCORE	0–5	6–7	8–9	10
	You need help	Fair	Good	Excellent

Have pupils who made mistakes explain their solutions to problems so you may determine causes of difficulties (computational errors, lack of understanding of problem situation or process, and so on). Plan remedial work as needed.

Present diagnostic test of skills and understandings taught in Chapter 4, with page references for remedial work. Record results of test on pupils' progress cards.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Change to decimals: Practice Pages

1. 25% .25   77% .77   6% .06   2% .02   126

2. $6\frac{1}{2}$% .065 $9\frac{1}{4}$% .0925 $5\frac{3}{4}$% .0575 $15\frac{1}{2}$% .155   142

3. 280% 2.80   300% 3.00   185% 1.85   1256% 12.56   146

Change to per cents:

4. .18 18%   .27 27%   .08 8%   .30 30%   126

5. .25 25%   .50 50% $.37\frac{1}{2}$ $37\frac{1}{2}$% $.66\frac{2}{3}$ $66\frac{2}{3}$%   133, 135

Find the answers:

6. 6% of $25 $1.50   32% of $150 $48.00   71% of $220 $156.20   128, 129

7. 25% of $60 $15   50% of $236 $118   75% of $184 $138   132, 133

8. $62\frac{1}{2}$% of $56 $35   40% of $125 $50 $33\frac{1}{3}$% of $216 $72   135, 136

9. $2\frac{1}{2}$% of $32 $.80 $7\frac{3}{4}$% of $100 $7.75 $11\frac{1}{3}$% of $108 $12.24   142, 144

10. 400% of $15 $60   150% of $128 $192   325% of $124 $403   145, 146

Find the answers to the nearest cent:

11. 3% of $4.25 $.13 $12\frac{1}{2}$% of $9.00 $1.13   52% of $7.35 $3.82   138

Find a 25% discount (1) and the net price (2) of articles with these marked prices:

12. $275.00 (1) $68.75 (2) $206.25 $118.80 $29.70 $89.10 $13.68 $3.42 $10.26   137

Estimate these answers. (1) Then find exact answers: (2)

13. 11% of $60 (1) $6 (2) $6.60   49% of $54 $27 $26.46   33% of $18 $6 $5.94   139

Pupils who do well on test might like to work on original projects, problems, and so on. Help others to determine causes of errors. Reteach as needed before assigning practice pages.

See Guide for specific aims of Chapter 5.

Chapter 5

Measuring Electricity

Teach how electricity is measured and how its cost in running certain appliances is computed (pages 151-153).

1. Electric light bulbs that are used in reading lamps are often marked 75W, 100W, or 150W. A bulb marked 100W requires 100 **watts** of electric power. Electric appliances are also marked to show how many watts of electric power they require. Have pupils check markings of electric appliances at home and make chart showing appliances and markings.

2. Electric light bills are based on the amount of electricity used. The amount of current is measured in **watt-hours.** Stress. A 100-watt bulb uses 100 watt-hours of electricity in 1 hr.; in 2 hr. it uses 200 watt-hours; in 10 hr. it uses 10 × 100 watt-hours, or 1000 watt-hours. 1000 watt-hours are called Stress meaning **1 kilowatt-hour;** kilo means 1000. Since 1 watt-hour of electricity is a very small amount, electric current is usually sold by the kilowatt-hour (kw-hr).

3. If an electric light bulb marked 150W is burned for 20 hr., how many watt-hours of electricity does it use? 3000 How many kw-hr does it use? 3

Display bulbs that are marked 50W, 75W, and so on. Emphasize that marking shows how many watt-hours are used if bulb burns for 1 hour.

Do ex. 1–2 with pupils. Be sure all pupils understand meanings of W, kw-hr, and watt-hour. Give more oral practice in changing watt-hours to kw-hr. Have pupils complete ex. 3–8.

Measuring Electricity

1. If an electric light bulb marked 75W is burned 5 hr., what part of 1 kw-hr does it use? .375

▶ In 5 hr. this bulb uses 5 × 75 watt-hours, which is 375 watt-hours. It takes 1000 watt-hours to make 1 kw-hr, so 375 watt-hours are $\frac{375}{1000}$ of 1 kw-hr, or .375 kw-hr. To change watt-hours to kilowatt-hours, divide the number of watt-hours by 1000.

2. Change to kw-hr: 850 watt-hours; .85 1750 watt-hours. 1.75

3. If a 40-watt lamp is burned for 6 hr., how many watt-hours does it use? 240 how many kw-hr? 24 How many kw-hr does a 60-watt lamp use in 5 hr.? .3 in 8 hr.? .48 in 20 hr.? 1.2

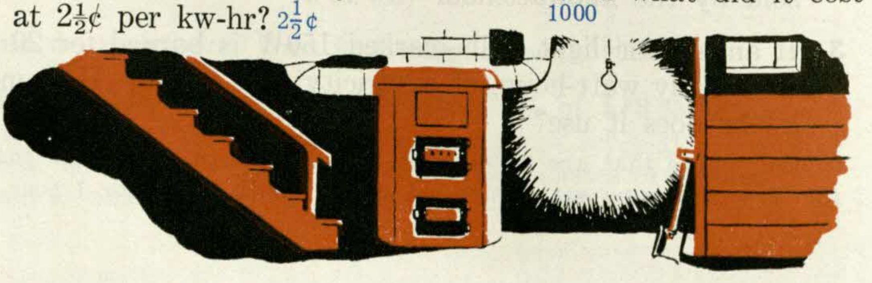
4. When Mr. Adams went away on vacation, he forgot to turn off a 75-watt bulb in his garage, so it burned day and night for 20 whole days. How many kw-hr of electricity did it use 36 and what did it cost at 5¢ per kw-hr? $1.80

5. How many hours could you burn a bulb marked 50W on 1 kw-hr of electricity? 20

6. At Henry's home there are two electric bulbs in the living room that are marked 100W and one that is marked 150W. If all three of these bulbs are turned on for 4 hr., how many kw-hr of electricity are used? 14 At 3¢ per kw-hr, how much does this amount of electricity cost? 4.2¢

7. Mary's father bought for their living room an air conditioner that requires 900 watts of electric power. If they use their air conditioner for a total of 80 hr. in one month, how many kw-hr of electricity will be used? 72 At 3¢ per kw-hr, what will it cost? $2.16

8. Mr. Brown left the cellar light on from 9:45 P.M. until 7:45 A.M. the next day. It was a 100-watt bulb. How many watt-hours of electricity did he waste 1000 and what did it cost at $2\frac{1}{2}$¢ per kw-hr? $2\frac{1}{2}$¢



In connection with this work, point out that electric appliances that produce heat (toasters, stoves, irons) use much more electricity per hour than those with electric motors (fans, clocks).

Using Electric Appliances

1. Here are the names of some electric appliances, with the number of watts each requires:

Electric clock	2 watts	Toaster	900 watts
Refrigerator	300 watts	Table stove	800 watts
Vacuum cleaner	300 watts	Waffle iron	700 watts

If the clock runs continuously during a 30-day month, how many watt-hours of current does it use? 1440 How many kw-hr does it use? 1.44

2. The refrigerator motor runs about 8 hr. every day. How many kw-hr does it use in a 30-day month? 72 See ex. 1. Find the cost of this electricity at 3¢ per kw-hr. $2.16

3. Each week the vacuum cleaner is used ½ hr. a day for 5 days and on the sixth day it is used for 3 hr. How many kw-hr does the cleaner use in 4 weeks? See ex. 1. 6.6

4. For breakfast each day the toaster is used for 12 min. and the table stove for 15 min. See ex. 1. In a 30-day month, how many kw-hr are used in this way? 11.4 What does the electricity cost at 4¢ per kw-hr? 45.6¢

5. Bobby makes waffles for Sunday morning breakfast, using the waffle iron for 40 min. How many watt-hours are used on 3 Sunday mornings? 1400 How many kw-hr? 1.4 See ex. 1. What does this electricity cost at 4¢ per kw-hr? 5.6¢

6. To keep food frozen, a food locker uses 3 kw-hr per day. How many kw-hr does it use in 30 da.? 90 in 100 da.? 300

7. A washing machine (250 watts) takes 30 min. to wash one tubful of clothes. If Mrs. Green has 4 tubfuls at each washing, how many kw-hr does she use in 5 washings? 2.5 What does the electricity cost at 3¢ per kw-hr? 7.5¢

8. Mr. Williamson bought an electric heater which requires 1320 watts of electric power. If this heater is turned on 2 hr. each day, how many kw-hr of electricity will be used in a 30-day month? 79.2 What will the electricity cost at 2½¢ per kw-hr? $1.98

Have pupils compare markings of appliances used in their homes with ones given in ex. 1. Do some problems with the pupils and have them complete others if understanding is assured. Have volunteers explain solutions.

Present application of multiplication of fractions. Have each pupil bring to class one recipe that uses at least three fractions or mixed numbers. Then have each recipe changed to another that uses half the amount; to one that uses $1\frac{1}{2}$ times the amount.

Changing Recipes

1. A cook needs to know how to multiply and divide fractions in order to make more or less than the amount a recipe will make. She may need to find $2\frac{1}{2}$ times $\frac{1}{2}$ cup of milk. How much would this be? $1\frac{1}{4}$ cups

2. Peggy wants to make $\frac{1}{2}$ the recipe for beef stew given below. How much of each ingredient should she use?

Beef Stew

$1\frac{1}{2}$ pounds lean beef $\frac{3}{4}$
$\frac{1}{4}$ cup shortening $\frac{1}{8}$
4 cups boiling water 2
$1\frac{1}{4}$ teaspoons salt $\frac{5}{8}$
a dash of pepper same

2 tablespoons flour 1
2 tablespoons water 1
1 cup sliced onions $\frac{1}{2}$
$\frac{1}{4}$ cup chopped green pepper $\frac{1}{8}$
2 cups canned tomatoes 1

3. Ann made $\frac{1}{2}$ the recipe for chocolate waffles given below. Find the amount of each ingredient she used. (1) How much of each should she use for $1\frac{1}{2}$ times the recipe? (2)

(1), (2) Chocolate Waffles

$1\frac{1}{2}$ cups sifted flour $\frac{3}{4}$, $2\frac{1}{4}$
2 teaspoons baking powder 1, 3
$\frac{1}{2}$ teaspoon salt $\frac{1}{4}$, $\frac{3}{4}$
$\frac{1}{2}$ cup sugar $\frac{1}{4}$, $\frac{3}{4}$
2 egg yolks, well beaten 1, 3

$1\frac{1}{2}$ cups milk $\frac{3}{4}$, $2\frac{1}{4}$
6 tablespoons melted butter 3, 9
$2\frac{1}{2}$ squares chocolate, melted $1\frac{1}{4}$, $3\frac{3}{4}$
1 teaspoon vanilla $\frac{1}{2}$, $1\frac{1}{2}$
2 egg whites, well beaten 1, 3

4. Jim likes to cook. He made $1\frac{1}{2}$ times this recipe for nut muffins. How much of each ingredient did he use?

Nut Muffins

2 cups sifted flour 3
$2\frac{1}{2}$ teaspoons baking powder $3\frac{3}{4}$
3 tablespoons sugar $4\frac{1}{2}$
$\frac{1}{2}$ teaspoon salt $\frac{3}{4}$

$\frac{1}{2}$ cup nut meats $\frac{3}{4}$
1 egg, well beaten $1\frac{1}{2}$ (2 small ones or 1 and 1 yolk)
1 cup milk $1\frac{1}{2}$
4 tablespoons melted butter 6

Post pupils' recipes and new ones on bulletin board for future reference.

Present set of improvement tests in subtraction.

Improving by Practice

Subtraction Test 3a.

Time: 4 min.

1.	77967 12469 65,498	27435 4786 22,649	70283 39295 30,988	82352 27593 54,759
2.	92142 4093 88,049	61248 51469 9779	81071 24198 56,873	45788 18746 27,042
3.	76300 7996 68,304	84826 46047 38,779	50174 22379 27,795	69620 10991 58,629

⑫

Subtraction Test 3b.

Time: 4 min.

4.	83603 9977 73,626	84166 34679 49,487	72190 55091 17,099	59112 26826 32,286
5.	65409 40953 24,456	94085 63479 30,606	48146 9603 38,543	20399 9124 11,275
6.	67253 19934 47,319	38513 31998 6515	63310 51897 11,413	56399 39689 16,710

⑫

Subtraction Test 3c.

Time: 4 min.

7.	41907 26023 15,884	31153 14878 16,275	59566 19679 39,887	61354 12489 48,865
8.	72308 2353 69,955	96421 55497 40,924	80101 12782 67,319	74089 45448 28,641
9.	68220 8358 59,862	69154 3366 65,788	84258 10805 73,453	54923 32567 22,356

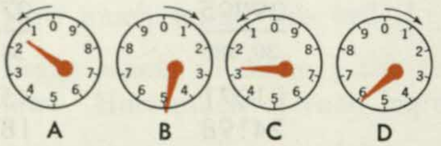
⑫

To the Pupil. This is the third set of Improvement Tests in subtraction. Try to get a score of 10 on each test.

Tests are to be given, scored, and recorded as others were (see pages 46-50, 57, 109). Have pupils compare scores with previous ones to note improvements or further needs. Plan remedial work based on results. Let others work on projects.

Teach how to read an electric meter and to figure cost of using given number of kilowatt-hours of electricity (pages 156–158).

Reading the Electric Meter

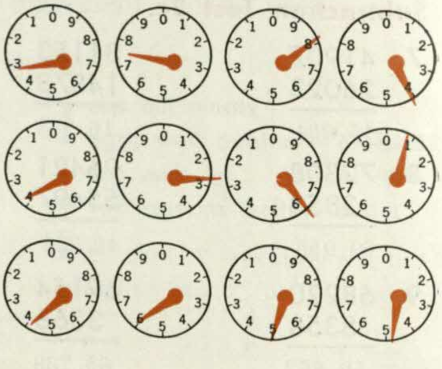


1. An electric meter measures the amount of electricity that has been used in a building. There are four small dials on an electric meter, as shown at the left. These four dials together give the number of kilowatt hours of electricity that have been used since the meter was first installed. To read the meter, read Stress: it like a four-figure number, beginning at the left dial. Dial A shows thousands, dial B shows hundreds, dial C shows tens, and dial D shows ones. The pointer on a dial is often between two numbers; in this case read the smaller number Stress. for that dial.

On A the pointer is between 1 and 2; read 1.
On B the pointer is between 5 and 6; read 5.
On C the pointer is between 2 and 3; read 2.
On D the pointer is between 6 and 7; read 6.

The dials above read 1526, which means that 1526 kw-hr of electricity have been used since the meter was put into use.

2. Read the number of kw-hr shown on these three sets of dials. The first set of dials shows the reading on March 31, the second set shows the reading on April 30, and the third set shows it on May 31. How many kw-hr were used during April? 476 How many were used during May? 385


2784
3260
3645

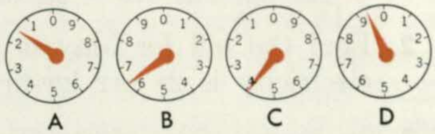
3. Suppose the dials on a meter show a reading of 4018 on June 30 and a reading of 4407 on July 31. How many kw-hr of electricity were used during July? 389

4. If the reading is 4733 on August 31, how many kw-hr were used during August? See ex. 3. 326

All explanation and work on this page should be discussed thoroughly with class. Emphasize that on dials A and C pointers move in counterclockwise direction; on B and D they move in clockwise direction. Give more practice on reading meters.

Reading the Dials Correctly

1. Each dial of a meter is related to the dial at its left. While the pointer on D makes a complete revolution, the pointer on C moves slowly ahead 1 space, just as the hour hand of the clock moves ahead 1 hour while the minute hand is making a complete revolution. Dials C and B are related in the same way and likewise dials B and A. If you understand this relationship, it will help you to read the dials correctly. Is the reading on the dials above 1649 or 1639? To answer this question, study dials C and D carefully, as is done below.



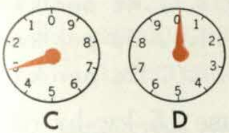


Fig. 1

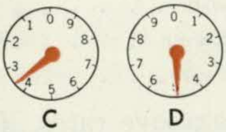


Fig. 2

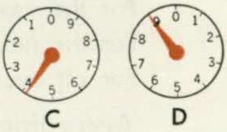


Fig. 3

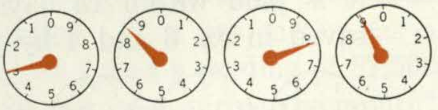
Suppose that pointer D starts on 0 and pointer C on 3, as in Fig. 1. When pointer D reaches 5, pointer C is halfway between 3 and 4, as shown in Fig. 2. When pointer D reaches 9, as shown in Fig. 3, pointer C is **nearly** at 4, but it will not reach 4 until pointer D reaches 10 (or 0). Since pointer D in Fig. 3 is still at 9, dial C is read 3; so the correct reading of dials C and D is 39 and not 49. If 49 were the correct reading, then pointer C would be close to 5, which is not the case. This shows that the meter reading at the top of the page should be 1639.

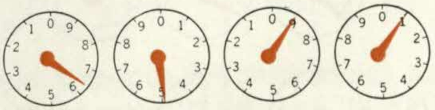
Stress.

Stress.

2. Read the two sets of dials at the right, and in each case watch carefully the two dials that may give trouble. When the pointer seems to be exactly on a number, look at the dial at its right in order to make the correct reading.

Stress.


2879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Follow development as given in ex. 1–2. Have pupils read meter in their homes and bring in copy of dials, showing positions of pointers. Discuss the more difficult cases with whole class. Give more practice as needed.

Paying Bills for Electricity

1. In ex. 2 on page 156, you found the number of kw-hr used in April was 476 kw-hr. At 2¢ per kw-hr, find the bill for electricity in April. $9.52

▶The bill was 476 × 2¢, or $9.52.

2. Find the bill for May in ex. 2 on page 156 if the rate for electricity is 2¢ per kw-hr. $7.70

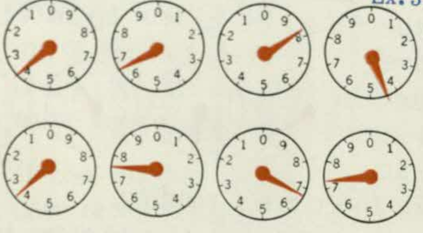
3. In many cities electricity is sold at decreasing rates per kw-hr. Then the more electricity you use, the less it costs per kw-hr. In Mill Creek these are the monthly rates:

For the first 15 kw-hr (or less)	85¢
For the next 35 kw-hr	5¢ per kw-hr
For the next 50 kw-hr	4¢ per kw-hr
For the next 100 kw-hr	3¢ per kw-hr
For all over 200 kw-hr.	2¢ per kw-hr

According to the above rates, if you use 15 kw-hr or less in any month, you must pay 85¢ for that month. In fact, if you are away a month and use no electricity, you still have to pay 85¢ for that month. If you use 70 kw-hr in a month, you consider 70 equal to 15 + 35 + 20. You pay 85¢ for 15 kw-hr, 5¢ per kw-hr for 35 kw-hr, and 4¢ per kw-hr for 20 kw-hr. How much in all do you pay? $3.40 At these rates, find the cost of 110 kw-hr. $4.90

4. In Mill Creek how much is the charge for 40 kw-hr in a month? $2.10 for 180 kw-hr? $7.00 for 220 kw-hr? $8.00 for 375 kw-hr? $11.10

5. In Mill Creek there is a 10% discount on an electric bill if it is paid within 15 days. Find the amount that can be saved in ex. 3 and 4 by paying these bills within 15 days. Ex.3: $.34, $.49, ex. 4: $.21, $.70, $.80, $1.11



6. Here are two sets of dials showing the reading of Mr. Brown's electric meter on November 30 and December 31. At the Mill Creek rates, find his bill. $3.92

Do ex. 1–3 with pupils. Point out that the more electricity one uses, the cheaper it gets. Discuss reasons for this with class. Have pupils investigate electric rates in your community and use them in problems.

Teach how gas is measured and how to find cost of given number of cubic feet of gas.

Paying Gas Bills

1. Many homes have gas stoves and gas furnaces. The amount of gas used is measured by a gas meter which measures the gas by the cubic foot (cu. ft.).

Some gas companies sell gas by the 1000 cu. ft. Suppose the gas consumed is 11,600 cu. ft. at a charge of \$1.25 per 1000 cu. ft. Divide 11,600 by 1000 by moving the decimal point 3 places to the left; this gives 11.6. Multiply \$1.25 by 11.6, which gives \$14.50, the amount of the bill for the gas used.

2. At \$1.25 per 1000 cu. ft., find the amounts of bills for these numbers of cubic feet of gas: 1800; 9400; 12,200; 10,400;
\$2.25 \$11.75 \$15.25 \$13.00
8600; 2500; 8880.
\$10.75 \$3.13 \$11.10

3. The gas company in one locality sells gas by the 100 cu. ft., with a decreasing **monthly** rate as follows:

First 200 cu. ft. or less	\$2.00	
Next 1,800 cu. ft.	.10	per 100 cu. ft.
Next 28,000 cu. ft.	.08	per 100 cu. ft.
Next 70,000 cu. ft.	.07	per 100 cu. ft.
All over 100,000 cu. ft.	.065	per 100 cu. ft.

At the above rates, find the monthly bills for these numbers of cubic feet of gas: 900; 2400; 6500; 11,600; 55,300;
\$2.70 \$4.12 \$7.40 \$11.48 \$43.91
120,000.
\$88.20

4. A gas meter is similar to an electric meter. A gas meter used in homes has three dials which show the number of 10,000's, 1000's, and 100's of cubic feet of gas that have been used since the meter was first installed. If a gas meter has a reading of 75,200 cu. ft. on March 1 and a reading of 87,500 cu. ft. on April 1, how much gas was used during the month? 12,300 cu. ft.
Compute the bill at the rates in ex. 3.
\$12.04

5. Find the amount of gas used in a month if the readings at the beginning and end of the month were 27,300 and 39,200. 11,900 cu. ft.
Compute the bill at the rates in ex. 3. \$11.72

Make sure pupils understand computations involved in ex. 1 and 3. Have pupils find out gas rates in your community and use them in problems.

Do first two problems orally, asking different pupils which numbers in the problems are not needed and why. Emphasize that only numbers used for solution of problem are to be used.

Which Numbers Are Not Needed?

In each problem tell which numbers are not needed to answer the question in the problem. Then work the problem:

1. At 8:30 A.M. Mr. Wood left home in his car to make a trip of 171 mi. The speed limit on the road is 50 mi. an hour; Mr. Wood drove at an average speed of 38 mi. an hour. How long did Mr. Wood drive on that trip?
8:30 A.M. and 50 mi. not needed; $4\frac{1}{2}$ hr.

2. Mary has a dress pattern that requires $3\frac{1}{2}$ yd. of material 39 in. wide. She bought a remnant of material that width containing $3\frac{7}{8}$ yd. It was marked down from $1.00 a yard to $.75 a yard. How much more material did Mary buy than she needed? 39 in., $1.00, and $.75 not needed; $\frac{3}{8}$ yd.

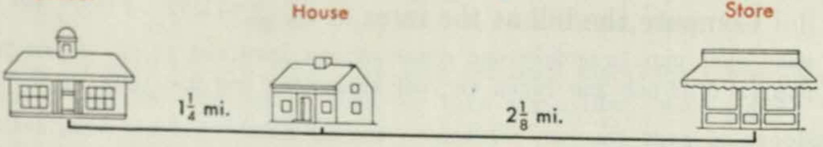
3. Bob was saving to buy a bicycle costing $42.50. When he had saved $13.50, he found he could buy an old bicycle for $16.75 and 2 new tires at $2.75 each. If he bought the old bicycle and 2 tires, how much did he pay for them?
$42.50 and $13.50 not needed; $22.25

4. The Browns had won $\frac{4}{5}$ of the 40 baseball games they had played, while the Reds had played 52 games and won 39 of them. How many games had the Browns won?
52 games and 39 not needed; 32

5. Miss Reed's class had a picnic yesterday. Sally had $1.25 when she went to the picnic. She paid $.38 as her share of the expenses of $12.16. She also spent $.14 for bus fare, $.25 for peanuts and ice cream, and $.35 for a boat ride. How much did Sally spend? $1.25 and $12.16 not needed; $1.12

6. By the piece, Mrs. Chase can buy 15 yd. of ribbon for $1.20. By the yard, she must pay $.50 for 5 yd. Find the cost of 1 yd. of ribbon bought by the piece. $.50 and 5 yd. not needed; $.08

7. Henry lives $1\frac{1}{4}$ mi. from school and $2\frac{1}{8}$ mi. from the store. He can walk to school in 20 min. and ride his bicycle there in 9 min. It takes him 40 min. to walk to the store. How much farther is it from Henry's house to the store than it is from his house to school?
20 min., 9 min., and 40 min. not needed; $\frac{7}{8}$ mi.



Teach the economy of buying food supplies in large packages if food can be used before it spoils.

Buying Wisely

Some shoppers buy more wisely than others and save money Stress. by doing so. One of the ways to save money is to buy food in large packages **if the family is large enough to use the food before it spoils.** Here are some prices of articles sold in small packages and also in large packages:

Article	Amount in small package	Cost	Amount in large package	Cost
Crackers	8 oz.	23¢	1 lb.	33¢
Shortening	1 lb.	35¢	3 lb.	97¢
Prunes	1 lb.	29¢	2 lb.	49¢
Cocoa	$\frac{1}{2}$ lb.	38¢	1 lb.	67¢
Maple sirup	1 pt.	$1.15	1 gal.	$7.00
Flour	5 lb.	69¢	25 lb.	$2.69

1. How much can be saved by buying a 1-pound box of crackers instead of buying an 8-ounce box today and another 8-ounce box next week? 13¢

2. Find the saving on 3 lb. of shortening, 8¢ on 2 lb. of prunes, 9¢ and on 1 lb. of cocoa 9¢ if they are purchased in large packages.

3. Mrs. Ward uses at least 1 gal. of maple sirup during the year. How much can she save by buying a 1-gallon can of sirup instead of buying enough pint bottles of sirup at different times to make 1 gal.? $2.20

4. If Mrs. Fox can use 25 lb. of flour before it spoils, how much can she save by buying a 25-pound bag rather than five 5-pound bags? 76¢

5. Another way to save money is to buy at a sale or when there is a special price. A shopper should be able to tell which is the better price. Which is the better price on oranges, 1 doz. at 35¢ or 9 oranges at 29¢? 1 doz. at 35¢

6. Mr. Case saves money on paint by buying it by the gallon instead of by the quart. How much does he save on 1 gal. of paint if a quart costs $1.29 and a gallon costs $4.42? $.74 What is the cheapest way he can buy 6 qt. of this paint? 1 gal. and 2 qt.

Discuss information above ex. 1 with pupils. Let them complete ex. 1-6. Have pupils find out local prices of the 6 articles mentioned at top of page and figure out saving effected by buying larger package.

Teach meaning of installment buying and show its advantages and disadvantages through problems (pages 162-163).

Buying on the Installment Plan

1. Mrs. Mills is planning to buy living room furniture and some kitchen equipment. The cash price of the furniture is $375. The installment price is as follows: a cash payment equal to 20% of the cash price and 12 monthly payments of $26.60 each. Find the actual price of this furniture if it is bought on the installment plan? $394.20 How much can Mrs. Mills save if she pays cash for the furniture? $19.20

2. The kitchen equipment Mrs. Mills wishes to buy costs $300. To buy this equipment on the installment plan she must make a cash payment of $100. Then she can make 6 monthly payments of $34.51 each,(1) or 12 monthly payments of $17.76 each,(2) or 18 monthly payments of $12.18 each.(3) Find the installment price by each of these three plans. How much can Mrs. Mills save if she pays cash instead of selecting the plan with 12 monthly payments? $13.12
(1) $307.06; (2) $313.12; (3) $319.24

3. If Mrs. Mills buys all the things in ex. 1 and 2 on the installment plan, how much does she need for both down payments? $175 How much in all will she have to pay each month if she selects the 12-month plan in ex. 2? $44.36 Do you think she should plan to do all this installment buying in one year? Give reasons for your answer.

4. Mr. Brown needs a new milking machine on his dairy farm. He can get one for $295 but he does not have that amount of cash to pay for the machine. The installment price is found as follows: a cash payment of $25 and 15 monthly payments of $20 each. Find the total amount Mr. Brown will pay for the milking machine if he buys it on the installment plan. $325 How much more is this than the cash price? $30 Do you think Mr. Brown would be wise to spend the extra money to buy the machine on the installment plan? Give reasons for your answer.

5. From a store in your city get figures for buying a television set on the installment plan. Name two advantages and two disadvantages of buying on the installment plan.

Point out that installment prices of articles are higher than cash prices. Lead pupils to see that if article is a necessary one, installment buying is justified; if unnecessary, it is better to wait until the cash is saved.

Payments on a Car

1. Miss Wood bought a new car on the installment plan. The price of the car was $2700, to be paid as follows:

1. Cash payment equal to $\frac{1}{3}$ of the cost of the car. How much was this cash payment? $900

2. Equal monthly installments for 1 yr. to cover the total amount of these 3 items: (a) balance due on the cost of the car, which is $1800; (b) insurance for 1 yr., which costs $60; and (c) a carrying charge, which is 5% of $1860, or $93. What is the total of these 3 items? $1953 To pay this total in 12 monthly installments, how much will each installment be? $162.75

Find the total amount that Miss Wood must pay for the car, including the insurance and the carrying charge. $2853

2. Another kind of car costs $3000. If the cash payment is $\frac{1}{3}$ of the cost, the insurance is $68, and the carrying charge is 5% of the sum of the insurance and the balance of the cost, find the monthly payment for 12 months. $180.95 Then find the actual amount a person would pay for the car if he bought it on the installment plan. $3171.40

3. Try to find the installment terms for buying a car from one of the automobile dealers in your town.

Discuss ex. 1 with pupils and explain (or have volunteer explain) the meaning of carrying charge and reason for it. In connection with ex. 3, have pupils make up problems for class to solve, based on their findings.

Present application of percentage through problems about budgets (pages 164-166). Discuss ex. 1 with pupils, the reasons for making a budget, items included, and other items that might have been included.

Making a Budget

1. When the Browns first began to keep house, they found that often by the end of the month they did not have enough money left to meet expenses. Then they decided to follow a **budget** plan which lays aside certain percentages of Mr. Brown's yearly income after taxes. Mrs. Brown kept an account of approximately $4000 spent last year, so she knows how much was used for food, clothing, and other items. The budget plan the Browns are trying to follow for the coming year is based on their expenses of last year. Here is their plan:

Rent	20%	Clothing	15%
Food	25%	Advancement	10%
Savings	15%	Reserve fund	7%
	Operating expenses 8%		

2. The budget allows 20% of $4000 for rent. How much is that for rent per year? How much is that per month? About $67

$800

3. The budget given in ex. 1 allows 20% for rent. If the Browns own a house and do not pay rent, can the item of rent be left out of the budget? Yes Will some other item be needed to take the place of rent? If so, what will it be?

Taxes and upkeep of the house will have to be added.

4. The budget of the Browns allows 25% of $4000 for food. How much is that per year? per month? About $83

$1000

5. The Browns were able to keep the food item as low as 25% because they had a large garden and did much canning; otherwise they would have allowed 30% for food. How

Have pupils complete ex. 2-11. Then discuss problems with pupils, letting volunteers explain their solutions. Have pupils note why budget must sometimes be modified.

Have pupils make out own budget for a week, with items given in per cents.

much would that have been? $1200 If they had allowed 30% for food, what would have happened to some of the other items in the budget? They would need to reduce other items by 5%. What item might they have changed in order to provide 30% for food? Reduce reserve fund by 5% or make reductions in clothing, advancement, and savings, totaling 5%.

6. How much per year did the Browns allow for clothes? $600 What would this average per month? $50 Are the purchases for clothing likely to be the same each month? No Is it allowable to spend more for clothing one month if one spends less another month? Yes At what time in the year is the greatest amount of money usually spent for clothing? In the fall

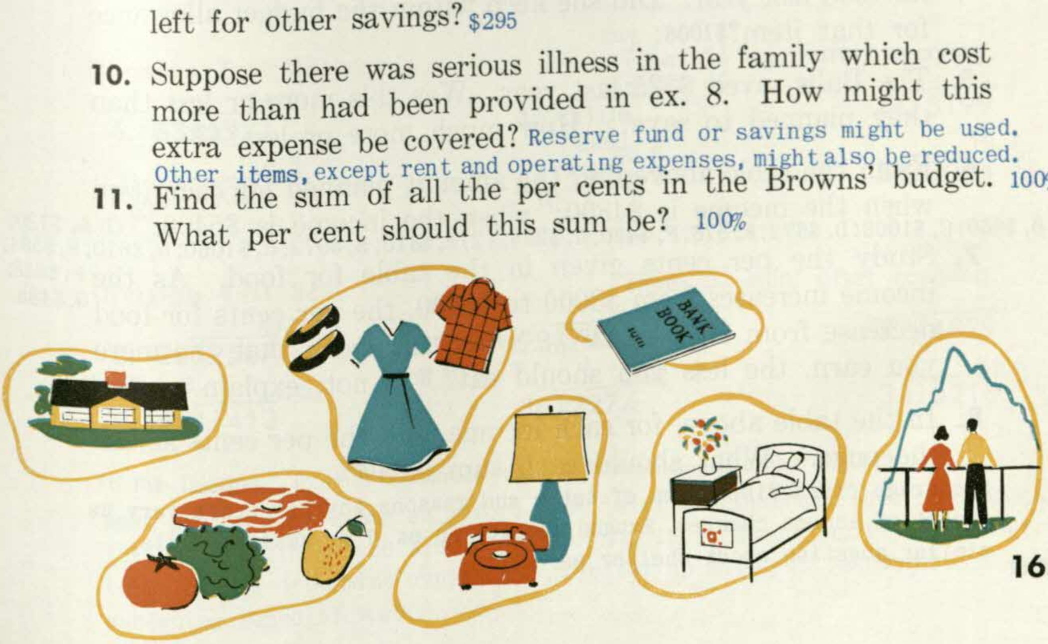
7. Operating expenses include the amounts spent for running the household, such as fuel, light, telephone, laundry. What is the average amount per month allowed for this item? About $27

8. Advancement includes education, books, church, health, medical and dental care, and recreation. How much per year is allowed for this? $400 Do you think the Browns can afford to spend $200 for a vacation trip? There would then be only $200 left for all the other important items under "advancement."

9. How much is allowed for savings per year? $600 This allowance includes money put in the savings bank, payments for life insurance, payments for bonds, etc. Mr. Brown pays $125 per year for life insurance and makes payments of $15 a month to purchase a bond. Do these payments use up all the amount planned for savings? No If not, how much is still left for other savings? $295

10. Suppose there was serious illness in the family which cost more than had been provided in ex. 8. How might this extra expense be covered? Reserve fund or savings might be used. Other items, except rent and operating expenses, might also be reduced.

11. Find the sum of all the per cents in the Browns' budget. 100% What per cent should this sum be? 100%



Fitting Budgets to Incomes

The per cents in a budget vary as the amount of income changes. The table below gives suggested budget per cents for annual incomes from $3000 to $5400 after taxes.

	A	B	C	D	E	F	G
Income	Savings	Shelter	Food	Clothing	Operating	Advancement	Reserve
$3000	8%	25%	30%	12%	10%	8%	7%
$3600	10%	22%	28%	13%	10%	9%	8%
$4200	12%	20%	25%	14%	12%	9%	8%
$4800	15%	20%	21%	14%	12%	10%	8%
$5400	15%	18%	20%	15%	11%	12%	9%

1. Mr. King earns $3000 a year. Using the per cents given above for that income, find how much Mr. King's family should plan to spend for each item in the budget. A, $240; B, $750; C, $900; D, $360; E, $300; F, $240; G, $210

2. Mr. Ball earns $80 a week. Choose the plan above that most closely fits Mr. Ball's yearly income; Use budget for $4200. then find how much Mr. Ball's family should spend for each item. A, $499.20; B, $832; C, $1040; D, $582.40; E, $499.20; F, $374.40; G, $332.80

3. Mr. Ball pays $90 a year for life insurance. Since this is part of his savings, how much more money should be saved during the year? $409.20 Suggest two things to do with this money. Saved in savings bank, savings bonds, savings-and-loan association, etc.

4. Last year Mr. Ball earned $3600. Mrs. Ball spent $950 for food last year. Did she keep within the budget allowance for that item? $1008; yes

5. The Balls saved $325 last year. Was this more or less than they planned to save? Less How much more or less? $35

6. From the table above find the amount planned for each item when the income is $4800; (1) when the income is $5400. (2) (1) A, $720; B, $960; C, $1008; D, $672; E, 576; F, $480; G, $384; (2) A, $810; B, $972; C, $1080; D, $810; E, $594; F, $648; G, $486

7. Study the per cents given in the table for food. As the income increases from $3000 to $5400, the per cents for food decrease from 30% to 20%. Does this mean that the more you earn, the less you should eat? No If not, explain. See Guide.

8. In the table above, for each income, add the per cents across the page. What should each sum be? 100%

Encourage class discussion of table and reasons why per cents vary as amount of income changes. Extend concept in ex. 7 by asking pupils similar question about shelter per cents.

Present set of improvement tests in multiplication and one in division.

Improving by Practice

Multiplication Test 3a.

Time: 4 min. after copying.

1.	2674	9735	8562	5893	8462	7493
	85	46	97	51	63	29 ⑥
	227,290	447,810	830,514	300,543	533,106	217,297

Multiplication Test 3b.

Time: 4 min. after copying.

2.	6852	3185	6413	1482	1253	9572
	72	18	39	64	57	23 ⑥
	493,344	57,330	250,107	94,848	71,421	220,156

Multiplication Test 3c.

Time: 4 min. after copying.

3.	1375	2607	9735	9407	3896	1864
	26	41	38	75	94	82 ⑥
	35,750	106,887	369,930	705,525	366,224	152,848

Division Test 3a.

Time: $5\frac{1}{2}$ min. after copying.

4. $\begin{array}{r}36.3\\81\overline{)2943}\end{array}$ $\begin{array}{r}93.8\\79\overline{)7408}\end{array}$ $\begin{array}{r}67.2\\35\overline{)2352}\end{array}$

5. $\begin{array}{r}83.1\\24\overline{)1995}\end{array}$ $\begin{array}{r}75.5\\48\overline{)3624}\end{array}$ $\begin{array}{r}64.7\\57\overline{)3686}\end{array}$ ⑥

Division Test 3b.

Time: $5\frac{1}{2}$ min. after copying.

6. $\begin{array}{r}94.0\\63\overline{)5921}\end{array}$ $\begin{array}{r}67.5\\46\overline{)3105}\end{array}$ $\begin{array}{r}43.2\\74\overline{)3199}\end{array}$

7. $\begin{array}{r}80.2\\29\overline{)2326}\end{array}$ $\begin{array}{r}89.6\\25\overline{)2240}\end{array}$ $\begin{array}{r}79.1\\38\overline{)3004}\end{array}$ ⑥

Division Test 3c.

Time: $5\frac{1}{2}$ min. after copying.

8. $\begin{array}{r}89.5\\28\overline{)2507}\end{array}$ $\begin{array}{r}81.0\\92\overline{)7450}\end{array}$ $\begin{array}{r}62.7\\77\overline{)4829}\end{array}$

9. $\begin{array}{r}31.4\\45\overline{)1413}\end{array}$ $\begin{array}{r}63.2\\36\overline{)2274}\end{array}$ $\begin{array}{r}94.5\\34\overline{)3213}\end{array}$ ⑥

To the Teacher. In ex. 4–9, instruct pupils to find quotients to nearest tenth.

Have pupils copy examples before test is given. Instruct them to copy carefully and legibly (see also pages 69, 87, 127). Group pupils who had errors to help them determine causes and to clear up difficulties before assigning remedial work.

Acquaint pupils with different services of United States Post Office Department and give problems relating to these services (pages 168–171).

United States Mail

1. Almost every person in the United States is served by the United States mail. There are four classes of mail, each with its own rate of postage. **First class** mail includes both regular mail and air mail. The postage for regular mail is 5¢ per ounce, and that for air mail is 8¢ per ounce. Any fraction of an ounce is counted as a full ounce; for example, $1\frac{1}{2}$ oz. counts as 2 oz. First class mail includes all letters, handwritten and typewritten material, sealed parcels, and post cards. The charge for post cards is 4¢ each. These charges refer, of course, to mail sent and delivered within the United States. How much postage should be put on a letter sent by regular mail and weighing $\frac{1}{2}$ oz.? 5¢ on one weighing $1\frac{1}{4}$ oz.? 10¢ $2\frac{1}{2}$ oz.? 15¢ 5 oz.? 25¢ $6\frac{1}{2}$ oz.? 35¢ What would be the postage in each case if sent by air mail? 8¢; 16¢; 24¢; 40¢; 56¢

2. The postage on a letter sent by regular mail was 20¢. The letter weighed between . 3 . oz. and . 4 . oz.

3. Joe wants to send a letter by air mail to South America. What will the postage be at 15¢ per **half-ounce,** or fraction thereof, if the letter weighs $\frac{3}{8}$ oz.? 15¢ $1\frac{1}{2}$ oz.? 45¢ $\frac{3}{4}$ oz.? 30¢

4. Mr. Weeks sent a letter from Denver to Japan by air mail. What was the postage at 25¢ per half-ounce, or fraction thereof, if the letter weighed $1\frac{1}{4}$ oz.? 75¢ if it weighed $1\frac{7}{8}$ oz.? $1.00

5. If you wish to have a letter delivered by special messenger as soon as it arrives in the city to which it is addressed, you can put on it a **special delivery** stamp which costs 30¢. The letter must also have the usual postage on it. Find the cost of sending a letter by special delivery air mail if the letter weighs $\frac{3}{4}$ oz.; 38¢ $1\frac{1}{2}$ oz.; 46¢ $2\frac{3}{4}$ oz. 54¢

Have pupils check current rates (first class, special delivery, registered mail, parcel post) and rework problems on these pages if there have been any changes.

1. When first class mail contains money or valuable papers, it can be sent by **registered mail** by paying an extra fee. These extra fees vary with the value of the registered mail and are shown at the right for values up to $600. These fees are in addition to the usual postage. If registered mail is lost, the sender is reimbursed for its value, up to the amount covered by the registry fee. What is the registry fee for mail valued at $4? $.60 at $35? $.75 at $260? $1.25 at $190? $1.00 at $500? $1.50

Value of Mail	Registry Fee
$ 0.01 to $ 10	$ .60
$ 10.01 to $100	.75
$100.01 to $200	1.00
$200.01 to $400	1.25
$400.01 to $600	1.50

2. If a package valued at $125 weighs $7\frac{3}{4}$ oz., find the cost, including postage, of sending it by registered first class mail. $1.40

3. By paying an extra 10¢ the sender of a registered letter can request a **return receipt** which is signed by the person receiving the letter. Find the total cost of sending a registered letter, weighing $2\frac{1}{4}$ oz. and valued at $80, by air mail if the sender requests a return receipt. $1.09

Find the cost of mailing registered letters with these weights and values by regular first class mail:

4. $2\frac{1}{2}$ oz., $22 $.90 $1\frac{1}{4}$ oz., $9.75 $.70 24 oz., $168 $2.20

5. 3 oz., $38 $.90 $2\frac{3}{4}$ oz., $120 $1.15 $9\frac{1}{2}$ oz., $58 $1.25

6. $1\frac{1}{2}$ oz., $75 $.85 $1\frac{3}{4}$ oz., $175 $1.10 $3\frac{1}{4}$ oz., $514 $1.70

7. Second class mail is mostly concerned with newspapers and periodicals. Third class mail includes certain kinds of printed matter, merchandise, photographs, seeds, and other articles weighing not over 15 oz. Packages weighing over 15 oz. should be sent by parcel post; see page 170. If you are interested in knowing more about second or third class mail, inquire at the post office or consult this year's *World Almanac.*

Have pupils set up a display or chart showing examples of the various kinds of mail, rates, weight and size regulations. In connection with ex. 7, urge students to report findings to class and bring in examples of second class mail.

Parcel Post

1. **Fourth class** mail is called **parcel post** and is used for packages weighing **over** 15 oz. and up to 70 lb. The size of the package is limited to 100 in. for the length **plus** the girth. In measuring a package, the distance in a straight line between the ends is taken as its length while the distance around the package at its thickest part is taken as its girth. Stress. A box 24 in. long, 12 in. wide, and 10 in. high has a **length** of 24 in. and a **girth** of 12 in. + 10 in. + 12 in. + 10 in., or 44 in. The length plus the girth is 68 in.

   Both the weight and size of a package are limited to smaller amounts if the package is sent between two first class post offices. Find out about this at the post office.
   Have pupils report findings to class.

Which of these packages are too large to be sent by parcel post?

2. 17″ × 8″ × 5″ 36″ × 18″ × 12″ 4′ × 1′ × $1\frac{1}{2}$′ Too large

3. 42″ × 6″ × 4″ 40″ × 17″ × 16″ Too large 6′ × $\frac{1}{2}$′ × $\frac{3}{4}$′ Too large

4. The cost of sending a package by parcel post depends upon its weight and the distance it is sent, as shown below:

Zone	Distance	One to Two Pounds	Each Additional Pound
Local	Local post office	24¢	2¢
1–2	Up to 150 mi.	33¢	5¢ to 10 lbs., 4¢ over 10 lbs.
3	150 to 300 mi.	35¢	6¢ to 15 lbs., 5¢ over 15 lbs.
4	300 to 600 mi.	39¢	8¢ to 5 lbs., 7¢ 6 to 50 lbs., 6¢ over 50 lbs.
5	600 to 1000 mi.	45¢	10¢ to 20 lbs., 9¢ 21 to 50 lbs., 8¢ over 50 lbs.
6	1000 to 1400 mi.	51¢	13¢ to 10 lbs., 12¢ 11 to 30 lbs., 11¢ over 30 lbs.
7	1400 to 1800 mi.	58¢	16¢ to 15 lbs., 15¢ 16 to 30 lbs., 14¢ 31 to 50 lbs., 13¢ over 50 lbs.
8	Over 1800 mi.	64¢	19¢ to 10 lbs., 18¢ 11 to 30 lbs., 17¢ 31 to 40 lbs., 16¢ over 40 lbs.

Find the cost of sending these packages to the zones given. Any fraction of a cent is counted as another cent:

	Weight	Zone	Weight	Zone	Weight	Zone
5.	8 lb.	3 $.71	16 lb.	6 $2.27	36 lb.	2 $1.77
6.	6 lb.	4 $.70	21 lb.	8 $4.14	66 lb.	4 $4.74

Discuss ex. 1 and have various packages available to reinforce understanding of how size is measured (length and girth). Then have pupils complete ex. 2–3. Discuss ex. 4 and have pupils do some more examples before assigning ex. 5–6.

7. Parcel post packages may be insured against loss or damage by paying an additional fee, as shown in the table at the right. Find the cost of sending a 10-pound package having a value of $65 to a city in zone 5 by insured parcel post. $1.55

Value	Insurance Fee
$0.01 to $10	10¢
$10.01 to $50	20¢
$50.01 to $100	30¢
$100.01 to $200	40¢

8. Books can be sent to any zone at a rate of 10¢ for the first pound and 5¢ for each additional pound. Find the cost of sending by parcel post a package of books weighing 39 lb. $2.00

9. It costs $9.79 to send a 15-pound package to zone 6 by **air parcel post.** About how many times as much is this as the cost by ordinary parcel post? When would this extra cost be justified? About $4\frac{1}{2}$ times

10. Parcel post is often used to send packages to foreign countries. The cost of sending a package to England is 90¢ for the first two pounds and 35¢ for each additional pound. Find the cost of sending to England a package weighing 3 lb.; $1.25 a package weighing 20 lb. $7.20

11. The cost of sending a package to France by air parcel post is $1.89 for the first 4 oz. and $.42 for each additional 4 oz. Find the cost of sending a package weighing 2 lb. $4.83

Do problems orally with class, letting different pupils explain solutions at board. Some pupils may have had experiences in mailing packages. Have them tell class about these.

Compare different ways of shipping and their costs. For ex. 1 divide class into 5 groups, assigning one method to each group. Have pupils find out new

Shipping by Express and Freight

rates by choosing two cities in different parts of the country.

1. Other means of making shipments are Railway Express, air express, air freight, rail freight, and truck freight. Try to find out as much as possible about shipment by each of these methods, including size of packages allowed, the limit of the weight, rates, time for transportation, etc.

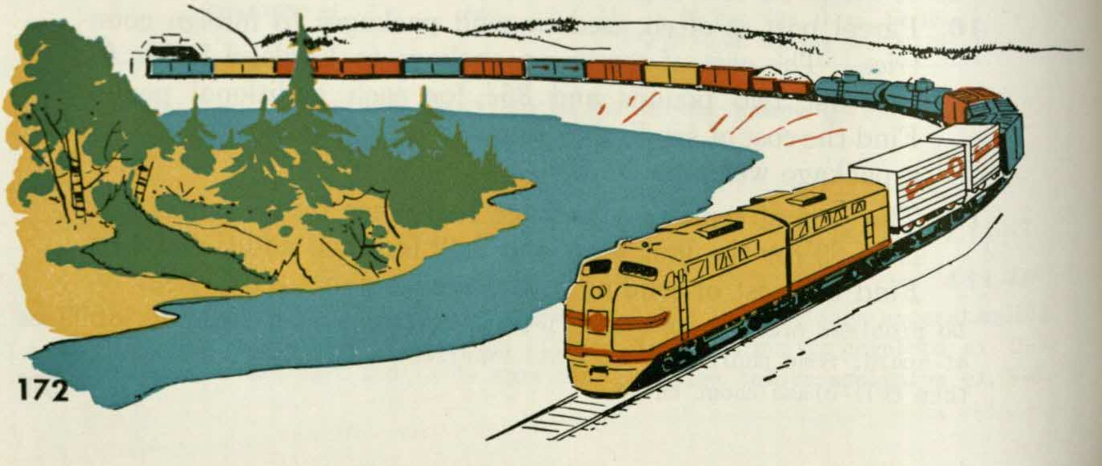
Have each group make two problems similar to ex. 3–5.

2. Tell which of the methods of shipment you think would be best for sending from California to Chicago each of the following: household furniture, flowers, an automobile, a small trunk, a piano, a bicycle, crates of strawberries.

3. Packages are often sent by Railway Express. The express rate between two certain cities is \$2.57 for a 5-pound package, \$3.59 for a 20-pound package, and \$7.04 for a 75-pound package. There is also a surcharge of 30¢ on each shipment. How much more would it cost to ship three 20-pound packages and three 5-pound packages, a total of 75 pounds, than to ship one 75-pound package? \$11.44

4. It costs \$18.75 per 100 pounds to ship a package from Chicago to Los Angeles by air freight. Find the total cost of sending a shipment weighing 300 lb. between these cities. \$56.25

5. Rail freight is an economical way to ship heavy articles. Freight rates depend upon the kind of article that is shipped as well as the distance. If the freight rate for furnaces between two cities is \$2.57 per 100 pounds, find the total cost of shipping a furnace weighing 450 lb. between these cities. \$11.57 The cost for 450 lbs. will be $4\frac{1}{2}$ times the rate for 100 lb. Why?



Acquaint pupils with telephone and telegraph services (pages 173–174).

Using the Telephone

1. There are two types of telephone service, one called **local** and the other, **long distance.** A person having a telephone at home or in an office can usually make any number of local calls for a certain monthly payment, except in the large cities. There are two kinds of long distance calls: the **station-to-station** type and the **person-to-person** type. Station-to-station means that the person calling is willing to talk with any person who answers the telephone, while person-to-person means that a certain person must be located and brought to the telephone. The cost of a person-to-person call is more than the station-to-station call. Why?

2. The rates for certain long distance calls are shown below. The first amount in each column is the rate for a 3-minute conversation; the amount in parentheses is the additional charge for each extra minute.

From New York to	Station-to-Station		Person-to-Person	
	Weekdays	Nights and Sundays	Weekdays	Nights and Sundays
Washington, D.C.	$ .80 (.20)	$ .60 (.15)	$1.10 (.20)	$ .90 (.15)
Chicago	1.45 (.40)	1.15 (.30)	2.10 (.40)	1.80 (.30)
Dallas	1.80 (.45)	1.40 (.35)	2.85 (.45)	2.45 (.35)
San Francisco	2.25 (.60)	1.75 (.45)	3.50 (.60)	3.00 (.45)

3. Mr. Fox telephoned from New York to Dallas at 11 A.M. on a person-to-person call and talked 12 min. Find the total charge including a 10% tax. $7.59 How much would he have saved by making this call in the evening at the person-to-person rate? $1.43 in the evening at the station-to-station rate? $2.58

4. Mr. Fox telephoned from New York to San Francisco on a Sunday on a station-to-station call. He talked 7 min. Find the cost including the 10% tax. $3.91 How much more would this call have cost if made at 4 P.M. on a weekday? $1.21 if made as a person-to-person call at 10 A.M. on a weekday? $2.58

5. Make up 5 problems, using the rates in ex. 2.

Discuss terms in ex. 1 with class, stressing meanings. If possible, have pupils investigate rates between their city and 3 or 4 other cities. Have them make a table like that in ex. 2 and use it in problems like ex. 3–4.

If possible, display telegram forms and give pupils practice in using them and figuring costs.

Sending Telegrams

1. To send a message by telegraph, you will probably choose one of these three types of service:

Emphasize reasons for using different types of service.

Full-Rate Telegram. This is a fast service, the message being sent immediately. The rates for these messages are for the first 15 words, with an extra charge for each additional word.

Day Letter. This service covers messages which may be delayed an hour or so. The rates for day letters are for messages of 50 words or less, with an extra charge for each additional group of 5 words or less.

Night Letter. This is an inexpensive overnight service for messages to be delivered the next morning. The rates for night letters are for messages of 50 words or less, with an extra charge for each additional group of 5 words or less.

2. The rate for a night letter of 50 words is less than that for a full-rate telegram of 15 words. The day-letter rate for 50 words is about 40% more than that for a full-rate telegram of 15 words.

Find the cost of these full-rate telegrams including tax:

	Number of Words	15 Words	Each Extra Word	Tax	
3.	20	$1.10	5¢	10%	$1.49
4.	19	1.45	6.5¢	10%	$1.88
5.	27	1.30	6¢	10%	$2.22

Find the cost of these night letters including tax:

	Number of Words	50 Words	Each Extra 5 Words	Tax	
6.	59	$1.05	7¢	10%	$1.31
7.	78	1.10	7.5¢	10%	$1.71

59 words count as 60 words; 78 words count as 80 words.

8. If the day-letter rate is $1.75 for 50 words plus 12¢ for each additional group of 5 words, what is the cost of a day letter of 60 words? of 76 words? Include 10% tax.

$2.19 $2.72

Have pupils visit telegraph office and secure rates from that office to several cities. Let them use these to make up and solve problems like ex. 3–8.

Review computation with decimals and per cents.

Decimals and Per Cents

Copy, add or subtract, and check:

1. $9.2 + 3.6 + 1.7 + 16.9$ 31.4 — $4218.3 - 3094.7$ 1123.6

2. $.17 + .58 + .96 + 4.33$ 6.04 — $4.3327 - 2.9984$ 1.3343

3. $26.231 + 5.025 + 9.418$ 40.674 — $16.375 - 14.759$ 1.616

4. $4.5 + 13 + 7.25 + 2.875$ 27.625 — \$400 − \$16.82 \$383.18

Multiply. Check by going over the work:

5. 12×6.218 74.616 — $.37 \times 66.38$ 24.5606 — $.05 \times .026$ .00130

6. 45×24.37 1096.65 — 1.9×17.52 33.288 — 1.6×4.25 6.800

7. 93×1.558 144.894 — $.58 \times 9.007$ 5.22406 — $.03 \times .004$ .00012

In ex. 8 and ex. 9, divide until there is no remainder. In ex. 10, find quotients to the nearest hundredth: Stress.

8. $204 \div .75$ 272 — $11.8 \div 2.5$ 4.72 — $8.37 \div .027$ 310

9. $2.17 \div 2.8$ .775 — $.111 \div .012$ 9.25 — $.297 \div .075$ 3.96

10. $.965 \div 3.4$ .28 — $217 \div 82.3$ 2.64 — $12.9 \div .031$ 416.13

Change to decimals (1) and then to per cents: (2)

(1) **11.** $\frac{9}{10}$ .9 — $\frac{7}{25}$ .28 — $\frac{3}{4}$ .75 — $\frac{12}{18}$ .66$\frac{2}{3}$ — $\frac{4}{5}$ .8 — $\frac{3}{8}$ .375 — $\frac{1}{6}$ .16$\frac{2}{3}$ — $\frac{17}{34}$ .5 — $\frac{24}{75}$ .32

(2) 90% — 28% — 75% — 66$\frac{2}{3}$% — 80% — 37$\frac{1}{2}$ % — 16$\frac{2}{3}$% — 50% — 32%

Find the answers. In ex. 14, find answers to the nearest cent:

12. 5% of \$128 \$6.40 — 62$\frac{1}{2}$% of \$152 \$95 — 200% of \$12.25 \$24.50

13. 3% of \$135 \$4.05 — 22$\frac{1}{4}$% of \$212 \$47.17 — 175% of \$14.60 \$25.55

14. 6% of \$2.98 \$.18 — 12$\frac{1}{2}$% of \$4.90 \$.61 — 215% of \$10.75 \$23.11

Estimate the answers; (1) then find the exact answers: (2)

(1),(2) **15.** 19% of 400 80, 76 — 12% of 1600 200, 192 — 76% of 2000 1500, 1520

16. 51% of 300 150, 153 — 24% of 280 70, 67.20 — 83% of 1200 1000, 996

17. 38% of 240 90, 91.20 — 67% of 333 222, 223.11 — 33% of 1500 500, 495

Check papers carefully and analyze errors. Return papers to pupils so they may find and correct any errors. Group pupils for reteaching or further review. Let others make up original problems using numbers in ex. 12–14.

Test covers problems with whole numbers, fractions, decimals, and percentage and can be used as mid-year problem test.

A Problem Test

1. How many feet does an automobile go in 3 sec. when it is traveling at a speed of 40 mi. per hour? 176

2. Find the cost of 38 kw-hr of electricity at the rate of 85¢ for the first 15 kw-hr and 5¢ per kw-hr for the rest. $2.00

3. Mr. Harris has been paying $780 a year to rent a house. Now the rent has been increased to $78 per month. Is this an increase of 10%, of 20%, or of 25%? 20%

4. Mrs. Smith drove 117 mi. on a trip to Short Hills. She left home at 8:15 A.M. and arrived in Short Hills at 11:30 A.M. What was her average speed for the trip? 36 mi.

5. A club committee paid $.40 a yard for ribbon for club arm bands. They bought enough ribbon to make 33 arm bands each 12 in. long. Find the cost of all the ribbon. $4.40

6. Jean needs 3½ yd. of material for a new dress. How much more will she pay for material at $1.50 a yard than for material at $1.34 a yard? $.56

7. Find Dick's monthly average on his arithmetic tests if his marks on the four tests were 81, 85, 92, and 94. 88

8. Ten years ago there were 20 Camp Fire Girls in Maybrook. Now there are 450% of that number. How many Camp Fire Girls are there in Maybrook now? 90

9. Mr. Page sold 340 bu. of apples. There were 100 bu. of "choice" grade at $4.25 a bushel and 215 bu. of "good" apples at $3.00 a bushel. The rest were "windfalls" at $1.25 a bushel. How much did he get for all the apples? $1101.25

10. Betty worked ¾ hr. after school each day for 4 da., 1 hr. on the fifth day, and 4½ hr. on Saturday. If she received $.60 an hour, how much did she earn? $5.10

Have pupils record scores on own graphs (page 41).

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Group pupils who had errors and have them explain their solutions. Try to determine if errors are caused by lack of understanding of problem situations, computations, using wrong process, and so on. Plan remedial work as needed.

Present diagnostic tests (pages 177-178) of skills taught thus far, with practice-page references. These pages may be used as mid-year test.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Divide and check the work:

			Practice Pages
1.	233,243 ÷ 46 $5070\frac{1}{2}$	209,632 ÷ 64 $3275\frac{1}{2}$	27-30
2.	261,965 ÷ 433 605	150,309 ÷ 293 513	32, 33
3.	884,764 ÷ 9617 92	653,184 ÷ 2016 324	36

Add. Check by going over the work:

4.	$2\frac{7}{8}$	$6\frac{1}{5}$	$4\frac{11}{12}$	$3\frac{3}{4}$	$1\frac{1}{2}$	54, 55 62
	$9\frac{11}{16}$	$\frac{9}{10}$	$7\frac{5}{6}$	$2\frac{1}{6}$	$2\frac{1}{3}$	
	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{4}$	$4\frac{1}{2}$	$1\frac{1}{5}$	
	$14\frac{1}{16}$	$9\frac{3}{5}$	16	$10\frac{5}{12}$	$5\frac{1}{30}$	

Subtract. Check by going over the work:

5.	$6\frac{3}{8}$	$9\frac{3}{4}$	$7\frac{2}{3}$	$8\frac{1}{2}$	$5\frac{2}{3}$	56
	4	$3\frac{1}{8}$	$4\frac{2}{3}$	$5\frac{3}{10}$	$3\frac{1}{2}$	
	$2\frac{3}{8}$	$6\frac{5}{8}$	3	$3\frac{1}{5}$	$2\frac{1}{6}$	
6.	8	$4\frac{1}{4}$	$7\frac{1}{10}$	$9\frac{1}{6}$	$6\frac{1}{8}$	58, 59 61
	$4\frac{9}{16}$	$2\frac{3}{4}$	$1\frac{3}{5}$	$3\frac{3}{4}$	$2\frac{5}{16}$	
	$3\frac{7}{16}$	$1\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{5}{12}$	$3\frac{13}{16}$	

Multiply. Check by going over the work:

7.	$3\frac{1}{2} \times 15$ $52\frac{1}{2}$	$24 \times 1\frac{1}{8}$ 27	$\frac{3}{4} \times 11$ $8\frac{1}{4}$	63, 64
8.	$\frac{3}{8} \times \frac{1}{2}$ $\frac{3}{16}$	$\frac{4}{5} \times \frac{5}{8}$ $\frac{1}{2}$	$\frac{9}{10} \times \frac{5}{12}$ $\frac{3}{8}$	65, 66
9.	$4\frac{1}{2} \times \frac{2}{3}$ 3	$6\frac{2}{3} \times 1\frac{3}{8}$ $9\frac{1}{6}$	$\frac{5}{6} \times 1\frac{1}{8}$ $\frac{15}{16}$	67

Divide. Check by going over the work:

10.	$15 \div \frac{5}{8}$ 24	$\frac{9}{16} \div \frac{3}{8}$ $1\frac{1}{2}$	$\frac{4}{5} \div \frac{1}{2}$ $1\frac{3}{5}$	72, 73
11.	$7\frac{1}{2} \div \frac{3}{10}$ 25	$3\frac{1}{3} \div \frac{5}{6}$ 4	$4\frac{1}{6} \div \frac{5}{8}$ $6\frac{2}{3}$	74
12.	$2\frac{1}{2} \div 15$ $\frac{1}{6}$	$2\frac{2}{3} \div 5\frac{1}{3}$ $\frac{1}{2}$	$\frac{9}{10} \div 1\frac{1}{2}$ $\frac{3}{5}$	75, 76

Results of tests should be noted on pupils' progress cards and compared with previous ones to determine further weaknesses and/or improvements. Use developmental material in text for reteaching if needed.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Practice Pages

Round off to the nearest tenth:

1. 6.82 6.8 4.67 4.7 15.29 15.3 36.08 36.1 104

Change to decimals:

2. $\frac{1}{4}$ .25 $\frac{7}{8}$ .875 $\frac{3}{25}$ .12 $\frac{11}{16}$ .6875 107

3. 37% .37 4% .04 $10\frac{1}{2}\%$ .105 178% 1.78 142, 146

Change to per cents:

4. .25 25% .5 50% .875 $87\frac{1}{2}\%$ $.33\frac{1}{3}$ $33\frac{1}{3}\%$ 133, 135

5. 2.9 290% 3 300% .045 $4\frac{1}{2}\%$ .0775 $7\frac{3}{4}\%$ 142, 146

Multiply. Check by going over the work:

6. $\begin{array}{r} .216 \\ 28 \\ \hline 6.048 \end{array}$ $\begin{array}{r} 4.39 \\ .07 \\ \hline .3073 \end{array}$ $\begin{array}{r} .005 \\ .02 \\ \hline .00010 \end{array}$ $\begin{array}{r} 5.56 \\ 3.14 \\ \hline 17.4584 \end{array}$ 96–100

Divide. If you continue to have a remainder, round off the quotient to the nearest hundredth: Stress.

7. $\begin{array}{r} .004 \\ 6\overline{).024} \end{array}$ $\begin{array}{r} 11.375 \\ 16\overline{)182} \end{array}$ $\begin{array}{r} 6.84 \\ 25\overline{)171} \end{array}$ $\begin{array}{r} 1.21 \\ 36\overline{)43.6} \end{array}$ 101–105

8. $\begin{array}{r} 5.3 \\ .8\overline{)4.24} \end{array}$ $\begin{array}{r} 1360 \\ .12\overline{)163.2} \end{array}$ $\begin{array}{r} .83 \\ 4.7\overline{)3.89} \end{array}$ $\begin{array}{r} 2550 \\ .014\overline{)35.7} \end{array}$ 111–113

Find the answers to the nearest cent:

9. 14% of $33 $4.62 42% of $65 $27.30 54% of $95 $51.30 128, 129

10. 75% of $52 $39 $33\frac{1}{3}\%$ of $78 $26 $62\frac{1}{2}\%$ of $72 $45 132–136

11. $4\frac{1}{4}\%$ of $12 $.51 $15\frac{1}{2}\%$ of $50 $7.75 $10\frac{2}{3}\%$ of $57 $6.08 142, 144

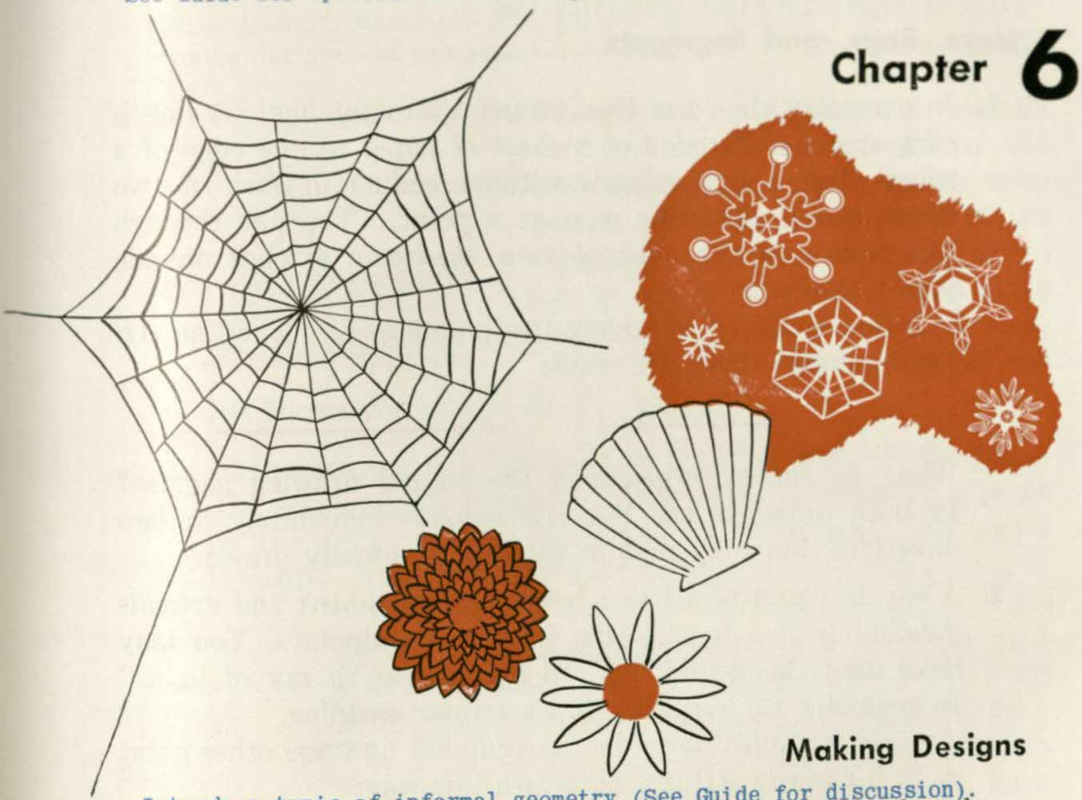
12. 200% of $9 $18 128% of $8 $10.24 375% of $12 $45 145, 146

13. 6% of $6.32 $.38 $37\frac{1}{2}\%$ of $15 $5.63 18% of $4.42 $.80 138

Try to determine causes of errors through individual conferences with pupils. Clear up difficulties before assigning practice pages. Mastery of these skills should be assured before beginning new work.

See Guide for specific aims of Chapter 6.

Chapter 6



Making Designs

Introduce topic of informal geometry (See Guide for discussion).

1. In nature you can see many beautiful designs that have geometric shapes. The spider web in the picture at the top of the page is a good example. Nature gives interesting geometric shapes to snowflakes, shells, and flowers. Many attractive designs on cloth and wallpaper contain geometric shapes. You can also see geometric forms when you look at houses or large buildings, at furniture, and at airplanes.

2. You see a **circle** when you look at a wheel of an automobile, at a dinner plate, or at the top of a waste basket. Give as many examples of circles as you can. Display and discuss instruments listed in ex. 3.

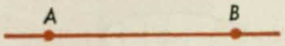
3. Geometric designs can be drawn by using a pair of compasses to make circles, a protractor to measure angles, and a draftsman's triangle to construct perpendicular and parallel lines. You will learn in this chapter how to use each of these drawing instruments. Require all pupils to have compasses, protractor, ruler, and sharp pencil in class the next day. Right triangles may be made in class from pieces of cardboard.

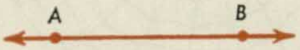
Have several examples of lines, rays, and line segments on separate cards or sheets for class identification.

Lines, Rays, and Segments

1. In geometry the term **line** means a straight line. A line is suggested by the edge of a sheet of paper or the edge of a ruler. But a line extends without ending in each of two directions; it does not stop at a point. The line through two points on the floor of your classroom extends outside your classroom.

A line is named by giving two points on it. The line AB can be drawn these two ways:

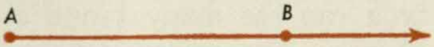




What do the arrowheads in the second drawing suggest? In both drawings the line AB extends indefinitely in two directions, but only part of the line is actually drawn.

2. A **ray** is a part of a line. It has one **endpoint** and extends indefinitely in one direction from this endpoint. You may have used the word "ray" in the phrase "a ray of light." In geometry the term ray has a similar meaning.

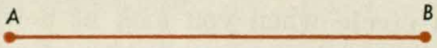
A ray is named by giving its endpoint and one other point on it. The ray AB can be drawn this way:



The point A is the endpoint of this ray.

3. The part of a line between two points on it, including the two **endpoints**, is called a **line segment** or a **segment.** The sides of a rectangle are segments.

A segment is named by giving its endpoints. The segment AB is drawn below:



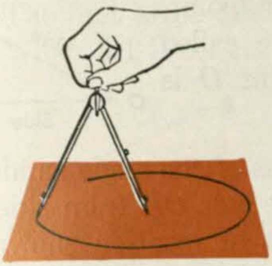
4. Give some illustrations of lines, rays, and segments.

5. Sometimes the term "line" is used for any one of these three: line, ray, and segment. It is better to use the correct term for a ray and for a segment. However, when you encounter the term "line" you can usually tell from the situation in which it is used whether it means line, or ray, or segment.

After they can identify lines, rays, segments, and endpoints correctly, have pupils reproduce them with proper labels. Reproduction is usually more difficult than identification, but both are necessary.

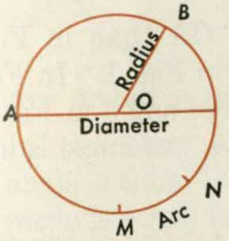
Explain how to draw a circle and give practice in it. Introduce terms, radius (radii), diameter, arc. Begin class chart of geometric terms, showing definitions and examples of each.

Drawing Circles



1. The circle is one of the most frequently used geometric figures. When you draw a circle you use a pair of compasses. Put the sharp point of the compasses on the point you wish to use as center of the circle and move the pencil point of the compasses completely around. The picture at the left shows you how to do it.

2. In the figure shown below, point *O* is the center of the circle. The segment *OB*, which is drawn from the center to a point on the circle, is called a **radius** of the circle. The plural is **radii.** Any circle has many radii, all of the same length, just as a wheel has many spokes of the same length. *AC*, which passes through the center, is a **diameter** and is twice as long as a radius. A circle can have many diameters, each passing through the center. A part of the circle, such as the part *MN*, is called an **arc** of the circle. A **semicircle** is half of a circle. The arc *ABC* is a semicircle.



Stress meaning of semicircle as "half circle."

3. Draw a circle with a radius of 2 in. First open your compasses so that the points are 2 in. apart; then draw the circle. What is the length of the diameter of this circle? 4 in.

2in. 3 in. 5 in.

4. Draw circles having radii equal to 1 in., $1\frac{1}{2}$ in., $2\frac{1}{2}$ in., and 3 in. What is the length of each diameter?

6 in.

5. Draw three circles having the same center and radii equal to $1\frac{1}{4}$ in., $1\frac{1}{2}$ in., and $1\frac{3}{4}$ in.

6. Draw six circles of different sizes. Then draw one diameter for each circle and measure its length. Give as much practice as needed so all pupils will be proficient in this skill.

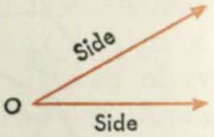
7. On the chalkboard draw a circle, using chalk and a piece of string. Your teacher will show you how to do it.

When discussing terms in ex. 2, point out and demonstrate that all radii and all diameters of same circle are equal. Have pupils verify this. Familiarize pupils with technical instructions: "With A as center, draw a circle with a 2-inch radius."

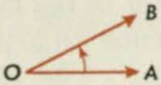
Teach the meaning of angle, certain definitions relating to angles, and method of reading angles (pages 182–183).

Studying Angles

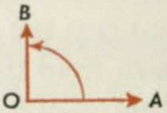
1. When two rays have the same endpoint, they form an angle. The rays are called the **sides** of the angle and endpoint O is called the **vertex** of the angle.



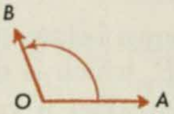
2. In the figures below think of the sides of the angle as hinged together at O so that you can rotate side OB from side OA as shown by the arrow; the amount of rotation shown by the arrow indicates the size of the angle.



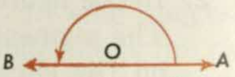
Acute angle
Fig. 1



Right angle
Fig. 2



Obtuse angle
Fig. 3



Straight angle
Fig. 4

In Fig. 2, OB has rotated more from OA than in Fig. 1, so the angle in Fig. 2 is larger than that in Fig. 1. In Fig. 3, the arrow shows still more rotation, so the angle in Fig. 3 is larger than the angle in Fig. 2. In Fig. 4, the angle is larger than in Fig. 3; why?

Emphasize.

The size of an angle is indicated by the amount of rotation of one side of the angle from the other side.

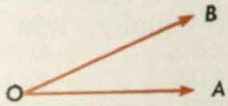
3. The name of an angle depends upon the amount of rotation that has taken place. If the amount of rotation is a **quarter** of a complete revolution, as shown above in Fig. 2, the angle is called a **right angle.** If the rotation is **half** of a complete revolution, as in Fig. 4 above, the angle is called a **straight angle;** in this case the sides of the angle lie in a straight line. A straight angle is twice as large as a right angle because there is twice as much rotation. An angle which is **less** than a right angle, as in Fig. 1, is called an **acute angle,** while an angle **greater** than a right angle but **less** than a straight angle, as in Fig. 3, is an **obtuse angle.** Acute means pointed; obtuse means blunt.

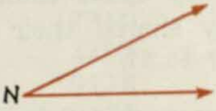
4. Draw 3 acute and 3 obtuse angles of different sizes.

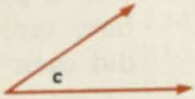
Follow development as given. Use chalkboard diagrams to explain different types of angles in ex. 2–3. Also illustrate meanings of angles and amount of rotation by spreading legs of compasses.

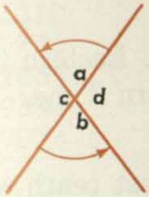
Reading Angles

1. The first angle below may be read angle *AOB* or angle *BOA*. When 3 letters are used to name an angle, the letter at the vertex is written between the other two letters. Often one letter is enough to name an angle; in such cases, a capital letter is written at the vertex or a small letter is written inside the angle as shown above.

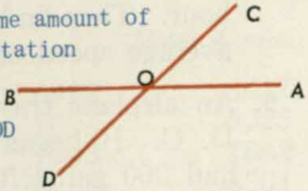




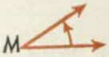


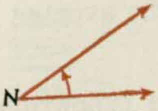


2. When two lines intersect, they form 4 angles, as shown here. Angles *a* and *b* are called **opposite** angles. Find another pair of opposite angles. c and d It is easy to see that angles *a* and *b* are equal because each angle is formed by the same amount of rotation. Are angles *c* and *d* equal? Yes Why? Same amount of rotation



3. In the figure at the right, which angle is equal to angle *AOC*? BOD Which angle is equal to angle *COB*? AOD What kind of angle is *COB*? Obtuse





4. The size of an angle is not affected by how long you draw its sides, because the sides of an angle are rays which extend indefinitely from the vertex of the angle. The size of an angle depends only upon the amount of rotation. Since angles *M* and *N* both have the same amount of rotation, these angles are equal. Draw an angle that is larger than angle *N*; that is smaller than angle *N*.

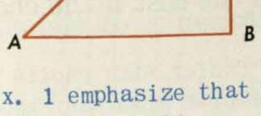
Stress.

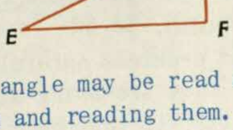
5. How many right angles are there in each figure below? Read each right angle, using 3 letters for each angle:

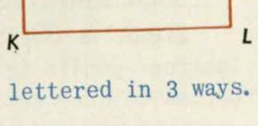
ABC; BCD

EFG

KLM; LMN; MNK; NKL







In ex. 1 emphasize that an angle may be read and lettered in 3 ways. Give practice on labeling angles and reading them.

Review problem solving involving mixed operations and concepts.

More Problems

1. Jack and George sell chickens and eggs. Last month they sold 16 doz. eggs at \$.45 a dozen and 32 lb. of chicken at \$.43 a pound. They spent \$9.82 for feed, egg boxes, carfare, etc. If they shared their profit equally, how much did each boy get? \$5.57

2. An electric table stove has 3 heats. It uses 1000 watts per hour when the switch is turned to "high," 500 watts when turned to "medium," and 250 watts when turned to "low." If the stove is used $\frac{3}{4}$ hr. on "high," 2 hr. on "medium," and 1 hr. on "low," how many watt-hours of current are used? 2000 Find the cost at 3¢ per kw-hr. 6¢

3. A jet plane left Dayton, Ohio, at 10:35 A.M. Eastern Time and landed in New York at 11:41 A.M. Eastern Time. Find the time for this flight. 1 hr. 6 min.

4. Change the time found in ex. 3 to the nearest tenth of an hour. Then find, to the nearest tenth of a mile per hour, the average speed of the plane if the distance is 544 mi. 494.5

5. An airplane traveled 8198 mi. from Guam to Washington, D. C. It began the trip with 11,110 gal. of gasoline and had 300 gal. left when it landed in Washington. Find, to the nearest tenth of a gallon, the average number of gallons of gasoline used per mile. 1.3

6. Miss Brown bought a portable typewriter that was priced at \$107.50. She was allowed \$15.00 for the old typewriter she turned in. If Miss Brown paid \$25.00 down and \$4.50 a month, how many months did it take her to pay for the typewriter? 15

7. In 7 hr. 50 min. how much farther can an airplane fly at an average speed of 240 mi. per hour than a train can travel at an average speed of 48 mi. per hour? 1504 mi.

8. At 56¢ a pound, find the cost of chopped steak for 33 persons, allowing $\frac{1}{4}$ lb. per person. \$4.62 Find the cost if the chopped steak is 68¢ a pound. \$5.61

Instruct pupils to read problems carefully. Confer with pupils who had errors to determine if they are using all problem aids they know; if they are computing incorrectly; if they have difficulties in interpreting problem situations.

Present another set of improvement tests in addition. Present oral review of percentage.

Improving by Practice

Addition Test 4a.

Time: 4 min.

1. 7280	7297	7674	9475	2995
3075	6089	8096	5183	7205
6964	5821	7710	1455	1236
2990	8665	4407	7046	1815
2425	6032	9118	5309	2839
4319	4930	9624	2188	1537 ⑤
27,053	38,834	46,629	30,656	17,627

Addition Test 4b.

Time: 4 min.

2. 7187	4405	1399	7627	8861
5310	5746	6606	2319	5229
5776	3303	7731	2451	9529
2063	1280	8148	6420	5072
9594	8048	8132	2993	9870
9078	4324	1358	6429	5067 ⑤
39,008	27,106	33,374	28,239	43,628

Addition Test 4c.

Time: 4 min.

3. 3512	7125	6439	8422	5607
5707	5046	1329	2094	6025
4429	1328	9347	9545	7842
4437	3939	6085	5036	6301
8817	1368	5956	1697	6378
4106	3064	4606	7280	2296 ⑤
31,008	21,870	33,762	34,074	34,449

Oral Practice

4. How much is 25% of 48? $33\frac{1}{3}\%$ of 63? 75% of 60? 45
12 21

5. How much is 100% of 39? 125% of 40? 300% of 8? 24
39 50

6. Which is more, 60% of 40 or $\frac{5}{8}$ of 40? $\frac{5}{8}$ 65% of 90 or $\frac{2}{3}$ of 90? $\frac{2}{3}$

7. Round off to the nearest cent: \$27.998; \$8.007 \$8.01
\$28.00

8. Estimate 63% of \$88; 49% of \$72; 76% of \$160. \$120
\$55 \$36

9. Change to a decimal: $2\frac{1}{4}\%$, $18\frac{1}{2}\%$, $5\frac{3}{4}\%$, $9\frac{1}{2}\%$, 125%. 1.25
.025 .185 .0575 .095

Tests are to be given, scored, recorded as others were (see pages 46–50, 99, 143). Use ex. 4–9 to spot-check for class or individual weaknesses. Reteach if necessary.

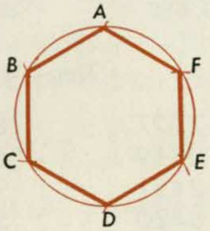
Teach meaning of and how to draw a hexagon, using the circle. Teach how to make designs based on the hexagon (page 187).

Drawing a Hexagon

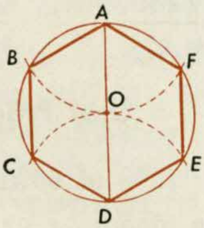
Stress.

1. A regular hexagon is a figure having six equal sides and angles. The easiest way to construct a hexagon is first to draw a circle and then to draw the hexagon inside the circle, using one of the methods given below.

2. **First Method.** Spread the points of the compasses so that they are 1 in. apart. Then draw a circle; this circle will have a radius of 1 in. Keeping the points of the compasses still 1 in. apart, select any point on the circle, such as *A*; using *A* as center, draw an arc cutting the circle at *B*. With this same radius and with *B* as center, draw another arc cutting the circle at *C*. Repeat this process around the circle, using *C* as center to draw arc *D*, then using *D* as center to draw arc *E*, and so on. If you do the work carefully, the last arc, which has *F* as center, will cut at point *A* where you started. The circle is now divided into 6 equal arcs. With your ruler, connect the 6 points with segments and you will have a **regular hexagon,** *ABCDEF*. The sides of this hexagon are each 1 in. long.



3. **Second Method.** This method is shorter than the method above. First, draw the circle with a radius of 1 in., using *O* as center. Next draw lightly any diameter, such as *AD*; this diameter passes through point *O*. Using point *A* as center, and a radius of 1 in., draw a long arc *BOF* cutting the circle at *B* and *F*. Then, with the same radius, use *D* as center and draw a long arc *COE* cutting the circle at *C* and *E*. These long arcs both pass through the center *O*. Draw segments connecting *A*, *B*, *C*, *D*, *E*, and *F* and you will have a hexagon. Why is the second method better than the first one? See Guide for discussion.



4. Draw hexagons by each of the above methods, using circles having radii of $1\frac{1}{2}$ in.; using circles having radii of 2 in.

Emphasize meaning of regular hexagon as given in ex. 1. You should experiment with the two methods before presenting them to class. When pupils do ex. 4, observe carefully to be sure all are doing work correctly.

Note that in designs 2, 3, 7, and 9, the hexagon must actually be drawn; but in designs 4, 5, 6, and 8, it is sufficient to mark the 6 points that divide the circle into 6 equal parts.

Using the Hexagon in Designs

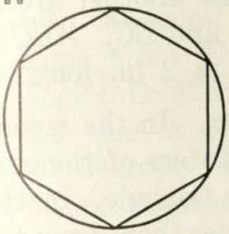
Study each design carefully; then copy it:

To copy designs 2 and 3, first draw hexagons in circles as shown in design 1. In design 2, first draw the hexagon and then join every other point on the circle to make the triangle. The triangle in design 2 is called an **equilateral triangle** because the three sides are equal in length.

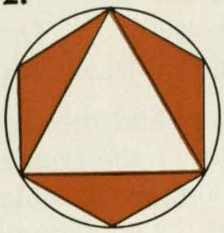
Stress.

Stress.

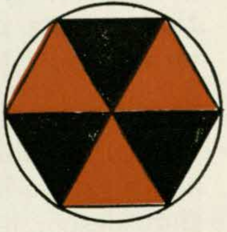
1.



2.



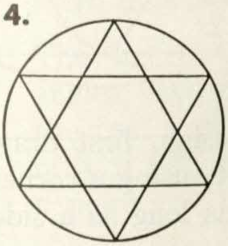
3.



Are the red triangles in design 2 equilateral triangles? No Are all the triangles in design 3 equilateral triangles? Yes

To copy designs 5 to 9, start as in design 4. Before drawing the segments in design 4, draw a hexagon to get the 6 points on the circle. In design 6, you will need to draw some segments that will finally be erased. Have pupils draw lines lightly.

4.



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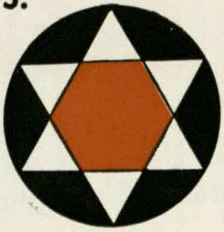

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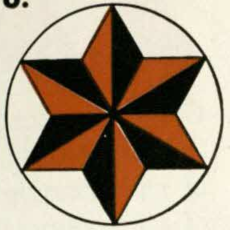

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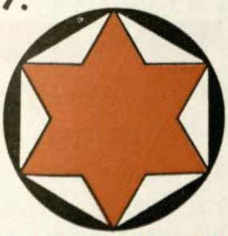

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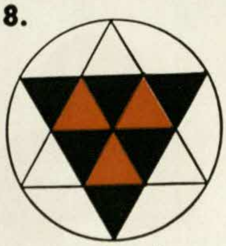
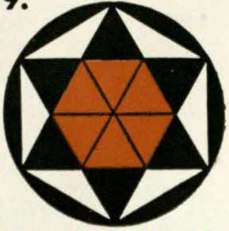


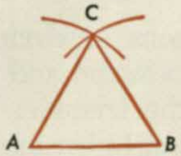
9.



Decide on the order of steps in drawing each of these designs. Have pupils follow your directions and check work carefully. When designs are completed urge pupils to construct some original designs.

Teach how to draw equilateral triangle not inscribed in a circle and how to use it in making designs. Have pupils follow directions carefully in ex. 1–3.

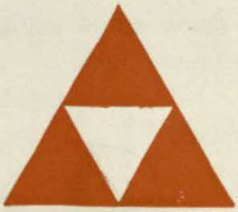
Drawing Equilateral Triangles

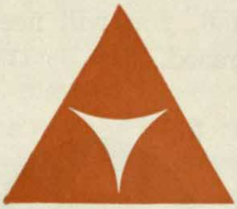


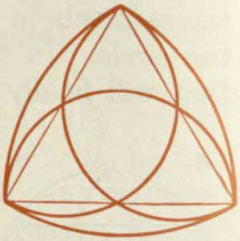
1. On page 187 you learned how to draw an equilateral triangle by dividing the circle into 6 equal parts and then joining the alternate points. You can also draw an equilateral triangle as follows: Draw *AB* 2 in. long. With a radius of 2 in. and with *A* as center, draw an arc above the line. Then with *B* as center and with the same radius, draw another arc so that it cuts the first arc at *C*. Draw *AC* and *BC*. *ABC* is an equilateral triangle in which each side is 2 in. long.

2. Study the designs below and then copy them. In the second design, use the corners of the triangle as centers of the arcs and a radius half as long as the side of the triangle. In the third design, first draw the equilateral triangle; then draw the outside arcs, using the corners of the triangle as centers; next draw the inside arcs, using the mid-points of the sides as centers; then erase the triangle.

A

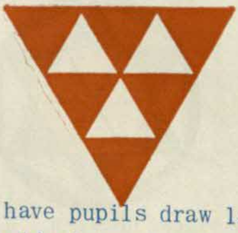


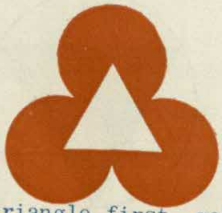


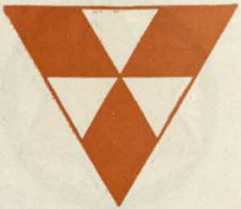


3. Copy the designs below. In the second design, first draw the equilateral triangle; then draw each circle, using a corner of the triangle as center and a radius half as long as a side of the triangle.

B







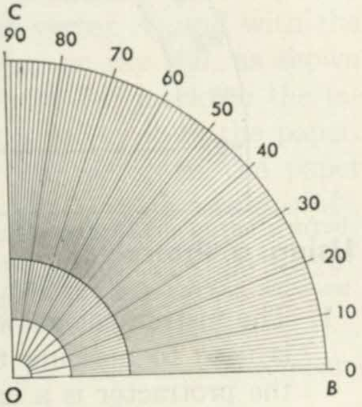
In A have pupils draw large triangle first, making each side 2 in. long. Then midpoints of sides may be found and connected easily. In B make sides 3 in. or $1\frac{1}{2}$ in. long to facilitate division into 3 equal parts.

Teach the measurement of angles and use of protractor (pages 189-191).

Measuring Angles

1. The size of an angle can be measured by a unit called the **degree,** just as the length of a segment can be measured by a unit called the **inch.** A **degree** is a very small angle equal to $\frac{1}{90}$ of a right angle.

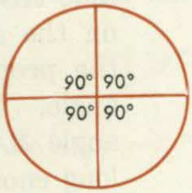
Stress.



In the figure at the right, *BOC* is a right angle which is divided into 90 small angles, all of the same size. Each of these small angles is called a **degree.** You see that a right angle contains 90 degrees, or 90°. The sign ° means **degree** or **degrees.**

2. In the figure shown above, the 90 small angles or degrees in angle *BOC* also divide the arc *BC* into 90 small equal arcs. Each of these small arcs is called a **degree** of arc. You see that the **degree** is a unit for measuring arcs as well as angles. Emphasize.

3. In the figure above, the arc *BC* contains 90°. What part of a whole circle is arc *BC*? $\frac{1}{4}$ How many degrees of arc are there in a whole circle? 360°

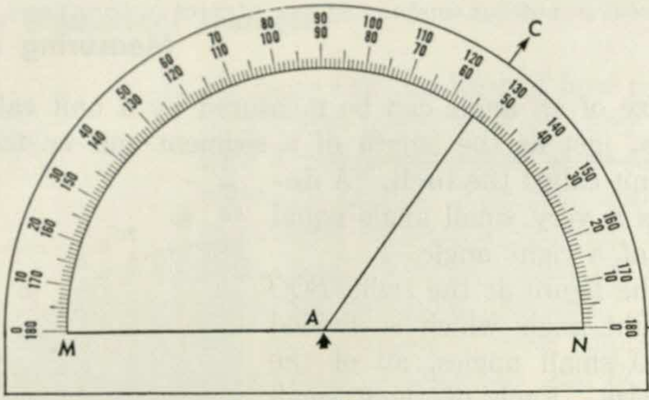


4. The figure at the right shows that four angles, each containing 90°, can be drawn at the center of a circle. You see that a total of 4 × 90°, or 360°, may be drawn around a point.

5. On page 186, you learned to draw a hexagon by dividing the circle into 6 equal arcs. How many degrees are there in each of these 6 arcs? 60°

6. When the hands of the clock are at 2 o'clock, how many degrees are there in the angle formed by the hands? 60° When the hands are at 6 o'clock, how many degrees are there in the angle formed by the hands? 180° When the hands are at 3 o'clock, how many degrees are there in that angle? 90°

Emphasize that degree refers to each of the 90 small angles formed at the vertex of a right angle and to each of the 90 small arcs into which an arc of a right angle is divided.



Using a Protractor

Before using protractors have pupils examine them. Point out the two scales used. Explain that this enables us to measure angles (up to 180°) in various positions.

1. The instrument shown above, which is called a **protractor**, is used to measure the size of an angle. Since the top of the protractor is a half-circle (semicircle), the arc is divided into 180°. The center of the circle is at point *A* at the tip of the arrow. *A* is also in the middle of *MN*, which is the diameter of the inner semicircle.

2. To measure an angle, such as angle *BAC* shown above, place the protractor over angle *BAC* so that the arrow of the protractor is at the vertex *A*, and the edge *MN* of the protractor lies over side *AB* of the angle, as shown above. Side *AB* of the angle should pass through the "zero" mark on the scale of the protractor. Stress. The protractor is now in the proper position so that you can read the size of the angle. You see that side *AC* points to 54°; this shows that angle *BAC* contains 54°. If either side of the angle is not long enough to meet the protractor, extend the side. Stress.

3. Draw 5 large angles of different sizes and measure each one with your protractor.

4. With your ruler, draw a large triangle and measure each of its 3 angles with your protractor. Then find the sum of the 3 angles. Is the sum equal to or close to 180°? The sum of the 3 angles in **any** triangle equals 180°. Stress.

5. With your ruler, draw 3 triangles of different shapes. Measure the angles in each triangle and find their sum. Sums should be close to 180°.

Emphasize that the center of the protractor must be placed exactly over the vertex of the angle being measured, and side AB must pass through the zero mark. Give much practice in class in measuring angles.

Teach how to bisect an angle using protractor or compasses.

Drawing Angles with a Protractor

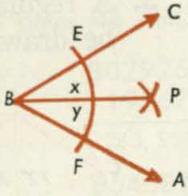
1. Suppose you wish to draw an angle of 54°. First draw ray AB, which will be one side of the angle; point A will be the vertex of the angle. Place your protractor over ray AB with the arrow of the protractor at vertex A, and with the "zero" mark on the inside scale lying on ray AB, as shown at the top of page 190. Next find the 54° mark on the inside scale of the protractor and put a dot by it on the paper. Call this dot D. Then take the protractor off the paper and draw AD. You now have angle BAD, which contains 54°.

2. With your protractor, construct angles of 60°, 90°, 120°, 45°, 73°, 156°, 43°, 110°, 15°, 175°, 150°.

3. **Bisecting an angle with a protractor.** To bisect an angle means to divide it into 2 equal angles.

▶ To bisect an angle, first measure it with a protractor. Suppose it measures 70°. $\frac{1}{2}$ of $70° = 35°$. Before removing the protractor from the angle, place a dot on the paper opposite the 35° mark on the scale. Then remove the protractor and draw a ray through the dot from the vertex of the angle; this ray bisects the angle, dividing it into 2 angles of 35° each.

4. **Bisecting an angle with compasses.** By this method you can bisect an angle without knowing its size.



▶ To bisect angle ABC, place the sharp point of the compasses on B as center. With any convenient radius, draw an arc cutting the sides of the angle at E and F. Then with EF as radius, draw an arc having E as center and another arc having F as center. These arcs cross at P. Draw BP, which bisects angle ABC, making angle x equal to angle y. BP is the bisector of angle ABC.

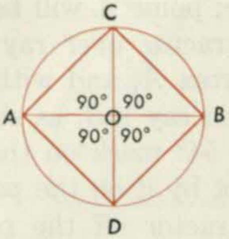
5. With your protractor, draw an angle of 98° and bisect the angle with your compasses. Then, with your protractor, measure the two angles obtained by bisecting 98°. Is each angle 49°? Do the same with angles of 84°, 118°, 140°, 90°.

Have pupils draw angle in ex. 1, under your direction. Check to see that all do it correctly. Give much practice on this in class (ex. 2), before assigning homework. Follow similar procedure for ex. 3-5.

Teach how to use a protractor to inscribe squares, octagons, and pentagons in circles. Add new words (inscribe, octagon, pentagon) to class chart.

Octagons and Pentagons

1. Draw a square in a circle.

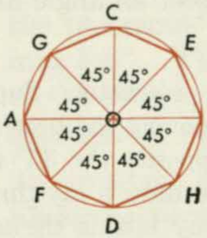


▶ Draw a circle and a diameter *AB*. Then, with your protractor, draw angle *BOC* equal to 90°, using *OB* as one side of the angle and point *O* as the vertex of the angle. After drawing *CO*, extend it to *D*. The 4 angles around *O* are each 90°. Why? The circle is now divided into 4 equal parts by points *A, C, B, D*. Draw *AC, CB, BD*, and *DA* and you will have a square which is **inscribed** in a circle.

Be sure pupils understand meaning of term "inscribed."

2. A regular octagon, which is a figure of 8 equal sides, can be drawn in a circle as follows:

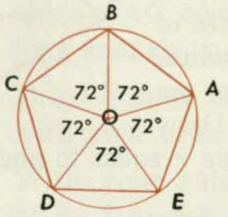
Stress.



▶ Draw a circle with diameters *AB* and *CD* as in ex. 1, making the 4 angles at the center each equal to 90°. With the compasses, bisect angle *BOC*, which will give *EO* as the bisector of the angle; extend *EO* to *F*. Angle *EOB* equals 45°; why? In the same way, bisect angle *COA* and extend the bisector *GO* to *H*. You now have 8 points on the circle which divide it into 8 equal parts. Connect the 8 points with segments and you will have a regular octagon.

Stress.

3. A regular pentagon, which is a figure of 5 equal sides, can be drawn in a circle as follows:



▶ Draw a circle and a radius *OA*. Since there are to be 5 equal sides, there must be 5 equal angles at the center of the circle. Each of these angles will be $\frac{1}{5}$ of 360°, or 72°. With your protractor, draw angle *AOB* equal to 72°, using *OA* as one side of the angle and *O* as its vertex. Next draw angle *BOC* equal to 72°, using *OB* as one side of the angle. Then draw angles *COD* and *DOE*, each equal to 72°. Do you need to draw angle *EOA*? To check your work, measure angle *EOA*; does it equal 72°? Join points *A, B, C, D, E* and you have a regular pentagon.

Have pupils draw all their circles with a radius of at least 2 in. Have them follow instructions, step by step, under your supervision. Observe pupils carefully to see that all are doing work correctly.

Present set of improvement tests in subtraction.

Subtraction Test 4a.

Time: $4\frac{1}{2}$ min.

1.	825527 587535 237,992	320661 144683 175,978	591704 295956 295,748	668014 249879 185,135	
2.	731123 493765 237,358	851668 91799 759,869	701967 375206 326,761	875354 300886 574,468	
3.	142897 46752 96,145	627011 412269 214,742	844329 376343 467,986	875973 195775 680,198	⑫

Subtraction Test 4b.

Time: $4\frac{1}{2}$ min.

4.	360920 104091 256,829	599352 245668 353,684	483546 283033 200,513	612577 123679 488,898	
5.	851235 56479 794,756	400072 159196 240,876	530128 94985 435,143	653033 136786 516,247	
6.	722879 229958 492,921	264541 196311 68,230	818398 479593 338,805	984494 459714 524,780	⑫

Subtraction Test 4c.

Time: $4\frac{1}{2}$ min.

7.	753699 94667 659,032	411194 251468 159,726	241061 189279 51,782	302926 19367 283,559	
8.	790050 394679 395,371	788006 213313 574,693	573999 396339 177,660	654581 397513 257,068	
9.	543097 449456 93,641	993866 703495 290,371	423573 153252 270,321	456076 374981 81,095	⑫

To the Pupil. This is the fourth set of Improvement Tests in subtraction. Did you get a score of 10 on each test?

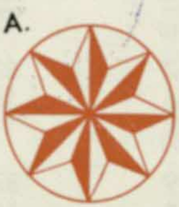
Test should be given, scored, recorded as others were (see pages 57, 109, 155). Have pupils compare scores with previous ones and note needs on own charts. Plan remedial activities as needed.

Teach pupils how to draw designs based on inscribed octagons and pentagons.

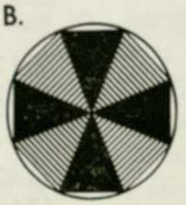
Drawing Designs

1. Copy each design in a circle having a radius of $1\frac{1}{2}$ in. Design B is an octagon. To draw designs C and D, first divide the circle into 5 equal parts as in ex. 3 on page 192.

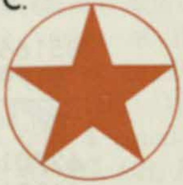
A.



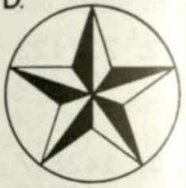
B.



C.



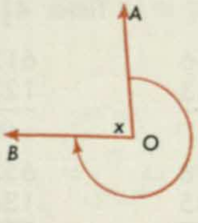
D.



2. Angles larger than 180° are often used. In angle AOB below, side OB has rotated more than $\frac{3}{4}$ of a full revolution, as shown by the arrow; in this case, side OB rotated **toward the right** in the direction in which the hands of a clock move; this is called the **clockwise** direction.

Stress.

Stress meaning.

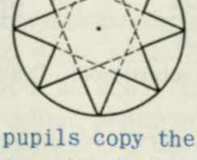


The rays OA and OB form two angles, one being the larger angle shown by the arrow and the other being the smaller angle marked x. You see that these two angles together equal 360°. You cannot measure an angle larger than 180° with your protractor but you can measure angle x. If angle x equals 85°, the larger angle AOB must equal $360° - 85°$, or 275°. Draw 3 angles each larger than 180°; then measure each angle in degrees, using the method just described. Give more practice in measuring angles larger than 180°, both in class and for homework.

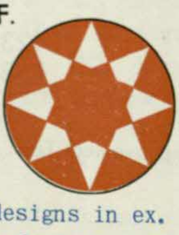
3. In the figure in ex. 2, suppose that side OB had rotated from OA **toward the left;** then the direction of rotation is called **counterclockwise.** Explain why.

4. Copy each design. First divide each circle into 8 equal parts. Design E will make F. Design G will make H.

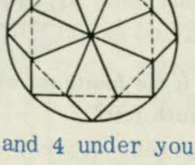
E.



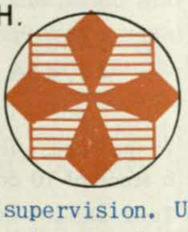
F.



G.



H.



Have pupils copy the designs in ex. 1 and 4 under your supervision. Urge them to make their own designs, using octagons and/or pentagons. Use clockface to illustrate concepts in ex. 2-3.

Review work in estimating answers, in problem and abstract form.

Estimating Answers

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At the end of each problem there are three numbers. Select the number you think is nearest to the right answer. Then work the problem and compare your answer with the number selected.

1. Three boys shared the cost of buying a tent for \$14.95, a camp stove for \$10.83, and an electric lantern for \$8.77. What was each boy's share of the cost? \$9.00 \$10.00 \$11.00 \$9.85
2. How much change did Mrs. Brown get from \$20 when she bought $3\frac{7}{8}$ yd. of linen at \$1.20 a yard and $2\frac{1}{8}$ yd. of wool cloth at \$3.92 a yard? \$7.00 \$8.00 \$9.00 \$7.02
3. Last summer Mr. Martin drove 5940 mi. in 15 weeks. Find the average number of miles he drove his car per week. 375 mi. 400 mi. 425 mi. 396
4. Tom sold 19 doz. eggs at 52¢ a dozen. How much did he get for these eggs? \$9.00 \$10.00 \$11.00 \$9.88
5. Mr. King wants to have the oil in his car changed when the speedometer shows 6000 mi. Now it shows 5495.8 mi. How many more miles can he drive before the oil is to be changed? 600 mi. 550 mi. 500 mi. 504.2
6. At the market Mrs. Ward spent \$2.12 for meat, \$.89 for vegetables and fruit, and \$1.96 for groceries. How much did she spend all together? \$4.00 \$4.50 \$5.00 \$4.97

Make an estimate for each answer (1) and then find the exact answer. (2) Give quotients to the nearest tenth:

\$7, \$6.60

(1), (2) **7.** 9.8 × 40.1 400, 392.98 9.9 ÷ 4.8 2, 2.1 12% of \$55
\$30, \$30.03

8. $9\frac{7}{8} \times 81$ 810, $799\frac{7}{8}$ 39 × 91 3600, 3549 33% of \$91
\$25, \$24.48

9. 97 + 996 1100, 1093 $4\frac{3}{4} \times 11\frac{1}{2}$ 55, $54\frac{5}{8}$ 48% of \$51
\$15, \$15.34

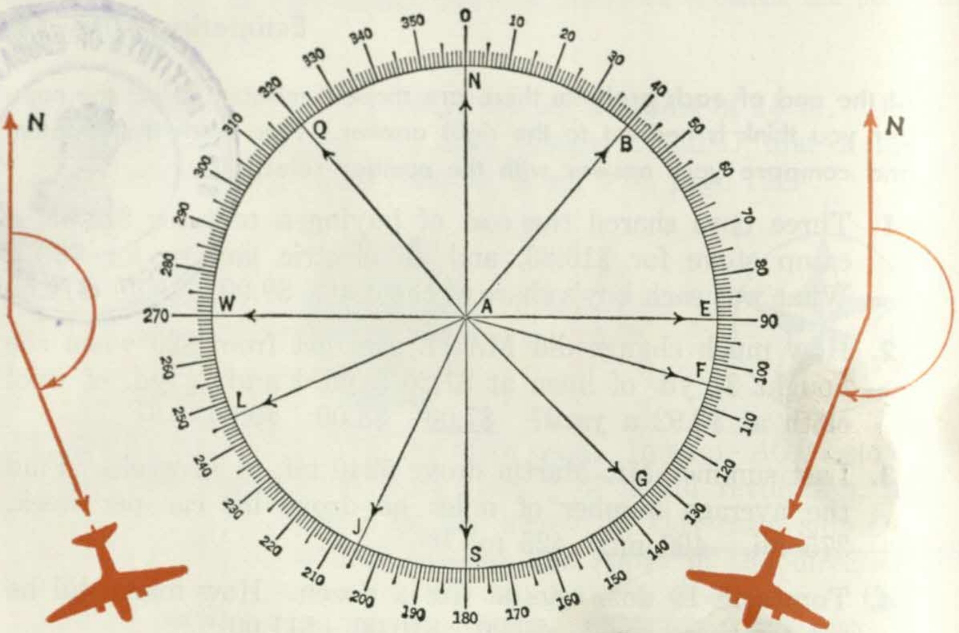
10. 79 ÷ 9.8 8, 8.1 79 × 71 5600, 5609 26% of \$59
\$9, \$8.74

11. 91 − 19 70, 72 11 × \$2.95 \$33, \$32.45 19% of \$46
\$15, \$15.12

12. 2.9 × 9.2 27, 26.68 31 × \$1.99 \$60, \$61.69 63% of \$24

Have pupils explain how they estimated each answer and why they think the estimated answer is more or less than the exact answer. Some pupils may need re-explanation of estimation and more practice in it.

Present application of the use of angles in navigation (pages 196–197).



Piloting an Airplane

1. The pilot of an airplane has to know the direction of the straight line in which he should fly his plane. In air navigation, the **direction** of a line, which the pilot calls the **course** of his plane, is expressed by giving the angle which the line makes with a line pointing north. These angles are always read in the **clockwise** direction from 0° to 360°. In the above figure, *AN* points north; therefore the direction of *AN* is 0°. If a plane flies north, its course is 0°. If a plane flies east, in the direction *AE*, its course is 90°. If it flies south, in the direction *AS*, its course is 180°. If it flies west, in the direction *AW*, its course is 270°.

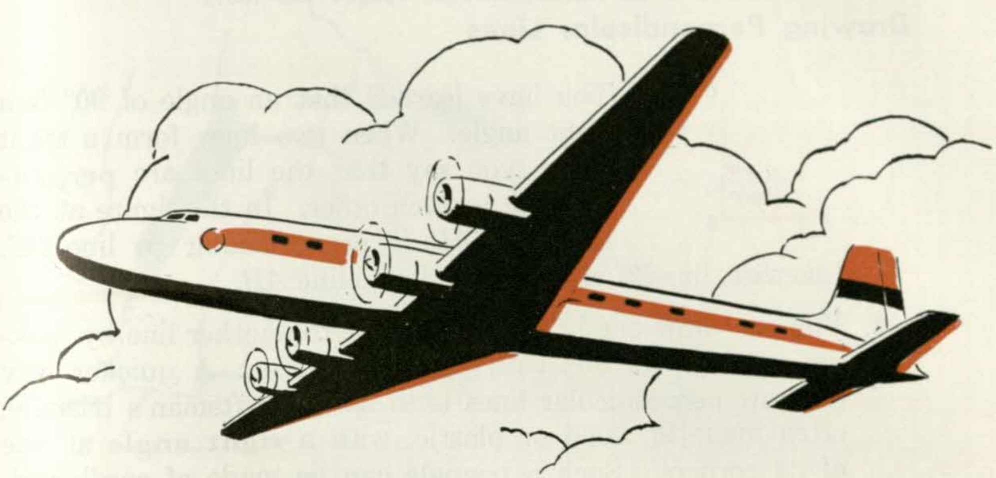
Stress.

▶ You might be tempted to say that **west** is a direction of 90° because angle $NAW = 90°$ if measured toward the left from 0°; but this would be wrong because the directions of all lines must be measured in the **clockwise** direction, which means measuring **toward the right** from 0°. The correct direction of *AW* is 270°.

Stress.

2. What is the course of a plane if it flies in the direction *AB*? 40°
if it flies in the direction *AF*? *AJ*? *AL*? *AQ*? 320°
105° 205° 246°

See Guide for instructions on how to make circular protractor. Discuss ex. 1 thoroughly with pupils, emphasizing meaning of "course" and how angles are read (clockwise).



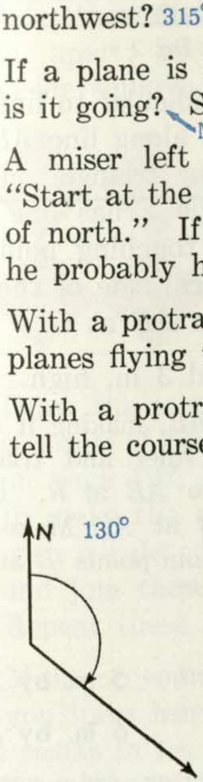
3. What is the course of a plane if it flies southeast, as shown by *AG* in the figure on page 196? 135 You see that southeast is the direction halfway between east and south. What is the course of a plane that flies southwest? 225° of one that flies northwest? 315°

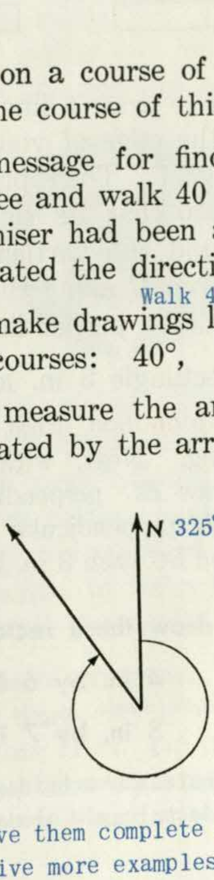
4. If a plane is flying on a course of 360°, in what direction is it going? North State the course of this plane in another way. 0°

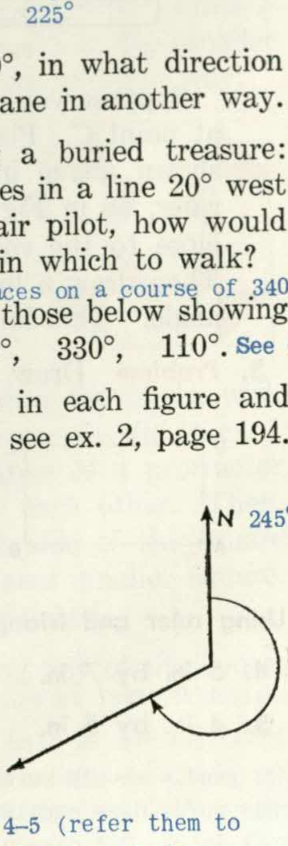
5. A miser left this message for finding a buried treasure: "Start at the oak tree and walk 40 paces in a line 20° west of north." If the miser had been an air pilot, how would he probably have stated the direction in which to walk? Walk 40 paces on a course of 340°.

6. With a protractor, make drawings like those below showing planes flying these courses: 40°, 195°, 330°, 110°. See Guide.

7. With a protractor, measure the angle in each figure and tell the course indicated by the arrow; see ex. 2, page 194.



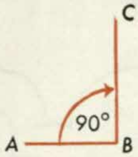




Do ex. 2 with class. Have them complete ex. 4–5 (refer them to diagram on page 196). Give more examples of this type before assigning ex. 6–7.

Teach pupils how to draw perpendicular lines and to use such lines in drawing rectangles and making designs (pages 198-199).

Drawing Perpendicular Lines



1. You have learned that an angle of 90° is a right angle. When two lines form a right angle, you say that the lines are **perpendicular** to each other. In the figure at the left, line *AB* is perpendicular to line *BC*. Likewise, line *BC* is perpendicular to line *AB*.

2. You can draw one line perpendicular to another line by making an angle of 90° with your protractor. A quicker way to draw perpendicular lines is to use a draftsman's triangle, often made of wood or plastic, with a **right angle** at one of its corners. Such a triangle can be made of cardboard. The way to use a draftsman's triangle is shown below:

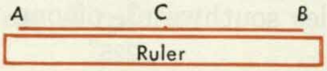


Fig. 1

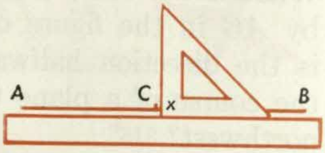
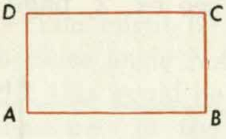


Fig. 2

Suppose you wish to draw a line perpendicular to line *AB* at point *C*. Place the edge of your ruler along line *AB*, as shown above in Fig. 1. Then place your triangle on the ruler, as in Fig. 2, so that its right angle x has one side close to the ruler and the vertical side touching point *C*. Then draw a line from *C* along the vertical side of the triangle. This line will be perpendicular to *AB* at *C*.

Stress.

3. Problem Draw a rectangle 5 in. long and 3 in. high.



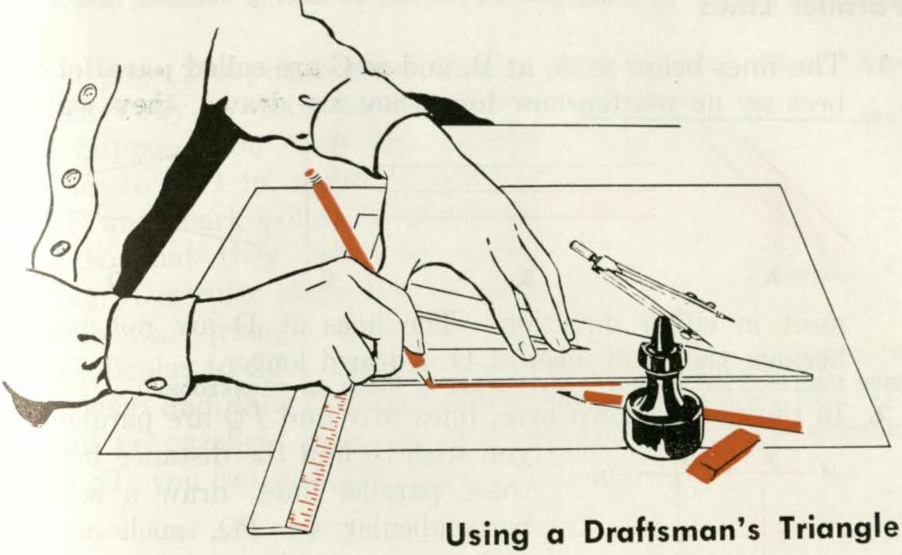
Explanation First draw *AB*, making it 5 in. long. Next, with your ruler and triangle, draw *BC* perpendicular to *AB* at *B*. Draw *AD* perpendicular to *AB* at *A*. Make *AD* and *BC* each 3 in. long. Join points *D* and *C*.

Using ruler and triangle, draw these rectangles:

4. 5 in. by 7 in. 4 in. by 6 in. 5 in. by 5 in.

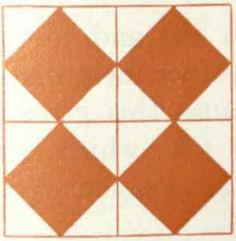
5. 4 in. by 5 in. 3 in. by 7 in. 6 in. by 7 in.

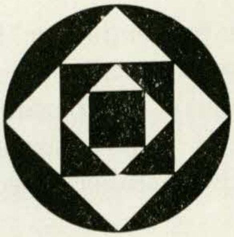
All pupils should have a draftsman's triangle, a ruler, and a pencil for this work. Have pupils slide triangle along thicker edge of ruler. Check work in ex. 3-5 carefully.

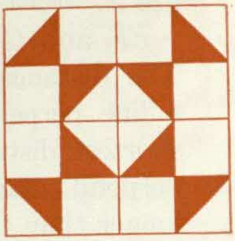


Using a Draftsman's Triangle

1. Draw the first design below, using your ruler and draftsman's triangle. Make the large outside square 2 in. on a side; then divide each side into 4 equal parts by points $\frac{1}{2}$ in. apart; join these points by lines to form the smaller inside squares. Draw the third design in the same way.







2. Draw the second design above. First draw the circle, with a radius of $1\frac{1}{2}$ in. Next inscribe a square as you did in ex. 1 on page 192, but use your triangle instead of a protractor to make the diameters perpendicular to each other. Then, with a ruler, find the mid-point of each side of the square and join these mid-points to form the next smaller square. Repeat these steps until the design is finished.

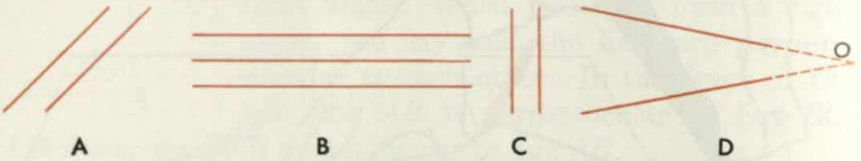
3. Make up some designs of your own, using any of the figures you have learned to draw. Have pupils make at least 8 designs.

Have pupils follow directions in ex. 1-2 (give help as needed). Pupils might enjoy making a special geometry scrapbook in which to keep examples of various figures and angles, how to make them, designs, and so on.

Teach meaning of parallel lines. Explain how to draw and use such lines in drawing rectangles and in making designs (pages 200–201).

Parallel Lines

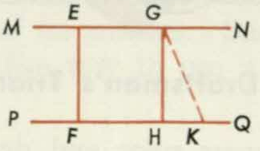
1. The lines below at A, at B, and at C are called **parallel** lines Stress. because no matter how long they are drawn, they will not



meet in either direction. The lines at D are not parallel because they will meet at O if drawn longer.

Have pupils find examples of parallel lines in classroom.

2. In the figure shown here, lines *MN* and *PQ* are parallel. If you wish to find the distance between these parallel lines, draw a segment perpendicular to *PQ*, such as *FE*, and measure its length; you will find *FE* to be $\frac{1}{2}$ in. long, so you say that the parallel lines are $\frac{1}{2}$ in. apart. If you draw another segment perpendicular to *PQ*, such as *HG*, it also will be $\frac{1}{2}$ in. long. So you see that the distance between these parallel lines is the same regardless of the point at which you measure it. Emphasize.



EF and *GH* are perpendicular both to *PQ* and to *MN*. The distance between parallel lines is always measured on a line perpendicular to the parallels because this gives the shortest distance. It is not measured on *GK*, which is not perpendicular to the parallels, because the segment *GK* is longer than *GH*.

Parallel lines are the same distance apart at all points.

3. Draw two lines parallel to each other and $\frac{3}{4}$ in. apart.

▶ First draw *PQ* as in the figure in ex. 2. At point *F* draw *FE* perpendicular to *PQ* and $\frac{3}{4}$ in. long. At point *H* draw *HG* perpendicular to *PQ* and $\frac{3}{4}$ in. long. Draw a line through points *E* and *G*. Then *EG* is parallel to *PQ* and $\frac{3}{4}$ in. above it.

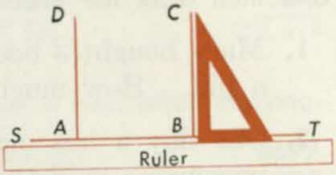
4. With your ruler and triangle, draw parallel lines that are 1 in. apart; that are 2 in. apart; that are $\frac{3}{4}$ in. apart; that are $2\frac{1}{2}$ in. apart. See ex. 3.

In ex. 1 point out that parallel lines may be vertical lines, horizontal lines, or oblique lines. Explain meaning of oblique and have pupils look up definition. Be sure pupils make drawings carefully and accurately.

After discussing and completing ex. 1–4, urge pupils to make own designs and to find examples of geometric designs (wallpaper, linoleum patterns, tiles, and so on).

Drawing Parallel Lines

1. On page 200 you learned how to draw parallel lines. Another way to draw parallel lines is as follows:



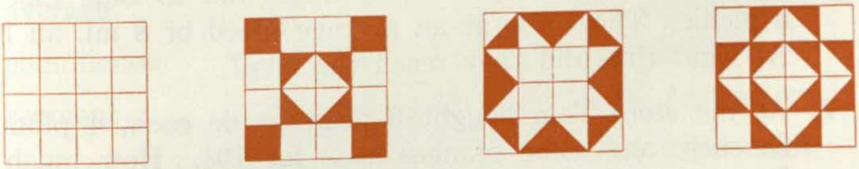
Suppose you wish the parallel lines to be 1 in. apart. Draw line ST and mark points A and B on it so that they are 1 in. apart. Lay your ruler along line ST and, with your triangle, draw AD perpendicular to ST. Then slide the triangle along the ruler to point B and draw BC perpendicular to ST. Lines BC and AD will be parallel. Since BC and AD are both perpendicular to ST, you can state this fact:

If two lines are each drawn perpendicular to a third line, the two lines are parallel.

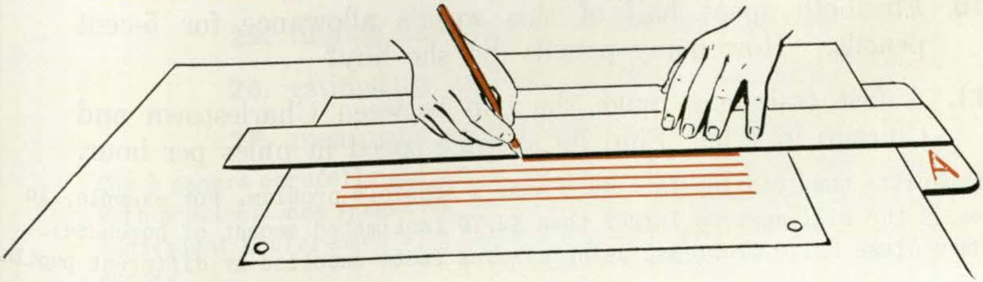
Stress after demonstration.

2. Draw five pairs of parallel lines, using the method shown above. Draw four lines parallel to each other.

3. With your ruler and triangle, draw 3 squares like the first one below. Then copy the designs in these squares.



4. The picture below shows how parallel lines are drawn with a T-square, which is a long flat ruler with a piece of wood, marked A, attached perpendicularly to it. As part A slides along the edge of the drawing board, lines drawn along the edge of the ruler will be parallel. Bring a T-square and drawing board to class and show how to use them. Try to have several sets of this equipment available for pupils to use.



Review problem solving and problems in which facts are missing.

What Fact Is Missing?

In each problem there is a fact you need to know before you can work the problem. Supply a fact that will make a good problem and then work the problem:

1. Mary bought 3 books at $.89 each. She paid for them with a bill. How much change did she get?
2. One day a bus driver collected $83.45 in fares. If all the passengers paid the same fare, how many did he carry?
3. Busses to Charlestown pass the school on the hour and half hour. How long will Mary need to wait for a bus if she reaches the bus stop 6 min. after school is out?
4. David sold 62 newspapers after school on Monday, 50 on Tuesday, 65 on Wednesday, and 53 on Thursday. Find the average number of newspapers per day David sold on the 5 school days.
5. If Mr. Ward can drive to Boston in 45 min., how many miles will he go in 1 hr. 15 min. at the same speed?
6. After school the Bicycle Club rode 6 mi. to Blue Lake for a picnic. They rode at an average speed of 8 mi. an hour. At what time did they reach the lake?
7. At the store Tom bought 3 pears at 6¢ each, 4 plums at 3¢ each, and some oranges at 3 for 10¢. How much did Tom pay for the fruit?
8. Mrs. Hall rode in a taxi from home to the bus station. She paid for her ride at the rate of 35¢ for the first $\frac{1}{4}$ mi. and 5¢ for each extra $\frac{1}{4}$ mi. How much did the taxi ride cost?
9. In the running broad jump Joe can jump $10\frac{1}{2}$ ft. How much farther is this than he could jump last year?
10. Elizabeth spent half of this week's allowance for 5-cent pencils. How many pencils did she buy?
11. A fast train can make the trip between Charlestown and Chicago in 7 hr. Find its average speed in miles per hour.

Emphasize that missing fact must make a sensible problem. For example, in ex. 1 the bill must be larger than $2.70 (estimated amount of purchase). Have class solve problems, using missing facts supplied by different pupils.

Present set of improvement tests in multiplication. Review some arithmetic terms.

Improving by Practice

Multiplication Test 4a.

Time: 4 min. after copying.

1. 4692	7369	4375	2054	
792	685	143	268	④
3,716,064	5,047,765	625,625	550,472	

Multiplication Test 4b.

Time: 4 min. after copying.

2. 8154	8307	9612	2697	
356	147	894	273	④
2,902,824	1,221,129	8,593,128	736,281	

Multiplication Test 4c.

Time: 4 min. after copying.

3. 9602	3896	8315	8731	
194	683	752	428	④
1,862,788	2,660,968	6,252,880	3,736,868	

Plan for redevelopment of meanings of terms in ex. 4-27, based on class or individual weaknesses.

The Language of Arithmetic

Show that you know what each word or group of words means by using it in a sentence or by giving an example of it:

4. average
5. commission
6. segment
7. 25% off
8. meter
9. radius
10. discount
11. ray
12. net price
13. hexagon
14. estimate
15. thousandths
16. kilowatt-hour
17. parallel lines
18. protractor
19. right angle
20. perpendicular
21. nearest tenth
22. commutative principle for multiplication
23. finding the least common denominator
24. buying on the installment plan
25. finding the answer to the nearest cent
26. estimating answers to avoid mistakes
27. measuring angles with a protractor

Check papers carefully and note kinds of errors. Have pupils compare results with previous ones (pages 69, 127, 167) and record scores on graphs. Through individual conferences with pupils, try to determine causes of errors.

Extend topic of scale drawing to more difficult type and include reading distances on map (pages 204–206).

Scale Drawing

1. Bill, Jerry, and Tom wanted to make a scale drawing of their garden plot so they could plan the garden before sowing the seeds. The garden is in the shape of a rectangle 30 ft. long by 20 ft. wide. Here is each boy's way of making the scale drawing:

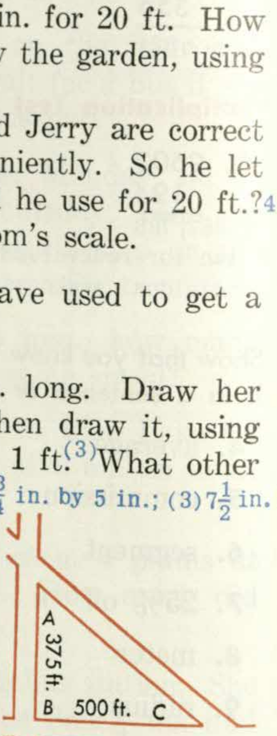
Bill used squared paper, letting 1 square = 5 ft. He used 4 squares for 20 ft. How many did he use for 30 ft.? 6 Draw the garden Bill's way.

Jerry let $\frac{1}{2}$ in. = 10 ft. So he used 1 in. for 20 ft. How many inches did he use for 30 ft.? $1\frac{1}{2}$ Draw the garden, using Jerry's scale.

Tom said that the drawings of Bill and Jerry are correct but much too small to work with conveniently. So he let 1 in. = 5 ft. Then how many inches did he use for 20 ft.? 4 for 30 ft.? 6 Make the drawing, using Tom's scale.

2. Name 3 other scales that Tom might have used to get a drawing of convenient size.

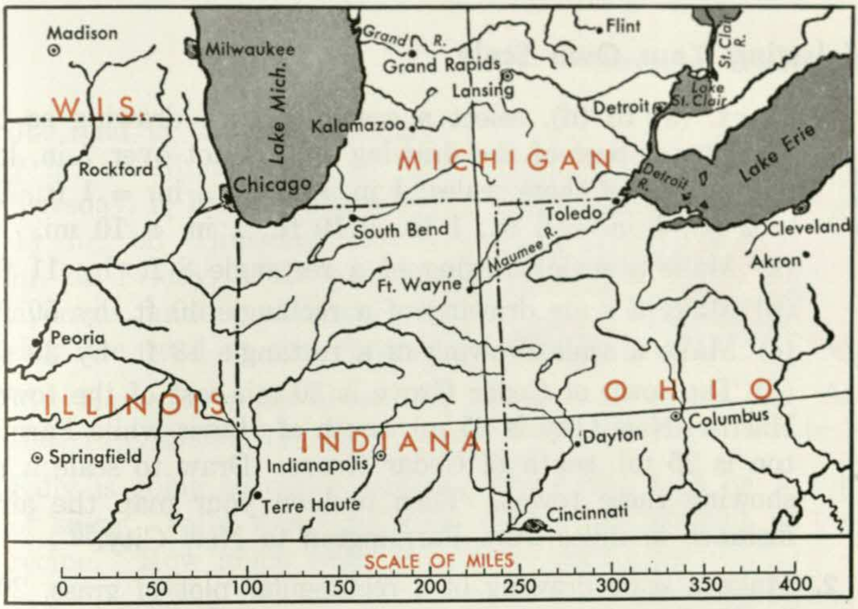
3. Mary's garden is 15 ft. wide and 20 ft. long. Draw her garden, using a scale of 1 in. = 5 ft. (1) Then draw it, using $\frac{1}{4}$ in. = 1 ft. (2) Also draw it, using $\frac{1}{2}$ in. = 1 ft. (3) What other scale could you use? (1) 3 in. by 4 in.; (2) $3\frac{3}{4}$ in. by 5 in.; (3) $7\frac{1}{2}$ in. by 10 in.



4. This picture shows a triangular block *ABC* in the town where Joe lives. Angle *B* is 90°. *AB* is 375 ft. long and *BC* is 500 ft. long. Make a scale drawing of this block, letting 1 in. = 100 ft. How many feet will be represented by $\frac{1}{4}$ in.? 25 by $\frac{3}{4}$ in.? 75 by $\frac{1}{2}$ in.? 50 When your drawing is finished, find the length of *AC* in inches. $6\frac{1}{4}$ Then tell how many feet long *AC* is. 625

5. On a sheet of paper measuring $8\frac{1}{2}$ in. by $10\frac{1}{2}$ in., make a scale drawing of a cottage 28 ft. square with a porch 8 ft. by 28 ft. Leave a border $\frac{3}{4}$ in. wide all around the drawing. What scale do you use for the drawing? 1 in. = 4 ft.; see Guide.

Have pupils make all scale drawings as called for on these pages. Discuss ex. 1–3 with class. Have pupils do ex. 4–5 independently. Ask them to make simple scale drawing of home or school.



Using the Scale on a Map

1. The scale at the bottom of the map is used for finding distances. To find the airline distance between Akron and Toledo, lay the edge of a sheet of paper on the map so that it touches the dots representing these two cities. Make marks on the paper opposite these dots. Then place the paper along the scale, with one mark at the zero point, and read the distance opposite the other mark. The distance should be 110 miles, correct to the nearest 10 miles.

2. Find, to the nearest 10 miles, the airline distance between the following cities: (1) Springfield and Dayton, 300 (2) Madison and Peoria, 160 (3) Indianapolis and Flint, 260 (4) Flint and Peoria, 350 (5) Columbus and Dayton, 60 (6) Indianapolis and Peoria, 200 (7) Lansing and Dayton, 200 (8) Madison and Columbus, 390 (9) Lansing and Ft. Wayne, 120 (10) Toledo and Madison. 320

3. Put your ruler on the scale under the map. About how many miles are represented by 1 in.? 100 by 2 in.? 200 by 3 in.? 300

4. Find two cities on the map that are about 100 miles apart; (1) that are about 350 miles apart. (2)

(1) Cincinnati and Columbus;
(2) Springfield and Columbus

Do ex. 1 and part of ex. 2 with pupils to be sure all understand how to use the scale. Have them complete ex. 3-4. Let pupils bring to class highway maps that have printed scales, and show how the scales are used.

Selecting Your Own Scale

1. In ex. (a) to (d), select a scale for each drawing so that the longest part of the drawing will be not over 7 in. long. Choose one of these scales: $\frac{1}{4}$ in. = 1 ft., $\frac{1}{2}$ in. = 1 ft., 1 in. = 2 ft., 1 in. = 5 ft., 1 in. = 10 ft., 1 in. = 10 mi.

(a) Make a scale drawing of a rectangle 8 ft. by 11 ft.

(b) Make a scale drawing of a rectangle 40 ft. by 50 ft.

(c) Make a scale drawing of a rectangle 18 ft. by 30 ft.

(d) The town of Cedar Grove is 30 mi. east of the town of Harris. New City is 65 mi. south of Harris while Farmington is 25 mi. south of Cedar Grove. Draw to scale a map showing these towns. Then find on your map the airline distance in miles from Farmington to New City. 50

2. Make a scale drawing of a rectangular plot of grass, 20 ft. by 35 ft., with a border of flowers 2 ft. wide around the grass plot and outside of it. What will be the length of the longest part of the drawing if the scale is $\frac{1}{4}$ in. = 1 ft.? $9\frac{3}{4}$ in. if the scale is $\frac{1}{8}$ in. = 1 ft.? $4\frac{7}{8}$ in. if it is $\frac{1}{2}$ in. = 1 ft.? $19\frac{1}{2}$ in. Which scale will you use and why?

3. Make a floor plan of your classroom, putting in the doors and windows and the teacher's desk. The drawing should be made on a sheet of paper measuring about 8 in. by 11 in. Select a convenient scale that will give a drawing covering most of the sheet of paper.

4. Walter's father has a garage that is 36 ft. long and 24 ft. wide. Walter drew a floor plan of the garage, making the drawing 9 in. long and 4 in. wide. His drawing was wrong; it should have been 9 in. long and 6 . . . in. wide, or 6 . . . in. long and 4 in. wide. What scale was used if the drawing was 9 in. long? 1 in. = 4ft. if it was 4 in. wide? 1 in. = 6ft.

5. A rectangular garden 18 ft. long and 15 ft. wide is represented by a scale drawing $4\frac{1}{2}$ in. long and $3\frac{3}{4}$ in. wide. What scale was used in making this drawing? 1 in. = 4ft. What scale would be used if the drawing were 6 in. long and 5 in. wide? 1in. = 2ft. if the drawing were 9 in. long and $7\frac{1}{2}$ in. wide? 1 in. = 3 ft.

Discuss problems with class and bring out that scale selected depends on size of object or area to be shown and space available for the drawing. Stress importance of carefulness and accuracy in making drawings.

Review Problems

1. Joe read in the school library $\frac{3}{4}$ hr. on Monday, $1\frac{1}{2}$ hr. on Tuesday, $1\frac{1}{4}$ hr. on Wednesday, $1\frac{1}{2}$ hr. on Thursday, and 1 hr. on Friday. What was the average number of hours per day Joe read in the library? $1\frac{1}{5}$

2. Mr. Harris averages about $38\frac{1}{2}$ mi. an hour in his car. At that rate, about how many miles would he drive in 8 hr.? 308

3. Sally is going to make cookies. The recipe calls for $\frac{2}{3}$ cup of sugar. Sally wants to make only $\frac{3}{4}$ the amount of the recipe. How much sugar should she use? $\frac{1}{2}$ cup

4. Mrs. Chase is cooking dinner for her family. She wants the roast to be done by 6 P.M. She has a $6\frac{1}{2}$-pound roast. If she cooks the meat 20 min. to the pound, at what time should she put the roast in the oven? 3:50 P.M.

5. Here is a record of the money Bob received and spent during a two weeks' period. Put it in the form of a cash account. He had \$.65 on hand on Feb. 3. He received \$.35 on Feb. 4 for doing an errand; \$1.00 on Feb. 7 for shoveling snow; \$.90 on Feb. 8 for working after school; \$1.50 on Feb. 11 for helping Mrs. Wood; \$.75 on Feb. 16 for selling old papers. Bob spent \$.55 on Feb. 5 for movies; \$.20 on Feb. 9 for school supplies; \$1.49 on Feb. 13 for book; \$.50 on Feb. 15 for ticket to school play. Find the balance at the end of the day on Feb. 17. \$2.41

See Guide

6. When Susan makes cocoa, she uses $\frac{2}{3}$ cup of milk for each cup of cocoa. How many cups of milk does Susan need if she wants to make 9 cups of cocoa? 6 if she wants to make 15 cups of cocoa? 10

7. Mrs. Williams bought 2 doz. eggs at \$.55 a dozen, $1\frac{1}{2}$ lb. of meat at \$.69 a pound, a bag of oranges for \$.49, vegetables for \$.45, and a box of cookies for \$.39. Find the total amount of her bill. \$3.47

Observe pupils as they work to note evidences of hesitation and frustration. Group pupils who did poorly to help them analyze causes of errors. Have them explain their solutions so you may determine sources of difficulties.

Present review of work on informal geometry taught in Chapter 6.

Chapter Review

1. With your protractor, draw an angle of 136°. Bisect this angle with your compasses. How many degrees should there be in each half of the angle? Check your work by measuring each half with your protractor. 68°

2. Draw a regular pentagon in a circle. Then draw a 5-pointed star in the circle.

Add to class chart.

3. A regular **decagon** has 10 equal sides. To draw a regular decagon in a circle, what size should you make each angle at the center of the circle? 36°

4. Construct an equilateral triangle *ABC*. Then with the draftsman's triangle, draw a line through *A* perpendicular to *BC*. Also draw a line through *C* perpendicular to *AB*, and one through *B* perpendicular to *AC*. Do these perpendiculars all cross at the same point? They should do so.

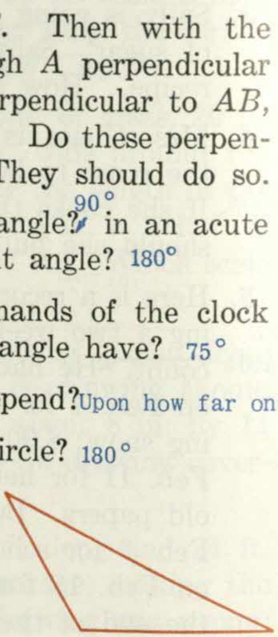
5. How many degrees are there in a right angle? 90° in an acute angle? Less than 90° in an obtuse angle? More than 90° in a straight angle? 180°

6. What kind of angle is formed by the hands of the clock at 3:30? An acute angle How many degrees does that angle have? 75°

7. Upon what does the size of an angle depend? Upon how far one side of the angle is rotated from the other side

8. How many degrees are there in a semicircle? 180°

9. Draw a large triangle shaped like the one at the right. Measure the 3 angles in your triangle. Do the 3 angles add up to 180°? Yes Should they do so? Yes



10. The direction called **northeast** is halfway between north and east. In air navigation, a plane flying northeast is flying a course of how many degrees? 45° State in degrees the direction of a plane flying northwest; 315° southeast. 135°

11. An airplane pilot flies on a course of 225°. With your protractor, make a drawing showing this course. How do you draw an angle of 225°? What is another way to describe a direction of 225°? Southwest

Check papers carefully and analyze errors. If most pupils seem weak in a particular area, topic may need to be retaught to whole class. Use developmental material in text related to pupils' difficulties.

A Problem Test

1. George worked for Mr. Johnson during the summer vacation. For the first 3 weeks he received \$9.75 a week and for the other 7 weeks his pay was increased to \$11.25 a week. Find George's average weekly pay for the summer. \$10.80

2. Mr. Ford's rent is \$68.00 per month. If it is raised 15%, how much more will his rent be for next year? \$122.40

3. A ball of yarn weighs $\frac{1}{2}$ oz. and sells at \$.59. Find the cost of $\frac{1}{4}$ lb. of this yarn. \$4.72

4. Mrs. Bell receives her bill for gas and electricity once every two months. These are the amounts she paid for the past year: \$10.86, \$9.58, \$10.52, \$12.72, \$9.80, \$11.44. Find the average cost of Mrs. Bell's gas and electricity per month. \$5.41

5. Find the cost of $15\frac{1}{2}$ yd. of cloth at \$.89 a yard. \$13.80

6. Mrs. Case wanted half of a ham that weighed 13 lb. 11 oz. When the butcher cut the ham and weighed one piece, it weighed 6 lb. 6 oz. Mrs. Case decided to buy the other piece at \$.64 a pound. How much did it cost? \$4.68

7. Mr. Fox's car used 35 gal. of gasoline for a trip of 588 mi. How many miles did it average per gallon? 16.8

8. The school store sells 50 sheets of paper for 23¢. The paper is bought in boxes of 500 sheets at \$1.78 per box. For how much more per box is the paper sold than it cost? \$.52

9. An airplane that flies $1\frac{3}{4}$ mi. in $\frac{1}{2}$ min. is flying at what speed in miles per hour? 210 mph

10. Mrs. Smith bought the following articles at the grocery store: 5 lb. potatoes at 10 lb. for 39¢, 3 lb. oranges at 13¢ a pound, $1\frac{1}{2}$ lb. string beans at 15¢ a pound, and $\frac{1}{2}$ lb. butter at 67¢ a pound. Find the amount of her bill. \$1.16

Have pupils record scores on graphs.

SCORE	0–5	6–7	8–9	10
	You need help	Fair	Good	Excellent

Instruct pupils to read problems carefully. Through pupils' explanations of their solutions, try to determine causes of errors. Group pupils for specialized work on problem solving. Let others work on original problems or projects.

Present diagnostic test of skills taught or retaught thus far with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Practice Pages

Divide and check the work:

1. $294\overline{)2823293}$ 9603 R11 $318\overline{)2745930}$ 8635 — 32, 33

Find the answers and check:

2. $4\frac{3}{10} + 6\frac{1}{2} + 1\frac{4}{5}$ $12\frac{3}{5}$ $7\frac{1}{4} + 2\frac{1}{6}$ $9\frac{5}{12}$ — 55, 62

3. $8\frac{3}{8} - 6\frac{5}{8}$ $1\frac{3}{4}$ $9\frac{1}{5} - 7\frac{7}{10}$ $\frac{3}{5}$ $5\frac{1}{6} - 3\frac{1}{8}$ $2\frac{1}{24}$ — 56–61

4. $1\frac{5}{6} \times 15$ $27\frac{1}{2}$ $\frac{9}{10} \times \frac{2}{3}$ $\frac{3}{5}$ $3\frac{1}{3} \times 1\frac{7}{8}$ $6\frac{1}{4}$ — 63–67

5. $1\frac{1}{8} \div 1\frac{1}{2}$ $\frac{3}{4}$ $24 \div 3\frac{1}{5}$ $7\frac{1}{2}$ $3\frac{3}{5} \div \frac{9}{10}$ 4 — 72–76

Change to decimals:

6. $\frac{5}{8}$ .625 $\frac{13}{20}$ .65 $\frac{13}{16}$ .8125 $\frac{21}{25}$ .84 — 106, 107

7. 6% .06 $12\frac{1}{2}\%$ .125 $2\frac{1}{4}\%$ .0225 137% 1.37 — 142, 146

Multiply and check the work:

8. $\begin{array}{r}9.39\\18\\\hline 169.02\end{array}$ $\begin{array}{r}68.1\\7.9\\\hline 537.99\end{array}$ $\begin{array}{r}1.16\\.012\\\hline .01392\end{array}$ $\begin{array}{r}1.04\\.094\\\hline .09776\end{array}$ — 96–100

Divide. If you continue to have a remainder, find the answer correct to the nearest tenth:

9. $8\overline{)134}$ 16.75 $7\overline{).056}$ .008 $35\overline{)938}$ 26.8 $47\overline{)59.3}$ 1.3 — 101–105

10. $.7\overline{)6.72}$ 9.6 $.18\overline{)444.6}$ 2470 $7.2\overline{)5.33}$ .7 $.125\overline{)157.5}$ 1260 — 111–113

Find the answers to the nearest cent:

11. 15% of \$13 \$1.95 $62\frac{1}{2}\%$ of \$36 \$22.50 $66\frac{2}{3}\%$ of \$39 \$26 — 128–136

12. $6\frac{1}{2}\%$ of \$24 \$1.56 300% of \$17 \$51 122% of \$31 \$37.82 — 142–146

13. 2% of \$7.71 \$.15 $12\frac{1}{2}\%$ of \$4.10 \$.51 19% of \$5.25 \$1.00 — 138

Emphasize importance of checking each example. After checking papers and analyzing errors, return them to pupils so they may find and correct errors. Clear up any difficulties before assigning remedial work.

See Guide for specific aims of Chapter 7.

Chapter 7

Buying a Boat

Introduce topic of comparison of numbers by division.

1. John bought an old rowboat from Mr. Green for \$36 and is paying for it at the rate of \$3 per week. John has made two payments on the boat. How many more weekly payments must John make to pay for the boat? 10

2. What part of the \$36 does John pay each week? What part of \$36 is \$3? $\frac{1}{12}$

▶ \$3 is $\frac{3}{36}$, or $\frac{1}{12}$, of \$36. So John pays $\frac{1}{12}$ of the cost each week.

3. When John made his second payment of \$3, he said, "I have now paid \$6 in all, so $\frac{6}{36}$, or $\frac{1}{6}$, of the total cost is paid." Was he right? Yes After he makes the third payment, how much in all will he have paid toward the \$36? \$9 What part of \$36 is \$9? $\frac{9}{36}$ Change the answer to lowest terms. $\frac{1}{4}$

4. How much in all will John have paid after the fourth payment? \$12 after the fifth payment? \$15 the sixth payment? \$18 the seventh payment? \$21 the eighth payment? \$24 What part of the \$36 was paid after each of these payments? $\frac{1}{3}$, $\frac{5}{12}$, $\frac{1}{2}$, $\frac{7}{12}$, $\frac{2}{3}$

First review how numbers are compared by subtraction: finding how much larger or smaller one number is than another. Then follow development as given, doing ex. 1-4 as class activity. Give further examples like ex. 3-4.

Teach how to compare a smaller number with a larger one by using

Comparing Numbers

division.

1. On page 211 you compared a number with a larger number by asking this question: "The smaller number is what part of the larger one?" You answered this question by writing the smaller number over the larger one to make a fraction. Then you changed the fraction to lowest terms.

Point out that division was used since $\frac{3}{36}$ means 3 ÷ 36.

2. On page 211 the amounts you compared were both expressed as dollars. If you wish to compare 25¢ with \$1, you must first change \$1 to 100¢. Then compare 25¢ with 100¢, which give $\frac{1}{4}$. This means that 25¢ is $\frac{1}{4}$ of \$1. When you compare two amounts, both must have the same unit. Stress.

3. What part of a yard is 1 ft. 6 in.? $\frac{1}{2}$

▶ These measures must both be expressed in inches. Change 1 ft. 6 in. to 18 in. and 1 yd. to 36 in. Then 18 in. is $\frac{18}{36}$ of 36 in. or $\frac{1}{2}$ of 36 in. So 1 ft. 6 in. is $\frac{1}{2}$ yd.

4. What part of a yard is 12 in.? $\frac{1}{3}$ What part of a yard is 24 in.? $\frac{2}{3}$ 27 in.? $\frac{3}{4}$ 30 in.? $\frac{5}{6}$ 2 ft. 3 in.? $\frac{3}{4}$ 1 ft. 3 in.? $\frac{5}{12}$

5. What part of an hour is 15 min.? $\frac{1}{4}$ What part of an hour is 40 min.? $\frac{2}{3}$ 48 min.? $\frac{4}{5}$ 50 min.? $\frac{5}{6}$

6. What part of a mile is 440 ft.? $\frac{1}{12}$ 528 ft.? $\frac{1}{10}$ 660 ft.? $\frac{1}{8}$ 880 ft.? $\frac{1}{6}$ 1320 ft.? $\frac{1}{4}$ 3960 ft.? $\frac{3}{4}$

7. What part of a ton is 100 lb.? $\frac{1}{20}$ 200 lb.? $\frac{1}{10}$ 500 lb.? $\frac{1}{4}$ 1200 lb.? $\frac{3}{5}$

8. What part of a foot is 1 in.? $\frac{1}{12}$ 5 in.? $\frac{5}{12}$ 4 in.? $\frac{1}{3}$ 6 in.? $\frac{1}{2}$ 8 in.? $\frac{2}{3}$

9. What part of a minute is 20 sec.? $\frac{1}{3}$ 25 sec.? $\frac{5}{12}$ 45 sec.? $\frac{3}{4}$

10. What part of a dollar is 10¢? $\frac{1}{10}$ 20¢? $\frac{1}{5}$ 25¢? $\frac{1}{4}$ 35¢? $\frac{7}{20}$ 40¢? $\frac{2}{5}$ 50¢? $\frac{1}{2}$ 60¢? $\frac{3}{5}$ 75¢? $\frac{3}{4}$ 85¢? $\frac{17}{20}$ 95¢? $\frac{19}{20}$

11. Jean has read 135 pages of a book containing 324 pages. What part of the book has she read? $\frac{5}{12}$

Find what part the smaller number is of the larger one:

12. 24, 60 $\frac{2}{5}$ 50, 75 $\frac{2}{3}$ 35, 40 $\frac{7}{8}$ \$48, \$120 $\frac{2}{5}$ \$175, \$350 $\frac{1}{2}$

13. 16, 72 $\frac{2}{9}$ 75, 90 $\frac{5}{6}$ 30, 45 $\frac{2}{3}$ \$54, \$144 $\frac{3}{8}$ \$110, \$165 $\frac{2}{3}$

Do ex. 1-4 with class. In ex. 2 be sure pupils understand why both amounts must be expressed in the same unit. While you give others special help as needed, ask better students to make up word problems using numbers in ex. 12-13.

Present set of improvement tests in division. Review problems in estimating answers.

Improving by Practice

Division Test 4a. Time: $5\frac{1}{2}$ min. after copying.

1. $\overset{91.5}{26\overline{)2379}}$ $\overset{68.0}{53\overline{)3606}}$ $\overset{58.2}{85\overline{)4947}}$

2. $\overset{71.8}{39\overline{)2800}}$ $\overset{75.5}{62\overline{)4684}}$ $\overset{62.3}{47\overline{)2928}}$ ⑥

Division Test 4b. Time: $5\frac{1}{2}$ min. after copying.

3. $\overset{53.9}{37\overline{)1995}}$ $\overset{40.3}{81\overline{)3264}}$ $\overset{72.8}{25\overline{)1820}}$

4. $\overset{69.1}{59\overline{)4076}}$ $\overset{50.9}{43\overline{)2189}}$ $\overset{96.5}{18\overline{)1737}}$ ⑥

Division Test 4c. Time: $5\frac{1}{2}$ min. after copying.

5. $\overset{63.4}{35\overline{)2219}}$ $\overset{3.9}{68\overline{)2986}}$ $\overset{93.5}{19\overline{)1776}}$

6. $\overset{81.5}{26\overline{)2119}}$ $\overset{47.3}{82\overline{)3881}}$ $\overset{85.5}{44\overline{)3760}}$ ⑥

To the Teacher. Instruct pupils to find the quotients to the nearest tenth.

To check understandings of this skill, have pupils explain how they estimated answers in ex. 7–10.

Estimating Answers

At the end of each problem there are three numbers. Select the number you think is nearest to the right answer. Then work the problem and compare the answer with the number selected:

7. Ellen bought 4 yd. of material at \$.98 a yard. How much did it cost? \$3.60 \$4.00 \$4.25 \$3.92

8. How much change is left from \$20 if you pay \$7.48 for a tennis racket, \$1.94 for tennis balls, and \$4.99 for tennis shoes? \$5.00 \$5.50 \$6.50 \$5.59

9. A train goes 389 mi. in 8 hr. What is its average speed in miles per hour? 45 mi. 50 mi. 55 mi. 48.6

10. Robert bought a fishing rod for \$3.95 and a line for \$.49. How much did he pay in all? \$4.25 \$4.50 \$4.75 \$4.44

Tests are to be given, scored, and recorded as previous ones were (see pages 87, 127, 167). Group pupils who had errors, to help discover causes from explanations of their work. Let others work on geometric designs, scale drawings, and so on.

Teach how to compare a number with a larger one by using a per cent (per cents in this case are whole numbers).

Comparing Numbers by Per Cents

1. Problem On a test of 20 examples, Joe worked 18 examples correctly. What per cent was his mark on the test? 90%

Explanation First find what part 18 is of 20. This gives $\frac{18}{20}$, which equals $\frac{9}{10}$. You know that $\frac{9}{10} = \frac{90}{100}$, or 90%. So Joe's mark was 90%.

2. Problem Ann worked 17 examples correctly out of 20 examples. What per cent was her mark? 85%

Explanation You have the fraction $\frac{17}{20}$ to change to a per cent. Since 20 can easily be made into 100 by multiplying by 5, you can multiply both numerator and denominator of $\frac{17}{20}$ by 5, which gives $\frac{85}{100}$. This gives 85% as Ann's mark. Check the work by finding 85% of 20; the result should be 17.

Have pupils explain answers for ex. 3–4, 6–7.

Find what per cent the first number is of the second:

3. 36, 48 75% 45, 60 75% 13, 50 26% 32, 64 50% 15, 20 75%

4. 38, 76 50% 12, 48 25% 42, 56 75% 6, 25 24% 48, 60 80%

5. Problem On a test containing 75 multiplication facts, Betty got 72 facts right. What per cent of the facts were right? 96%

Explanation First change $\frac{72}{75}$ to $\frac{24}{25}$. Then change $\frac{24}{25}$ to a fraction having 100 as its denominator; how do you do this? Betty's mark was 96%. Check the work by finding 96% of 75; it should be 72.

Tell the number to put in each space. Check the work: Stress.

6. 34 is $\overset{85}{. .}$% of 40 18 is $\overset{12}{. .}$% of 150 48 is $\overset{32}{. .}$% of 150

7. 21 is $\overset{28}{. .}$% of 75 30 is $\overset{24}{. .}$% of 125 85 is $\overset{68}{. .}$% of 125

Emphasize after understanding is assured.

To find what per cent one number is of another, find what fractional part the one number is of the other; change this fraction to hundredths and then to a per cent.

More Practice. See ㊺ on page 339. Use to reinforce skill.

In discussing ex. 1 try to get pupils to suggest how $\frac{9}{10}$ can be changed to a per cent (remind them that per cent is another way of expressing hundredths). Then discuss ex. 2. Ask pupils why we can multiply numerator and denominator by 5.

Teach comparison of two numbers by means of a per cent when the per cent ends in a fraction. Do ex. 1-4 with class.

Comparing Attendance Records

1. **Problem** Last month 37 of the 40 pupils in Jack's class were not absent from school any day of the month. What per cent of the class had a perfect attendance record? $92\frac{1}{2}\%$

Explanation In this problem, 37 is $\frac{37}{40}$ of 40. It is not easy to change $\frac{37}{40}$ to a fraction having 100 as a denominator, as you did on page 214. In such cases the fraction should be changed to a decimal and then to a per cent as shown below.

First change $\frac{37}{40}$ to a decimal. Since $\frac{37}{40}$ means 37 ÷ 40, write a decimal point and two zeros after 37 and divide by 40. Carry the quotient only to **hundredths** (2 decimal places) since **per cent** means **hundredths.** In this case there is a remainder of 20, which gives $\frac{20}{40}$, or $\frac{1}{2}$, as a fraction in the quotient. The quotient is $.92\frac{1}{2}$, so $\frac{37}{40} = .92\frac{1}{2}$. Now change $.92\frac{1}{2}$ to a per cent, which gives $92\frac{1}{2}\%$. This shows that 37 is $92\frac{1}{2}\%$ of 40, so $92\frac{1}{2}\%$ of the class had a perfect attendance. Check the work by finding $92\frac{1}{2}\%$ of 40; the result should be 37.

Stress why.

Stress.

Point out that this is the usual method of comparing 2 numbers by a per cent.

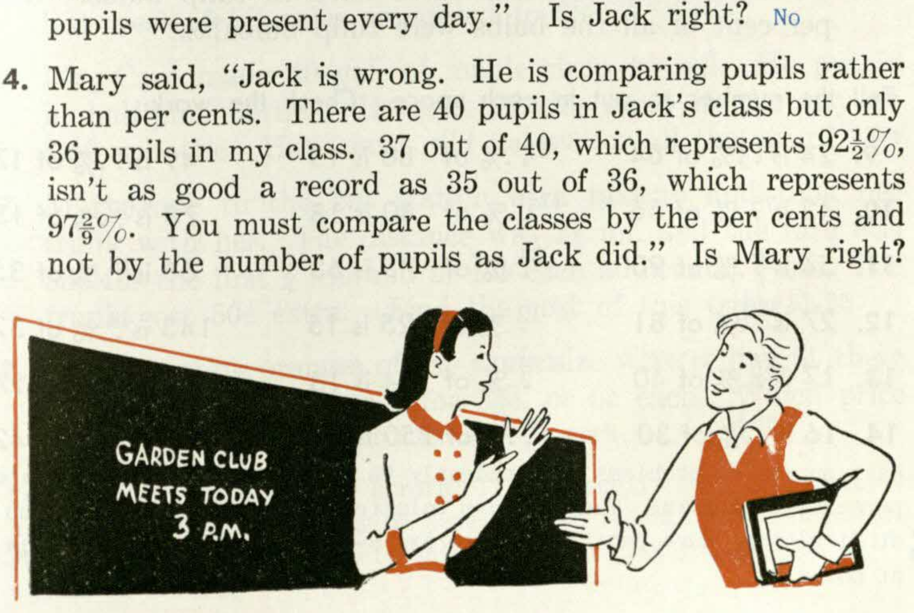
```
     .92½
 40)37.00
    36 0
     1 00
       80
       20
```

$\frac{20}{40} = \frac{1}{2}$

2. In Mary's class there are 36 pupils, 35 of whom were not absent any day last month. What per cent of Mary's class had a perfect attendance? $97\frac{2}{9}\%$

3. Jack said, "My class had a better record because 37 pupils were in school every day while in Mary's class only 35 pupils were present every day." Is Jack right? No

4. Mary said, "Jack is wrong. He is comparing pupils rather than per cents. There are 40 pupils in Jack's class but only 36 pupils in my class. 37 out of 40, which represents $92\frac{1}{2}\%$, isn't as good a record as 35 out of 36, which represents $97\frac{2}{9}\%$. You must compare the classes by the per cents and not by the number of pupils as Jack did." Is Mary right? Yes



Present problems and practice on comparing two numbers by using a per cent.

Problems and Practice

1. In Miss Black's room there are 32 pupils and 18 of them are girls. What per cent of the pupils are girls? $56\frac{1}{4}$%

2. Miss Black's room is in the Jefferson School. In the whole school there are 475 pupils and 247 of them are girls. What per cent of the pupils are girls? 52% In the whole school is the per cent of girls greater or smaller than in Miss Black's room? Smaller See ex. 1.

3. In the Jefferson School, what per cent of the pupils are boys? 48% See ex. 2.

4. In the Parker School, 840 pupils out of 875 are members of the Athletic Association. What per cent of the pupils are members? 96% What per cent are not members? 4%

5. Mr. Bell received $360 as his commission for selling a farm for $9600. What per cent of commission was he paid? $3\frac{3}{4}$%

6. At a furniture sale, chairs that formerly sold at $45 each are now selling at $39 each. At what rate of discount are these chairs selling? $13\frac{1}{3}$%

7. Mr. King planted 300 flower bulbs around his house last October. This spring only 18 of them did not grow. What per cent of the flower bulbs did not come up? 6%

8. Of the 300 flower bulbs there were 75 tulip bulbs. What per cent of all the bulbs were tulip bulbs? 25%

Tell the number to put in each space. Check the work:

9. 24 is $37\frac{1}{2}$ . .% of 64 — $16\frac{1}{4}$ . .% of 80 is 13 — 49 is 28 . .% of 175

10. 42 is 75 . .% of 56 — $22\frac{1}{2}$ . .% of 80 is 18 — 78 is 60 . .% of 130

11. 54 is 60 . .% of 90 — 84 . .% of 75 is 63 — 63 is 18 . .% of 350

12. 27 is $33\frac{1}{3}$ . .% of 81 — 12 . .% of 125 is 15 — 143 is 44 . .% of 325

13. 17 is $42\frac{1}{2}$ . .% of 40 — $12\frac{1}{2}$ . .% of 104 is 13 — 132 is 48 . .% of 275

14. 16 is $53\frac{1}{3}$ . .% of 30 — 26 . .% of 250 is 65 — 221 is 52 . .% of 425

Have pupils do problems independently to check their ability to interpret percentage problems. Then discuss solutions with class and clear up any difficulties. Have pupils complete ex. 9-14 and let volunteers show work at board.

Give review of problem solving of all kinds.

Problems with Several Steps

1. Miss Cross bought a radio for \$35. She paid 40% of the price when she got the radio and the rest in 6 equal monthly payments. How much did Miss Cross pay each month? \$3.50

2. A giant paper machine turns off rolls of newsprint paper at the rate of 1400 ft. a minute. How many miles of paper can it turn off in $\frac{1}{4}$ hr.? Find the answer to the nearest tenth of a mile. 5280 ft. = 1 mi. 4.0

3. Mr. and Mrs. Mason, Jean, and Bill are planning a trip to Washington. If they go by automobile, Mr. Mason estimates the average cost per mile for the trip to be 12¢. The road mileage to Washington is 356 mi. If they go by plane and use the family plan, Mr. Mason will pay the regular one-way fare, but Mrs. Mason, Jean, and Bill will pay $\frac{2}{3}$ fare. The regular one-way fare is \$20.10 plus 5% tax. Is it cheaper for the Mason family to go by plane or by automobile? By automobile How much cheaper? \$20.60

4. An airplane left Chicago at 8:30 A.M., traveling south. At the end of 49 min. it had covered 240 mi. If it continues at this rate, at what time will it arrive at an airport that is 1200 mi. from Chicago? 12:35 P.M.

5. Mr. Harris is paid \$60 a week and a commission of 5% on all his sales over \$100. How much in all did Mr. Harris get last week if his sales amounted to \$950? \$102.50

6. Mr. Case made 40 gal. of maple sirup to sell. He put it in half-pint bottles. He sold all this sirup in the city at \$.60 a bottle. How much did he get for all the sirup? \$384.00

7. Mary rode to the bus station in a taxicab and took her trunk with her. The distance was $2\frac{3}{4}$ mi. and the fare was 35¢ for the first $\frac{1}{4}$ mi. and 5¢ for each additional $\frac{1}{4}$ mi. The trunk cost 50¢ extra. Find the cost of this trip. \$1.35

8. In three stores oranges of the same size were selling at these prices: 50¢ a dozen, or 8 for 33¢, or 5¢ each. Which price was the cheapest? 8 for 33¢, at $4\frac{1}{8}$¢ each

Correct papers and analyze errors. If many pupils have difficulty with a particular problem it may be necessary to reteach topic involved. Pupils' explanations of their solutions will help you discover causes of errors.

Teach how to find rate of discount when purchase is made at reduced price.

Buying at a Discount

1. Betty waited to buy new skis until she could get them at a discount. At Mr. Carter's store she paid \$12.75 for a pair of skis marked \$15.00. What rate of discount did she get? 15%

2. Mr. Carter also has several pairs of skis for \$8.50 a pair. Formerly they sold for \$10.00 a pair. What is the rate of discount on these skis? 15%

3. If Joe buys new skis he also wants a ski pole. He can get a pole marked down from \$4.00 to \$3.00. What rate of discount will he get? 25%

4. Mr. Carter put an advertisement in the paper which included these items:

	Formerly	Now	
Snow Shoes	\$18.00	\$12.00	$33\frac{1}{3}$%
Ski Boots	\$10.00	\$8.40	16%
Ski Pants	\$9.50	\$7.60	20%
Ski Jacket	\$8.75	\$7.00	20%
Hockey Sticks	\$1.25	\$1.00	20%
Toboggans	\$16.00	\$10.00	$37\frac{1}{2}$%

Find the rate of discount on each of the articles above.

Point out that rate of discount means per cent of discount. Lead pupils to see which two numbers are to be compared (amount of discount and original price). Pupils might make up similar problems on advertisements of sales.

Teach comparison of two numbers by a per cent when nearest whole per cent is required (pages 219–220).

Finding Nearest Whole Per Cent

1. **Problem** Today Peter practiced throwing a basketball into the basket. Out of 38 trials, he threw the ball into the basket 25 times. What per cent of the trials were successful? Find the result to the nearest whole per cent. 66%

Explanation You see that $\frac{25}{38}$ of the trials were successful. First change $\frac{25}{38}$ to a decimal as shown at the right. After carrying the division to two decimal places, you still have a remainder, so carry the work to one more decimal place to see whether the result is nearer to .65 or to .66. Since the quotient is .657, the result, to the nearest hundredth, is .66, which means 66%. This shows, to the nearest whole per cent, that 66% of Peter's trials were successful. Check the work by going over it.

```
      .657
38)25.000
   22 8
    2 20
    1 90
      300
      266
```

Emphasize.→

To find a result correct to the nearest whole per cent, find the quotient to three decimal places; then round off to two places and change the two-place decimal to a per cent.

Change these decimals to the nearest whole per cent:

2. .284 28% .478 48% .215 22% .097 10% .009 1% .128 13%

3. Yesterday Peter threw the ball into the basket 11 times out of 12 trials. Find, to the nearest whole per cent, what per cent of the trials were successful. 92%

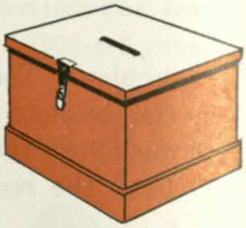
4. What per cent of your trials are successful if you throw the ball into the basket 31 out of 36 times? 86% 27 out of 43 times? 63% 9 out of 17 times? 53% 18 out of 29 times? 62%

5. Ann won 6 out of 11 sets of tennis and Jean won 5 out of 9 sets. Which girl had the better per cent of sets won? Jean Find each per cent to the nearest whole per cent. Ann, 55%; Jean, 56%

More Practice. See ㊻ on page 340.

When discussing ex. 1, emphasize that the quotient is expressed to one more decimal place than is needed in the answer. Ex. 2 may be done orally and more practice given if needed. Have pupils complete ex. 3–5.

Nearest Whole Per Cent



Find the answers to the nearest whole per cent:

1. In the election for president of the Athletic Association, the winner received 386 of the 425 votes cast. What per cent of the votes did he have? 91%

2. Jack saved $135 last year. Of this amount, he made $59 from the sale of airplane models. What per cent of Jack's savings was made from the sale of these models? 44%

3. Betty read that 9 days out of the past two weeks were rainy. What per cent of the past 14 days were rainy? 64%

4. On a test of 9 problems George worked 8 correctly. Find the per cent George had correct. 89%

5. Mrs. West has $36 a week to spend for housekeeping. Last week she spent $29 for food. What per cent of the money did she spend for food? 81%

6. Ann and Peggy played 7 games of checkers and Ann won 4 of them. What per cent of the games did Ann win? 57%

7. Mr. Marsh picked 482 bu. of apples. He sent 175 bu. to market, sold 225 bu. at his roadside stand, and put 82 bu. in storage. What per cent of the apples went in each way? 36%; 47%; 17%

8. Out of 185 days that school was held, there were only 6 days when John did not attend. What was John's per cent of attendance? 97%

9. At Eagle Bay Camp there were 130 boys and only 19 of them could swim before coming to camp. All the others learned to swim while at camp. What per cent of the boys learned to swim at camp? 85%

10. In an archery contest, Jim hit the bull's-eye 11 times out of 16. What per cent of Jim's arrows hit the bull's-eye? 69%

11. 4 is what per cent of 11? 36% What per cent of 72 is 53? 74%

12. 5 is what per cent of 16? 31% What per cent of 51 is 40? 78%

After pupils complete ex. 1-12 independently, have volunteers explain work at board. Encourage pupils to ask questions if they do not understand explanations. Group pupils for remedial work as needed.

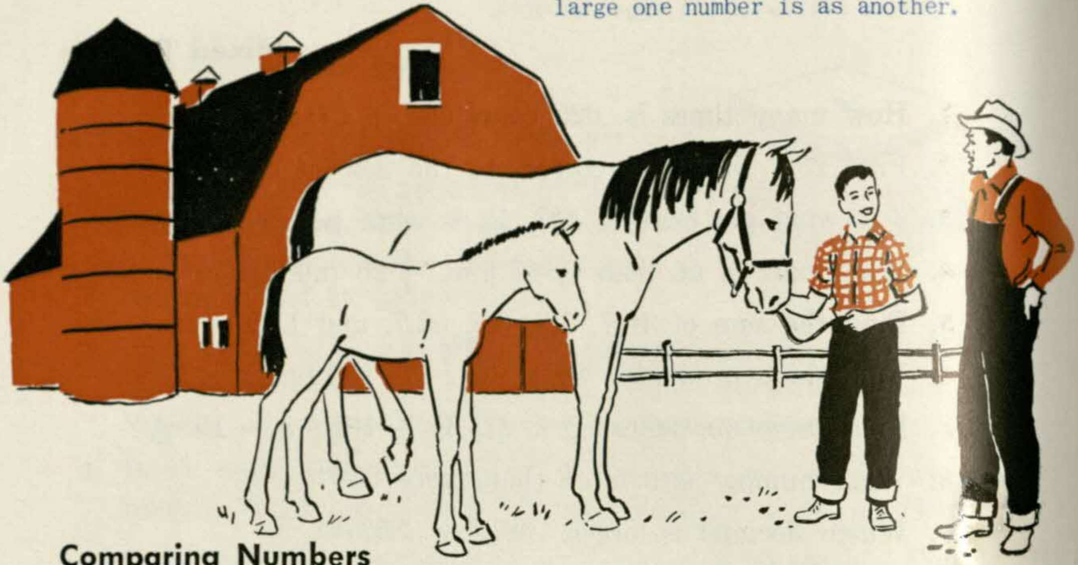
Present review of all types of work.

Mixed Practice

1. How many times is .025 contained in 341.55? 13,662
2. Find 25% of \$37.50 correct to the nearest cent. \$9.38
3. 6 is what per cent of 40? 12 is what per cent of 30? 40%
4. What part of an hour is 45 min.? $\frac{3}{4}$ 20 min.? $\frac{1}{3}$ 36 min.? $\frac{3}{5}$
5. Find the sum of 46.7, 3.8, 9.5, 32.7, and 11.5. 104.2
6. Find these products: $3\frac{2}{3} \times 16\frac{1}{2}$; $60\frac{1}{2}$ $5\frac{1}{4} \times 13\frac{1}{3}$; 70 $\frac{5}{8} \times 7$. $4\frac{3}{8}$
7. Find these quotients: $\frac{7}{8} \div \frac{1}{2}$; $1\frac{3}{4}$ $6\frac{3}{4} \div 10\frac{1}{8}$; $\frac{2}{3}$ $\frac{3}{4} \div 15$. $\frac{1}{20}$
8. What number is 9 more than twice 33? 75
9. Which decimal is larger, .0875 or .85? .85
10. 200 divided by 8 plus 15 equals what number? 40
11. Change $\frac{24}{37}$ to a decimal correct to the nearest tenth. .6
12. How many ounces are there in 4 lb. 11 oz.? 75
13. Find the average of $2\frac{2}{3}$, $7\frac{1}{6}$, $9\frac{1}{2}$, and $11\frac{1}{3}$. $7\frac{2}{3}$
14. Which is greater, 5% of 240 (12) or 8% of 175? (14) 8% of 175
15. Find the difference between 3993 and 9339; then divide the difference by 54. 99
16. How many hours and minutes is it from 10:15 A.M. on Jan. 5 to 8:08 A.M. on Jan. 6? 21 hr. 53 min.
17. What number is 10% more than \$250? \$275 What number is 10% less than \$250? \$225 What number is 10% of \$250? \$25
18. What per cent is 98 of 245? 40% What per cent, to the nearest whole per cent, is 81 of 93? 87%
19. What time is it in California when it is 8 A.M. in New York? 5 A.M. when it is noon in Illinois? 10 A.M.
20. Which group of coins amounts to more, 5 half dollars, 9 quarters, 3 dimes, 11 nickels, and 31 pennies or 2 half dollars, 15 quarters, 12 dimes, 6 nickels, and 17 pennies? The second group How much more? \$.51

Try to determine if errors were caused by carelessness, incorrect computation, lack of understanding, and so on. Plan for further review of topic or topics based on class or individual needs.

Teach how to compare two numbers by division by finding how many times as large one number is as another.



Comparing Numbers

1. The colt shown in the picture weighs about 95 lb. and the horse weighs about 950 lb. The horse is how many times as heavy as the colt? 10

2. Jim read about a man who was paid $6 a week in his first job and who now is paid $3000 a week. His present salary is how many times as large as his first salary? 500

3. Last year Mr. Wood raised only 320 bu. of peaches but this year he had 1680 bu. This year's crop of peaches is how many times as large as last year's crop? $5\frac{1}{4}$

4. An airplane averages 480 mi. per hour, while a train averages 45 mi. per hour. The airplane's speed is how many times that of the train? $10\frac{2}{3}$

Find the number to put in each space:

5. $250 is $\overset{200}{\ldots}$ times as large as $1.25.

6. 135 lb. is $\overset{30}{\ldots}$ times as heavy as $4\frac{1}{2}$ lb.

7. 960 mi. is $\overset{6\frac{2}{5}}{\ldots}$ times as far as 150 mi.

8. 44,880 ft. is $\overset{8\frac{1}{5}}{\ldots}$ times as high as 5280 ft.

9. $1,000,000 is $\overset{40}{\ldots}$ times as large as $25,000.

Before discussing ex. 1 illustrate some simple problems, such as "10 lb. is how many times as large as 5 lb." Use diagrams to lead pupils to see that you divide the larger number by the smaller one. Then do ex. 1-4 with pupils. Have them complete ex. 5-9.

Improving by Practice

Addition Test 5a.

Time: 4 min.

1.	10	90	83	27	74	37	87	51	
	99	26	65	42	97	11	72	95	
	29	47	64	15	35	48	30	62	
	50	65	94	80	32	21	27	96	
	18	20	62	24	57	96	31	60	
	75	48	20	26	31	20	51	89	
	66	19	83	97	33	83	80	76	
	48	41	80	75	87	27	93	14	
	59	10	27	59	96	59	10	59	⑧
	454	366	578	445	542	402	481	602	

Addition Test 5b.

Time: 4 min.

2.	31	20	34	69	96	98	54	56	
	60	35	54	30	17	54	62	76	
	74	15	46	72	80	45	25	48	
	97	24	50	43	83	52	16	27	
	43	18	79	13	80	60	71	70	
	48	56	80	70	96	91	78	92	
	52	79	96	49	84	40	60	51	
	45	93	28	13	52	17	93	20	
	81	37	99	19	90	98	75	11	⑧
	531	377	566	378	678	555	534	451	

Addition Test 5c.

Time: 4 min.

3.	88	24	82	90	84	80	97	26	
	53	66	44	37	93	65	95	72	
	55	81	20	60	20	49	78	46	
	40	18	19	41	86	39	59	40	
	37	30	39	13	95	77	32	71	
	76	32	80	27	89	24	64	88	
	62	50	36	55	37	20	23	45	
	83	49	18	50	60	11	70	13	
	86	25	55	33	70	31	11	16	⑧
	580	375	393	406	634	396	529	417	

To the Teacher. If individual pupils get low scores on Improvement Tests, such pupils may be assigned extra practice given on pages 325–328.

Pupils who do poorly on tests may need help with facts and with adding by endings. Remind pupils to record scores on own graphs. See pages 47, 99, 143, and 185 for other suggestions.

Teach how to compare larger number with smaller one by using per cents larger than 100% (pages 224–225).

Large Per Cents in Comparisons

1. You learned on page 145 that per cents larger than 100% represent numbers larger than 1: 150% = 1.50 or $1\frac{1}{2}$; 200% = 2; 350% = 3.50 or $3\frac{1}{2}$; 425% = 4.25 or $4\frac{1}{4}$.

Emphasize after understanding is assured:

To change a per cent to a whole number or a decimal, drop the sign % and move the decimal point two places to the left.

2. Change to whole numbers or decimals: 300%, 3 235%, 2.35 500%, 5 1400%, 14 375%, 3.75 280%, 2.8 145%, 1.45 825%, 8.25 205%. 2.05

3. In the same way, whole numbers or decimals larger than 1 represent per cents larger than 100%. For example, 7 = 700%; 4.75 = 475%; 3.1 = 310%; 13 = 1300%.

Emphasize after understanding is assured:

To change a whole number or a decimal to a per cent, move the decimal point two places to the right and annex the sign %.

4. Change to per cents: 6, 600% 3.45, 345% 12, 1200% 4.67, 467% $7\frac{1}{2}$, 750% 11.5. 1150%

Give further illustrations like ex. 5–6.

5. When you compare the number 75 with the smaller number 25, you divide the larger number by the smaller one as explained on page 222. In this case you find that 75 is 3 times as large as 25. You can also say that 75 is 300% of 25 because 3 times a number is the same as 300% of it.

6. Last year the Junior Red Cross had 120 members in our school; this year it has 300 members. This year's membership is how many times as large as last year's? $2\frac{1}{2}$ This year's membership is what per cent of last year's? 250%

7. Mary earns \$36 a week; Jane earns \$16 a week working part-time. Mary's wages are what per cent of Jane's? 225%

Have pupils tell how many times as large one number is as the other.

Find what per cent the first number is of the second:

8.	250, 100 250%	85, 85 100%	90, 72 125%	135, 120 $112\frac{1}{2}$%	390, 120 325%
9.	500, 375 $133\frac{1}{3}$%	63, 42 150%	88, 80 110%	173, 173 100%	351, 135 260%
10.	270, 135 200%	54, 45 120%	66, 48 $137\frac{1}{2}$%	432, 108 400%	279, 124 225%

More Practice. See ㊼ on page 340.

Ex. 1–3 should be discussed thoroughly with pupils. In ex. 1 show the intermediate step (150% = $\frac{150}{100}$ = 1.5). Follow similar procedure in ex. 3 (7 = $\frac{700}{100}$ = 700%). Emphasize that 200% of a number is the same as 2 times a number.

Gaining Speed in Typing

1. Joe has learned to use a typewriter. At first he could write an average of 20 words a minute. With practice he has learned to write 32 words a minute. His present speed is what per cent of his first speed? 160%

2. With practice Joe hopes to be able to write 55 words a minute by the end of the year. What per cent of Joe's original speed of 20 words a minute will the new speed be? 275%

3. If Joe is later able to write 75 words a minute, what per cent of his original speed will 75 words a minute be? 375%

4. One of the champion typists of the world typed at a speed of 144 words a minute. That speed was how many times as fast as Joe's present speed of 32 words a minute? $4\frac{1}{2}$ The champion's speed was what per cent of Joe's speed? 450%

5. A record was made one year when a typist using an electric typewriter made a speed of 149 words a minute. That was what per cent of Joe's speed of 32 words a minute? Give your answer to the nearest whole per cent. 466%

6. When Mary was learning to type, she wrote only 15 words per minute. Now she can write 36 words per minute. Her present speed is what per cent of her original speed? 240%

Do some problems with class, letting different pupils explain their solutions. Make up similar problems for pupils to solve independently.

The Language of Arithmetic

Read the statements below and tell the correct word to put in each space. Do not write in the spaces:

1. In a division example the answer is called the quotient; the number divided is the dividend; the number by which you divide is the divisor.

2. The third place to the right of the decimal point is called thousandths place.

3. When you say that 3 times $2.98 is about $9, you are estimating the answer.

4. When you change $\frac{5}{6}$ and $\frac{3}{4}$ both to 12ths, 12 is called the least common denominator.

5. When an article is sold at a reduced price, the amount of reduction is called the discount.

6. When you say that $\frac{3}{8} = .375$, you are giving the decimal that is equal to the fraction.

7. When you write the quotient 2.46 as 2.5, you are rounding off the quotient to the nearest tenth.

8. When you write $6 \div \frac{3}{4}$ as $6 \times \frac{4}{3}$, you have inverted the divisor and changed to the process of multiplication.

9. When you write $\frac{1}{4}$ as 25%, you have changed a fraction to a per cent.

10. When you write a whole number that has 10 digits in it, the left-hand digit is in billions place.

11. When you write a decimal that has 6 places, the right-hand digit is in millionths place.

12. The answer to a subtraction example is the remainder or difference; the number from which you subtract is the minuend; the number subtracted is the subtrahend.

13. If a fraction is divided by a whole number, the answer is less than the fraction. If any number is divided by a proper fraction, the answer is more than the number.

Review may be oral or written. If written, have pupils write only number of exercise and the missing word or words. Group pupils who had errors and have them explain answers to determine any need to review meaning of terms.

Teach how to find per cent of increase or decrease (pages 227–229).

Per Cent of Increase and Decrease

1. **Problem** The price of eggs per dozen was 48¢ in June and 60¢ in December. Find the per cent of increase in the price. 25%

   Explanation The difference between 60¢ and 48¢ is 12¢, which is the amount of increase. To find the **per cent of increase,** divide the amount of increase, which is 12¢, by the original price, which is 48¢. This gives $\frac{1}{4}$, or 25%. So the per cent of increase in the price of eggs from June to December was 25%.

2. **Problem** Suppose that the cost of eggs dropped from 50¢ to 42¢. What is the per cent of decrease? 16%

   Explanation The difference between 50¢ and 42¢ gives the amount of decrease, which is 8¢. Then find what per cent 8¢ is of 50¢, the original price. This gives $\frac{8}{50}$, or 16%. The per cent of decrease is 16%.

3. Mrs. Warner's rent was raised from $60 to $70 per month. What was the per cent of increase in the rent? $16\frac{2}{3}\%$

4. The cost of a railroad ticket was raised from $35.60 to $40.94. What was the per cent of increase in the cost? 15%

5. The price of a hat was reduced from $9.00 to $6.75. What was the per cent of decrease in the price of the hat? 25%

6. A book that used to sell at $3.00 now sells at $2.25. What per cent has the price decreased? 25%

7. The population of the United States was about 132 million in 1940 and about 179 million in 1960. Find, to the nearest whole per cent, the per cent of increase in the population during those 20 years. 36%

8. A farmer was able to increase the yield of corn per acre on his farm from an average of 40 bu. to an average of 88 bu. Find the per cent of increase in the yield of corn per acre. 120%

9. One year there were 43 automobile accidents in Fairfield. The next year there were 12 fewer accidents than the year before. Find, to the nearest whole per cent, the per cent of decrease in accidents. 28%

More Practice. See ㊽ on page 340. Use to reinforce skill.

First discuss meaning of increase and decrease. Then follow development as given in ex. 1–2. Emphasize that in finding per cent of increase or decrease we always divide by original number since it is the one increased or decreased.

The Cost of Living

1. Mrs. Hill goes to market once every week to buy supplies. She tries to buy as wisely as possible. Today she found that soup that usually sells at 17¢ a can could be bought at 6 cans for 93¢. If Mrs. Hill bought 6 cans for 93¢, what per cent less did she pay than she would have paid at 17¢ a can? Find the answer to the nearest whole per cent. 9%

2. Mrs. Hill found that the price of a dozen eggs had dropped from 63¢ to 59¢. Find the per cent of decrease in the price of a dozen eggs. 6%

3. There was a special price for steak today. It was 69¢ a pound instead of 79¢, which is the regular price. Mrs. Hill bought a steak weighing 3½ lb. How much less did it cost than it would have at the regular price? 35¢

4. In ex. 3, find the per cent of decrease in the price of 1 lb. of steak. Find the answer to the nearest whole per cent. 13%

5. Many grocery stores have special reduced prices at the end of the week. Look in a newspaper on Thursday or Friday and make a list of 5 articles with their regular price and their reduced price. Then find the per cent of decrease in the price of each article.

Have pupils complete these problems independently. Then discuss solutions with class. Be sure pupils compare amount of decrease with original price. For ex. 5 have pupils make chart showing items, original and reduced prices, per cents of decrease.

When discussing solutions, lead pupils to see that per cent of decrease can never be 100% (unless article is given away), while percent of increase can be more than 100%.

Price Changes

1. The table given below shows the increase or decrease in the prices of food from June to December one year. Using the figures in the table, find the per cent of increase or decrease in the price of each article. If necessary, find the answer to the nearest whole per cent.

Article	June Price	December Price	
1 lb. sugar	8¢	10¢	25% inc.
1 qt. milk	23¢	25¢	9% inc.
1 doz. eggs	53¢	57¢	8% inc.
1 lb. beef	55¢	62¢	13% inc.
5 lb. oranges	49¢	38¢	22% dec.
1 lb. white potatoes	4¢	5¢	25% inc.
Loaf of white bread	19¢	23¢	21% inc.
Chicken per pound	42¢	49¢	$16\frac{2}{3}\%$ inc.

2. The price of strawberries was reduced from 70¢ a quart to 42¢ a quart after there were more berries in the market. Find the per cent of decrease in the price of strawberries. 40%

3. In the summer tomatoes sold at 10¢ per pound. In the winter they sold at 29¢ per pound because they had to be shipped in. Find the per cent of increase in the price. 190%

4. One year at Thanksgiving time turkeys were plentiful and sold at 40¢ a pound. The next year turkeys were less plentiful and sold at 52¢ a pound. What per cent did the price increase? 30% If the price of turkeys drops from 56¢ to 49¢ per pound, what per cent is this drop? $12\frac{1}{2}\%$

5. By buying potatoes and eggs direct from a farmer, Mrs. Adams says that she can save about 10% of the price charged in the stores. Last year she spent about $45 in the stores for potatoes and eggs for her family. About what amount would she save by buying these things from a farmer? $4.50

Present introductory treatment of topic of profit and loss where only simple cases in finding profit, or per cent of profit, are given

Profit and Loss (pages 230–231).

1. Peter bought peanuts to sell at the school ball game and paid $6.25 for them. He sold them all and took in $12.20. The difference between $12.20 and $6.25 is $5.95, which Emphasize.→ seems to be Peter's gain. But Peter had some expenses connected with his sale of the peanuts. He paid $.30 for small paper bags for the peanuts, $1.50 to Joe for helping him, and $.40 for taxi fare to take the peanuts to the ball field. The sum of these **expenses** is $2.20 and this amount must be subtracted from $5.95 to give Peter's actual profit. You see that the profit was really $3.75. $5.95 is called the **gross profit** and includes both the **expenses** and the **profit.** Another name for **expenses** is **overhead,** which means such expenses as rent, heat, light, and wages of clerks.

Selling Price = Cost + Expenses + Profit

Sometimes the cost and the expenses together amount to more than the selling price. In such cases there is no profit and the difference between this amount and the selling price is the **loss.** Emphasize.

2. Jack bought a used radio for $9.50. He paid $1.50 for new tubes and $.10 for a new dial light bulb. Then he sold the radio for $15.00. What was Jack's profit? $3.90

3. Fred bought a dog for $15. He kept the dog for a month and then sold him for $25. During the month he paid $4 for dog food and $.50 to get the dog's hair clipped. Find Fred's expenses and his profit.
$4.50 $5.50

4. Mr. Case has some dresses that he is selling for $25.00 each. From his long experience he knows that 60% of his selling price represents his cost, that 30% of the selling price represents his overhead, and that 10% of the selling price is his profit. On each of these $25 dresses, find the cost, overhead, and profit. $2.50 $15.00
$7.50

5. Mr. Case reduced one of the $25 dresses to $19.50 for his clearance sale. Using the cost and overhead found in ex. 4, find his loss on this dress. $3.00

Use other examples to explain new terms in ex. 1. Be sure pupils understand that profit means actual gain. Have pupils label cost, expenses, selling price, and profit in ex. 2–4. Make up similar problems for class to solve independently.

Finding the Profit

1. Mr. Hall bought 50 baby chicks for $10.50. The expenses of raising the chicks for market amounted to $35. He lost 5 chicks but raised all the rest and sold them for $65.00. What was Mr. Hall's profit? $19.50 What per cent of the selling price was his profit? 30%

2. Mrs. Ward makes cookies to sell. Last month she paid $13.53 for materials for the cookies, $1.78 for cooking gas, $.19 for paper bags, and $2.50 to George for delivering cookies to her customers. She received $24.00 all together from the sale of cookies. How much was Mrs. Ward's profit $6.00 and what per cent of the selling price was the profit? 25%

3. Tom bought 6 bu. of apples at $2.25 per bushel. He sold them at $.75 a peck. He paid $.35 for baskets and bags and $1.00 to John, who helped him. What was Tom's profit? $3.15 What was the per cent of his profit on the selling price? 17½%

Find the profit (1) and the per cent of profit (2) on the selling price:

Article	Cost	Expenses	Selling Price	
(1),(2) **4.** Pair of gloves	$2.75	$1.75	$5.00	$.50, 10%
5. Man's sweater	$6.00	$2.75	$10.00	$1.25, 12½%
6. Wool dress	$9.75	$4.25	$17.50	$3.50, 20%
7. White shoes	$7.75	$3.50	$12.50	$1.25, 10%
8. Wool coat	$18.90	$9.45	$31.50	$3.15, 10%

More Practice. See (49) on page 340.

Emphasize that per cent of profit is found by comparing profit with selling price. Do. ex. 1-2 with class to be sure there are no difficulties. Have pupils complete ex. 3-8. If further teaching is needed, use "More Practice" examples.

Present oral review on percentage.

Oral Practice on Percentage

Tell what per cent each fraction equals:

1. $\frac{1}{10}$ 10% $\frac{1}{6}$ $16\frac{2}{3}\%$ $\frac{3}{4}$ 75% $\frac{7}{8}$ $87\frac{1}{2}\%$ $\frac{5}{6}$ $83\frac{1}{3}\%$ $\frac{1}{8}$ $12\frac{1}{2}\%$ $\frac{3}{5}$ 60% $\frac{1}{3}$ $33\frac{1}{3}\%$ $\frac{5}{8}$ $62\frac{1}{2}\%$ $\frac{1}{5}$ 20%

Tell two other ways to write each of the following:

2. $12\frac{1}{2}\%$ 50% 125% $37\frac{1}{2}\%$ $66\frac{2}{3}\%$ 15% 6% $4\frac{1}{2}\%$
.125, $\frac{1}{8}$ .5, $\frac{1}{2}$ 1.25, $1\frac{1}{4}$ .375, $\frac{3}{8}$ $.66\frac{2}{3}$, $\frac{2}{3}$ .15, $\frac{3}{20}$ .06, $\frac{3}{50}$ .045, $\frac{9}{200}$

Change per cents to decimals or fractions and find the answers:

3. 50% of 24 pupils 12 150% of 24 pupils 36

4. 25% of 24 pupils 6 300% of 24 pupils 72

5. Tell the result if \$40 is increased by 25%; \$50 if \$40 is decreased by 10%; \$36 if 50% more than \$40 is required. \$60

6. John earned \$2 last week and 50% more than that this week. How much did John earn this week? \$3

7. Tom bought a sweater marked \$5 for 20% less than that price. How much did Tom pay for the sweater? \$4

8. There were 800 tickets sold for the school play and 600 of these tickets were sold to pupils. What per cent of the total number were sold to pupils? 75%

9. Mr. Smith said, "I wish I could sell my house for 100% of the cost of it." The house cost Mr. Smith \$15,000. How much did he want to get for it? \$15,000

10. Mr. Carter said, "I wish I could sell my house for 100% more than it cost." If the cost was \$9000, how much did Mr. Carter want to get for the house? \$18,000

11. Joe bought a used trumpet for 25% less than it cost when new. If the trumpet cost \$64 when it was new, how much did Joe pay for it? \$48

12. On a science test, 10% of the pupils had marks of "excellent," 20% of them had marks of "good," 45% of them had marks of "fair," and the rest of them did not pass. What per cent of the pupils failed the test? 25%

If many pupils seem weak on particular part of this topic, reteaching may be needed for whole class. Use developmental material in text relating to pupils' difficulties. Try to clear up all difficulties now.

Present set of improvement tests in subtraction.

Improving by Practice

Subtraction Test 5a.

Time: 4 min.

1.	$529.36	$551.33	$858.50	$498.06	
	191.53	133.68	98.51	133.57	
	$337.83	$417.65	$759.99	$364.49	
2.	$668.01	$709.13	$663.26	$922.48	
	479.42	129.24	205.79	843.02	
	$188.59	$579.89	$457.47	$79.46	
3.	$320.75	$714.60	$784.12	$759.01	
	37.76	307.94	201.76	734.22	⑫
	$282.99	$406.66	$582.36	$24.79	

Subtraction Test 5b.

Time: 4 min.

4.	$315.19	$847.47	$514.34	$784.08	
	167.64	469.58	452.85	129.92	
	$147.55	$377.89	$61.49	$654.16	
5.	$756.79	$827.54	$991.40	$823.06	
	604.41	76.69	693.37	699.50	
	$152.38	$750.85	$298.03	$123.56	
6.	$198.16	$476.61	$952.83	$237.22	
	167.98	128.94	16.96	104.39	⑫
	$30.18	$347.67	$935.87	$132.83	

Subtraction Test 5c.

Time: 4 min.

7.	$501.93	$931.76	$818.40	$552.02	
	400.89	396.26	495.94	295.00	
	$101.04	$535.50	$322.46	$257.02	
8.	$597.13	$544.40	$630.14	$656.96	
	70.52	114.68	209.78	375.27	
	$526.61	$429.72	$420.36	$281.69	
9.	$807.47	$558.90	$803.28	$792.03	
	40.36	462.79	366.99	451.15	⑫
	$767.11	$96.11	$436.29	$340.88	

To the Teacher. If individual pupils get low scores on Improvement Tests, such pupils may be assigned extra practice given on pages 325–328.

Check papers carefully and analyze errors. Have pupils record scores on graphs. Try to determine causes of errors through individual conferences with pupils. Assign practice after difficulties are cleared up.

Present review of problem solving. Have pupils who did poorly explain their solutions so you may determine if problem situations, vocabulary, carelessness, and so on, caused errors.

Mixed Problems

1. Jack worked from 3:15 P.M. to 5:45 P.M. every day from Monday through Friday and from 8:30 A.M. to 1:00 P.M. on Saturday. He received 60¢ an hour for his work. How much did Jack earn in one week? $10.20

2. If Jack worked for 8 weeks during the spring and earned the same amount each week, what was the amount of his earnings? See ex. 1. $81.60

3. Martha bought 12 packages of seeds for her garden. For 3 packages she paid 20¢ each, for 5 packages she paid 10¢ each, and for the other packages she paid 15¢ each. Find the total cost of Martha's seeds. $1.70

4. In a period of five years the value of a building lot increased from $10,000 to $16,500. Find the per cent of increase in the value of this lot. 65%

5. Peter bought 500 stamps for his collection. He got 2 packages containing 100 stamps each at 50¢ a package, 2 packages containing 75 stamps each at 65¢ a package, and the rest in packages of 25 stamps each at 35¢ a package. How much did Peter pay for all the stamps? $4.40

6. From February through June Dick wants to make an average mark of 90 in arithmetic. In February his mark was 93, in March it was 86, in April it was 88, and in May it was 92. What mark must he get in June so that his average mark for the 5 months will be 90? 91

7. Which chair costs less money, one marked $58.50 and reduced $33\frac{1}{3}\%$ or one marked $50.00 and reduced $12\frac{1}{2}\%$? The first How much difference is there between the two sale prices? $4.75

8. A speed boat traveled 40 mi. in 1 hr. 6 min. Find, to the nearest tenth of a mile, the speed of the boat in miles per hour. (First change 1 hr. 6 min. to 1.1 hr.) 36.4 mph

Present written review of percentage.

Review of Percentage

Write these fractions and decimals as per cents:

1. $\frac{3}{4}$ 75% $\frac{2}{3}$ 66$\frac{2}{3}$% $\frac{7}{8}$ 87$\frac{1}{2}$% $\frac{4}{5}$ 80%   .15 15%   .6 60%   .09$\frac{1}{2}$ 9$\frac{1}{2}$%   2.73 273%

Write these per cents as decimals or whole numbers:

2. 17% .17   6% .06   12$\frac{1}{2}$% .125   245% 2.45   600% 6   130% 1.3

Find the answers:

3. 15% of $38 $5.70   2% of $150 $3.00   100% of $2450 $2450

4. 25% of $93 $23.25   6% of $325 $19.50   250% of $1500 $3750

5. 4$\frac{1}{2}$% of $46 $2.07   12$\frac{1}{2}$% of $472 $59   237% of $4320 $10,238.40

Find what per cent the first number is of the second. First find what part the first number is of the second and change the fraction to a per cent:

6. 12, 24 50%   8, 64 12$\frac{1}{2}$%   10, 30 33$\frac{1}{3}$%   27, 36 75%   68, 17 400%

7. 10, 15 66$\frac{2}{3}$%   5, 25 20%   15, 40 37$\frac{1}{2}$%   65, 39 166$\frac{2}{3}$%   16, 96 16$\frac{2}{3}$%

Find the answers to the nearest cent or nearest whole per cent:

8. 13% of $3.75 $.49   2% of $20.83 $.42   5 is what per cent of 17? 29%

9. 75% of $8.75 $6.56   7% of $47.72 $3.34   31 is what per cent of 38? 82%

10. 23% of $9.49 $2.18   6% of $74.46 $4.47   12 is what per cent of 35? 34%

11. A pair of skis marked $10.95 is sold at a discount of 33$\frac{1}{3}$%. Find the discount and net price.
$3.65 $7.30

12. A dealer bought a tennis racket for $5.95 and sold it for $10.00. His overhead expenses on this racket were $2.80. $1.25 Find the amount of his profit. Also find what per cent his profit was of the selling price. 12$\frac{1}{2}$%

13. John sold magazine subscriptions amounting to $120 and was allowed to keep 30% of this amount as his commission. What was his commission? $36

Use review as preparation for chapter review and diagnostic test. Check papers carefully and analyze errors. Help pupils to discover causes. Plan for remedial work as needed.

Present summary and review of work of Chapter 7.

Chapter Review

Find what part the first number is of the second:

1. 4, 16 $\frac{1}{4}$ 24, 32 $\frac{3}{4}$ 50, 100 $\frac{1}{2}$ 30, 36 $\frac{5}{6}$ 75, 100 $\frac{3}{4}$

2. 8, 12 $\frac{2}{3}$ 40, 50 $\frac{4}{5}$ 75, 225 $\frac{1}{3}$ 36, 90 $\frac{2}{5}$ 300, 450 $\frac{2}{3}$

Find what per cent the first number is of the second. In ex. 5, find answers to the nearest whole per cent:

3. 9, 12 75% 40, 20 200% 85, 125 68% 60, 60 100% 196, 112 175%

4. 9, 80 $11\frac{1}{4}$% 21, 56 $37\frac{1}{2}$% 45, 135 $33\frac{1}{3}$% 80, 64 125% 118, 800 $14\frac{3}{4}$%

5. 3, 13 23% 35, 93 38% 31, 127 24% 71, 81 88% 369, 115 321%

Tell what per cents are equal to these fractions:

6. $\frac{1}{4}$ 25% $\frac{9}{10}$ 90% $\frac{5}{8}$ $62\frac{1}{2}$% $\frac{1}{2}$ 50% $\frac{7}{16}$ $43\frac{3}{4}$ % $\frac{2}{3}$ $66\frac{2}{3}$% $\frac{5}{6}$ $83\frac{1}{3}$% $\frac{7}{20}$ 35% $\frac{1}{8}$ $12\frac{1}{2}$%

7. 1 ft. 6 in. is what part of a yard? $\frac{1}{2}$

8. 1 lb. 4 oz. is what part of 5 lb.? $\frac{1}{4}$

9. The cost of a pound of butter was increased from 56¢ to 84¢. Find the per cent of increase in the price. 50%

10. George earned $21.75 by selling magazine subscriptions totaling $72.50. Find the per cent of George's commission. 30%

11. Mr. Thomas sold a piano for $875 and made a profit of $125. What per cent of the selling price was his profit? Give the answer to the nearest whole per cent. 14%

12. Betty's present speed in typewriting is about 50 words a minute. When Betty first began to type, her speed was about 10 words a minute. Her present speed is about what per cent of her first speed? 500%

13. A trip of 3000 mi. is how many times as far as a trip of 125 mi.? 24 as a trip of 375 mi.? 8

14. The selling price of an expensive ring is $900. The ring cost the jeweler $485 and he allowed 12% of the selling price to cover his expenses. Find his profit. $307

Use review as preparation for diagnostic test and plan for reteaching as needed. Have pupils do computation aloud so you may determine extent of understanding.

A Problem Test

1. Mrs. Wood can buy a can of tomato juice for 15¢ or 6 cans for 75¢. If she can use 6 cans, how much will she save by buying them all at one time? 15¢

2. A train travels 42.9 mi. in $\frac{3}{4}$ hr. Find the speed of the train in miles per hour. 57.2

3. Mr. Lee sent a telegram of 19 words. It cost $1.20 for the first 15 words and $5\frac{1}{2}$¢ for each extra word. The Government tax was 10%. Find the cost of this telegram. $1.56

4. Mrs. Case can buy a sewing machine by paying $25 down and $6.75 a month for 12 months. How much will she pay in all for this machine? $106

5. Today Grace worked correctly 18 problems out of 20. Yesterday she worked correctly 9 problems out of 12. On which day was her mark better and how much better? Today, by 15%

6. If Bob can buy 100 Christmas cards for $3.75 and sell them all at 5¢ each, how much money will he make, provided he has no expenses? $1.25

7. The selling price of a book is $3.00. The book cost $1.75 and the expenses in connection with its sale are $.80. Find the profit $.45 and the per cent of profit on the selling price. 15%

8. Mr. Hunt left Greenfield at 9 A.M. for a trip of 305 mi. to Plum Beach. For the first $2\frac{1}{2}$ hr. he drove at the rate of 36 mi. per hour. Then he stopped $1\frac{1}{2}$ hr. for lunch. At what rate per hour did he drive for the rest of the trip if he reached Plum Beach at 6 P.M.? 43 mi.

9. How much more will our baseball club have to spend for 9 uniforms at $7.95 each than for 9 at $7.39 each? $5.04

10. If 4 oz. of cheese cost $.15, find the cost of 1 lb.? $.60

Have pupils record scores on graphs.

SCORE	0–5	6–7	8–9	10
	You need help	Fair	Good	Excellent

Hold individual conferences with pupils who had errors to ascertain reasons for them. Have the pupil explain his solution so you may determine if lack of information or understanding, incorrect computations, and so on, cause difficulties.

Present diagnostic test of skills taught in Chapter 7.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Practice Pages

Write as per cents:

1. 1.38 138% 4.25 425% 7 700% 11.73 1173% $1\frac{3}{4}$ 175% 224

Write as decimals or as whole numbers:

2. 300% 3 412% 4.12 100% 1 1550% 15.5 224

Find the answers:

3. 28 is what part of 84? $\frac{1}{3}$ of 32? $\frac{7}{8}$ of 40? $\frac{7}{10}$ 211, 212

Tell the numbers to put in the spaces. In ex. 6, find the nearest whole per cent:

4. 39 is . 75 .% of 52 39 is . $48\frac{3}{4}$.% of 80 214–216

5. 15 is . 125 .% of 12 80 is . 250 .% of 32 224

6. 11 is . 73 .% of 15 27 is . 51 .% of 53 219

Find the per cent of increase (inc.) or decrease (dec.) that the second number shows over the first number:

7. \$15, \$20 $33\frac{1}{3}$% inc \$200, \$400 100% inc. \$396, \$297 25% dec 227

8. \$30, \$18 40% dec \$1.50, \$1.75 $16\frac{2}{3}$% inc. \$4.80, \$6.60 $37\frac{1}{2}$% inc 227

The first number is the profit; the second is the selling price. What per cent is the profit of the selling price?

9. \$7, \$50 14% \$25, \$250 10% \$12, \$96 $12\frac{1}{2}$% 231

10. \$3, \$20 15% \$1.12, \$5.60 20% \$9.90, \$90 11% 231

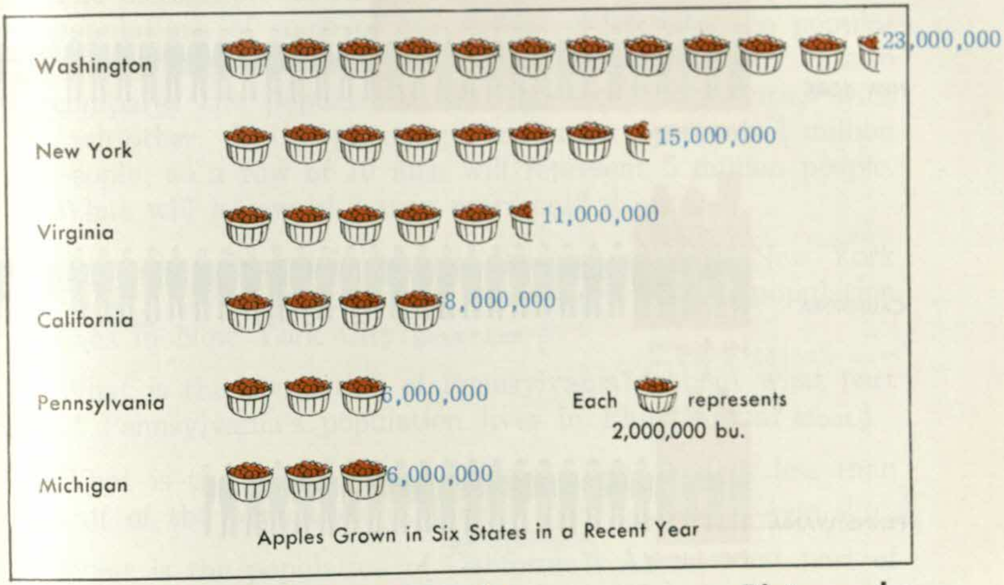
The first number is the marked price; the second is the reduced price. Find the per cent of discount:

11. \$9.36, \$7.02 25% \$6.50, \$4.55 30% \$82, \$41 50% 218

Compare results of test with previous reviews (pages 235–236). Plan for reteaching (individual or class basis) as needed before assigning practice exercises.

See Guide for specific aims of Chapter 8.

Chapter 8



Apples Grown in Six States in a Recent Year

Reteach reading of simple pictographs.

Pictographs

1. It is often effective to make comparisons by means of a **graph.** In graphs, quantities are represented by pictures or lines so that you can actually **see** the relationships.

2. One kind of graph, called a **pictograph,** is shown above. This pictograph compares apple production in six states in a recent year. Each symbol in the graph represents 2,000,000 bushels of apples. How many bushels of apples are represented by 4 symbols? 8,000,000 by 10 symbols? 20,000,000 by a half-symbol? 1,000,000 by 3½ symbols? 7,000,000

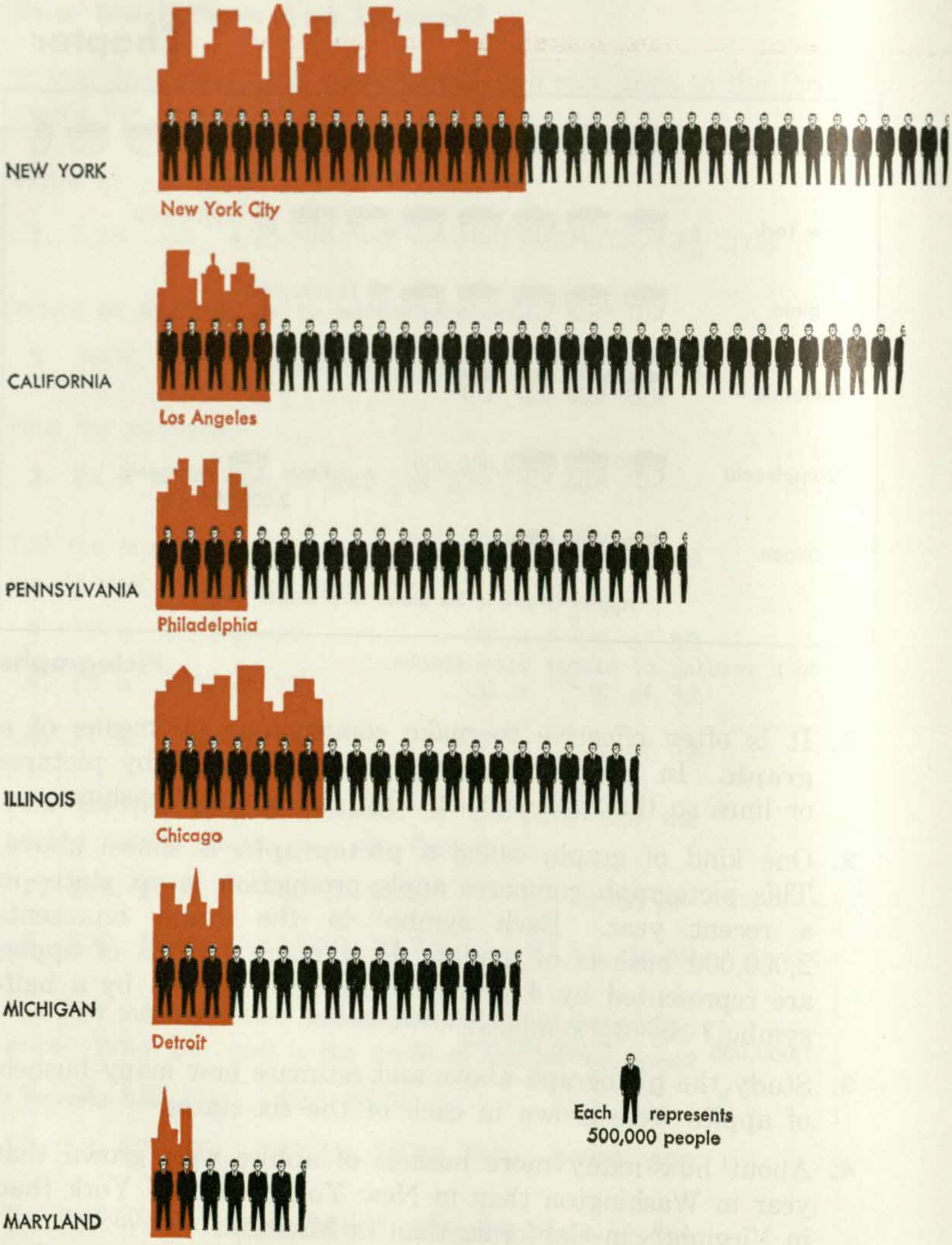
3. Study the pictograph above and estimate how many bushels of apples were grown in each of the six states. See answers on graph.

4. About how many more bushels of apples were grown that year in Washington than in New York? 8,000,000 in New York than in Virginia? 4,000,000 in California than in Michigan? 2,000,000

5. About how many times as many apples were grown in Washington as in Michigan? 4

First discuss importance and use of graphs in general. Then discuss meaning of pictograph and point out that it uses pictures to represent number facts. When discussing ex. 2, ask pupils what $\frac{1}{2}$ basket represents.

Extend topic of pictographs to more difficult cases (pages 240-241).



Populations of Our Largest Cities and Populations of Their States, 1960 Census

Discuss pictograph with the class, pointing out scale, title, and so on. Be sure they understand that total line of men represents population of the state, and the men enclosed in red picture represent population of the city.

1. The pictograph shown on the page at the left compares the populations of some of our largest cities with the populations of the states in which these cities are located. It also compares the populations of these states and cities with each other. In this pictograph each man represents $\frac{1}{2}$ million people; so a row of 10 men will represent 5 million people. What will a row of 7 men represent? $3\frac{1}{2}$ million

2. From the pictograph, what is the population of New York State? $16\frac{3}{4}$ million About what part of New York State's population lives in New York City? Less than $\frac{1}{2}$

3. What is the population of Pennsylvania? $11\frac{1}{2}$ million About what part of Pennsylvania's population lives in Philadelphia? About $\frac{1}{6}$

4. What is the population of Illinois? Does half or less than half of the population of Illinois live in Chicago? Less than $\frac{1}{2}$

5. What is the population of California? $15\frac{3}{4}$ million About what part of that population lives in Los Angeles? About $\frac{1}{6}$

6. What is the population of Detroit? $1\frac{1}{2}$ million of the entire state of Michigan? $7\frac{3}{4}$ million Compare these two populations. Detroit has about $\frac{1}{5}$ of Michigan's population.

7. The population of New York State is how many times that of Maryland? About $5\frac{1}{6}$ times

8. The population of New York City is how many times that of Detroit? $5\frac{1}{6}$ times

9. The numbers of bushels of sweet potatoes grown in four states in a recent year were: Louisiana, 8,800,000 bu.; North Carolina, 4,000,000 bu.; New Jersey, 3,000,000 bu.; Virginia, 2,800,000 bu. Make a pictograph to show these amounts. What will you use as a symbol? Give your graph a title and also tell how many bushels of sweet potatoes each symbol represents. Stress. See Guide.

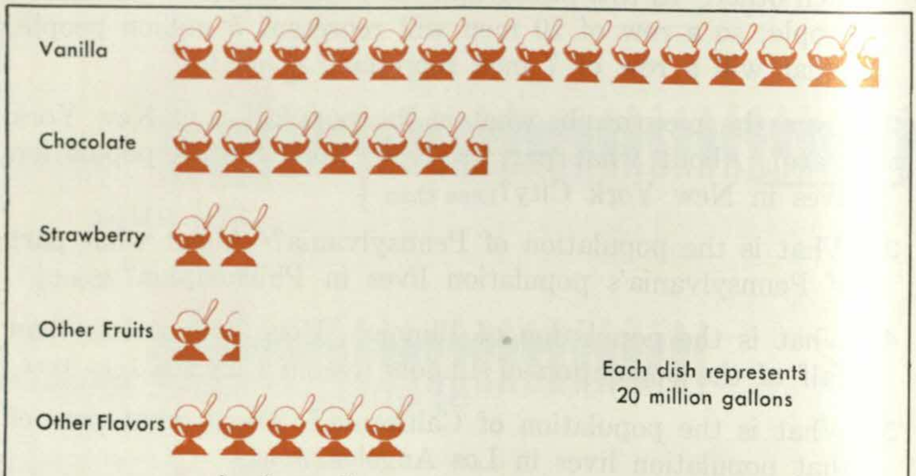
10. Make a pictograph to show the numbers of telephones in these cities: New York, 4,400,000; Los Angeles, 1,100,000; Chicago, 1,900,000; Cleveland, 700,000. See Guide.

Do ex. 1–3 with class. Then have them complete ex. 4–8. Note that fractions are to be given approximately. Have pupils also compare populations of other states and cities, not included in ex. 2–8.

Give more practice in reading and constructing a pictograph. Have pupils bring to class pictographs from newspapers or magazines, which they can explain to class and post on bulletin board.

Which Flavor Do You Prefer?

The pictograph below shows how much ice cream of each flavor was consumed in the United States in a recent year. Each dish of ice cream represents 20 million gallons; a half-dish represents 10 million gallons.



Ice Cream Consumption in the United States

1. Which flavor had the greatest consumption? Vanilla the next greatest consumption? Chocolate

2. How many gallons of vanilla ice cream were consumed that year? 290 million How many gallons of chocolate ice cream? 120 million How many gallons of strawberry? 40 million of other fruit flavors? 30 million of all other flavors? 100 million How many gallons in all were consumed? 590 million

3. About how many times as much vanilla as chocolate ice cream was eaten? 2 How many times as much vanilla as strawberry was eaten? 7 What part of the total consumption was vanilla ice cream? About $\frac{1}{2}$

4. Make a pictograph showing the amount of ice cream consumed if the following numbers of gallons were eaten: vanilla, 240 million; chocolate, 120 million; strawberry, 60 million; other fruit flavors, 40 million; all other flavors, 110 million. Let each dish represent 20 million gallons. See Guide.

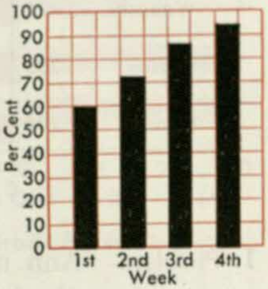
Ex. 1-3 may be done orally with class. Be sure all understand that the number of pictures must be counted and interpreted, using the scale, in order to find the number of gallons consumed.

Review and extend the reading and making of bar graphs. First review types of bar graphs (horizontal and vertical), and meaning and use of each.

Making Bar Graphs

1. **Problem** One month Jack got these marks on his arithmetic tests: first week, 60%; second week, 73%; third week, 87%; fourth week, 95%. Miss Hall asked Jack to make a bar graph to show his improvement. Emphasize that main purpose of graph is to provide easy way of comparing Jack's marks.

Explanation Jack found that if he let 1 square represent 10%, then 6 squares would represent 60%. To represent 73%, Miss Hall told him to use 7 whole squares to represent 70 and $\frac{3}{10}$ of the next square to represent 3. Stress. To show $\frac{3}{10}$ of a square, she told him to estimate it as closely as he could with his eye. To show 87%, Jack used 8 squares and $\frac{7}{10}$ of the next square; to show 95%, he used $9\frac{5}{10}$ squares, or $9\frac{1}{2}$ squares.



Squares: $7\frac{2}{5}$ $8\frac{3}{5}$ $7\frac{9}{10}$ $9\frac{4}{5}$ $8\frac{1}{5}$

2. How would Jack represent 74%? 86%? 79%? 98%? 82%?

3. Draw a bar graph to show Dick's marks in English for one week if he made 80% on Monday; 75% on Tuesday; 90% on Wednesday; 88% on Thursday; and 96% on Friday. See Guide.

4. Next month keep a list of your weekly marks in arithmetic, or in some other subject, and then make a bar graph to show whether your marks improved.

Draw bar graphs to show the following facts: See Guide.

5. In a track meet the competing schools made these points: Maple, 70; Brown, 85; Park, 90; Wells, 63; and Central, 57.

6. In the seventh grades of Westover Junior High School, the membership in each club is as follows: music, 68; dramatic, 47; radio, 43; girls' athletic, 80; boys' athletic, 85.

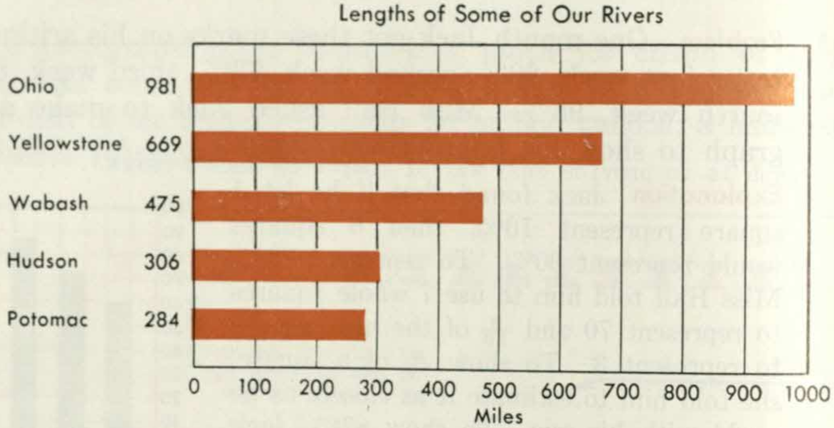
7. In a recent year the United States produced these per cents of the total world output of certain minerals: sulphur, 91%; petroleum, 52%; coal, 27%; copper, 33%; aluminum, 43%.

8. Try to find some bar graphs in newspapers or magazines and bring them to class. Have pupils interpret them.

In ex. 1 be sure pupils understand why 3 is represented as $\frac{3}{10}$ of next square. Discuss scale used and reasons for choice. Point out that it is not possible to determine exact values from graphs, only approximations.

Review and extend making of bar graphs with larger numbers.

Graphs with Larger Numbers



1. **Problem** Ann made a bar graph like the one above, which compares the lengths of some of our rivers. Read the graph and tell the length of each river. In this graph the marks and numbers at the bottom of the graph are called the **scale**; on this scale, each space represents 100. Contrast this scale with the one used on page 243, as to reasons for its use.

Explanation To show that the Ohio River is 981 mi. long, Ann used 9 spaces to represent 900 and .81 of the next space to represent 81. Since she cannot estimate .81 accurately by eye, she rounded off .81 to the nearest tenth, which gives .8. Then she made the end of the bar cover .8 of the last space. So the bar for 981 is 9.8 spaces long. A quick way to find that 981 is represented by 9.8 spaces is to divide 981 by 100, which gives 9.81, and then to round off 9.81 to the nearest tenth, which gives 9.8.

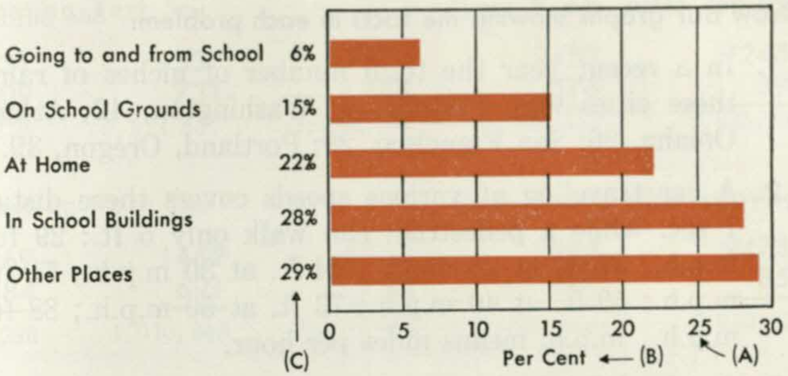
To represent 669 mi., the length of the Yellowstone River, Ann used 6.7 spaces, which she found as follows: She divided 669 by 100, which gave 6.69; then she rounded off 6.69 to the nearest tenth, which gave 6.7. To represent the length of the Wabash River, which is 475 mi., Ann used 4.8 spaces; show how she got 4.8. The bar showing the length of the Hudson River covers 3.1 spaces, while that for the Potomac covers 2.8 spaces. Show how Ann got 3.1 and 2.8. Be sure all pupils understand how to represent various lengths.

2. Draw a bar graph showing the number of pupils enrolled in the six schools in Johnstown: Lincoln, 475; Allen, 856; Knight, 452; Lake, 199; Harris, 706; Clark, 567. Follow the directions given above. Let 1 space = 100. See Guide.

When discussing ex. 1, point out that each bar is so drawn that its length is approximately correct. Also stress that the main reason for drawing a bar graph (even though lengths are given) is to make easy a comparison of lengths of rivers.

Provide some general directions for constructing bar graphs. Use other graphs from preceding pages or from newspapers to further illustrate these directions.

Hints on Making Graphs



Where Accidents Happen to Seventh Grade Pupils

1. Decide upon the scale, letting each space represent a number of convenient size. To represent numbers up to 100, try 10 for each space; for numbers up to 1000, try 100 for each space. For very large numbers, each space may represent 1000 or even 1,000,000. Give pupils practice in deciding on appropriate scale for various data, supplied by you.
2. Write the scale numbers directly opposite the lines that separate the spaces, as shown at A.
3. Under the scale, or at the side of it, write a statement telling what the scale numbers represent, as shown at B.
4. Bars should be arranged in some definite order. If the bars are not arranged in order of time, arrange them in order of length. Study the graph above.
5. Make all the bars the same width. It is good practice to make the spaces between the bars the same width as the bars themselves. Leave a space above and below the bars, if they are horizontal; or before and after, if they are vertical.
6. The number a bar represents is sometimes shown at the left end of the bar, as shown at C, if the bars are horizontal, or below the bar if they are vertical.
7. The graph should have a title as shown above.
8. Graphs should always be neat and attractive.

When number is to be represented approximately, instruct pupils to divide given number of value of each space and round off result correct to nearest tenth (if each space represents 1000, 9482 is represented by $\frac{9482}{1000}$ = 9.482 or 9.5 spaces).

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Give practice on making bar graphs.

Drawing Bar Graphs

Draw bar graphs showing the facts in each problem: See Guide.

1. In a recent year the total number of inches of rainfall for these cities was: Phoenix, 8; Washington, 42; Atlanta, 48; Omaha, 26; San Francisco, 29; Portland, Oregon, 39.

2. A car traveling at various speeds covers these distances in 1 sec. while a pedestrian can walk only 5 ft.: 29 ft. at 20 m.p.h.; 37 ft. at 25 m.p.h.; 44 ft. at 30 m.p.h.; 51 ft. at 35 m.p.h.; 59 ft. at 40 m.p.h.; 73 ft. at 50 m.p.h.; 88 ft. at 60 m.p.h. m.p.h. means miles per hour.

3. A school lunchroom asked the pupils to tell their favorite dessert. The result was as follows: ice cream, 722; cake, 580; fruit, 450; pudding, 376; pie, 257. Let 1 space = 100.

4. A college reported that these numbers of students were preparing for these occupations during a certain year: business, 987; law, 526; medicine, 416; and teaching, 172.

5. The number of miles of railroad in each of these states is: Ohio, 8419; New York, 7363; Georgia, 6017; Montana, 5038; Colorado, 4016. Let 1 space = 1000.

▶ To find the number of spaces to represent 8419, divide 8419 by 1000, which gives 8.419, and round off to the nearest tenth, which gives 8.4 spaces. Let each space be about $\frac{1}{2}$ in. long.

6. The altitudes of these cities are: Denver, 5221 ft.; Salt Lake, 4222 ft.; Pittsburgh, 1248 ft.; El Paso, 3710 ft.; Cheyenne, 6139 ft.; Dodge City, 2522 ft. Let 1 space = 1000.

7. The number of miles of rural delivery routes in these states recently was: Vermont, 8393; Connecticut, 8426; Montana, 7601; New Mexico, 3613; New Jersey, 9185.

8. The population of Washington, D. C., was 189,000 in 1890; 279,000 in 1900; 331,000 in 1910; 438,000 in 1920; 487,000 in 1930; 663,000 in 1940; 802,000 in 1950; 764,000 in 1960.

▶ Tell which of these values would be most convenient for each space on the scale: 10,000, 50,000, or 100,000. 100,000

Instruct pupils to follow directions as given on page 245. Have some of these graphs made in class so you may check pupils' work and give further help as needed (particularly in choosing scale).

Present sets of improvement tests in multiplication and division.

Improving by Practice

Multiplication Test 5a. Time: 5 min. after copying.

1. 8631	9537	2503	9452	7245	
609	731	482	576	390	⑤
5,256,279	6,971,547	1,206,446	5,444,352	2,825,550	

Multiplication Test 5b. Time: 5 min. after copying.

2. 8769	1462	8736	1268	9473	
384	691	725	407	850	⑤
3,367,296	1,010,242	6,333,600	516,076	8,052,050	

Multiplication Test 5c. Time: 5 min. after copying.

3. 4105	1896	9875	4973	6254	
437	681	209	564	720	⑤
1,793,885	1,291,176	2,063,875	2,804,772	4,502,880	

Division Test 5a. Time: 6 min. after copying.

4. 623)26860 43.1   782)45747 58.5

5. 475)42085 88.6   531)34921 65.8 ④

Division Test 5b. Time: 6 min. after copying.

6. 871)63783 73.2   426)22791 53.5

7. 953)25786 27.1   795)31164 39.2 ④

Division Test 5c. Time: 6 min. after copying.

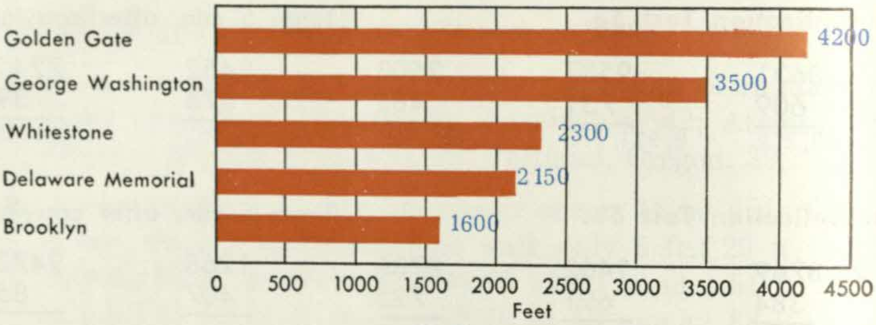
8. 345)18906 54.8   712)25872 36.3

9. 675)29272 43.4   596)15794 26.5 ④

To the Teacher. In ex. 4–9, instruct pupils to find quotients to nearest tenth. Instruct pupils to copy carefully and do all work neatly and legibly. After checking papers and analyzing errors, have pupils determine own scores and record them. Use "More Practice" pages for remedial work as needed.

Give pupils practice in reading bar graphs with very large numbers.

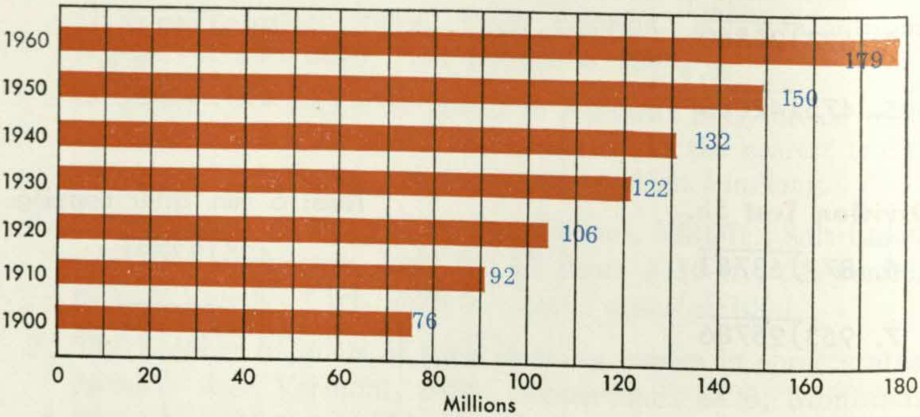
Reading Graphs



Lengths of Some Famous Bridges

Emphasize.

1. In reading graphs, your results must be approximate. Study the scale carefully to find the value of each space. What does each space equal in the above graph? 500 ft. About how many feet long is each bridge? See answers on graph. The Golden Gate Bridge is 4200 ft. long; on the first bar, what part of the last space was used to represent 200 ft.? $\frac{2}{5}$ Would the first bar be the same length if the bridge were 4215 ft. long? Yes Would it be the same length if the bridge were 4189 ft. long? Yes



Population of the United States, 1900-1960

2. In the second graph, only every other space is numbered; what is the value of each space? 10 million Tell what number might have been used between 0 and 20. 10 Note that 20 on the scale means 20,000,000. Tell the approximate population of the United States for each year shown. See answers on graph.

Instruct pupils to give answers to nearest 100 or nearest 50 in ex. 1. In ex. 2 suggest that answers be given in millions. Although only one answer is given for each bar, you may accept any other answer that is approximately correct.

Present review of fractions and general problem solving (pages 249-250).

Fraction Review

1. John started to walk $3\frac{1}{4}$ mi. to the airport. He had gone $\frac{3}{4}$ mi. when a farmer gave him a ride to Mayfield, which is only $\frac{3}{8}$ mi. from the airport. How many miles did John ride with the farmer? $2\frac{1}{8}$

2. Last Saturday a group of Camp Fire Girls went on a hike to Bald Mountain. They walked $2\frac{3}{4}$ mi. to Eagle Lake, $1\frac{1}{2}$ mi. more to the trail up Bald Mountain, and $1\frac{1}{4}$ mi. up the trail to the top. How many miles in all did they walk to get to the top of the mountain? $5\frac{1}{2}$

3. On a map the airline distance between two cities is $1\frac{3}{8}$ in. How many miles apart are these cities if the scale of the map is $\frac{1}{4}$ in. = 10 mi.? 55

Add. Check by going over the work:

4.	$7\frac{3}{16}$	$1\frac{4}{5}$	$6\frac{1}{2}$	$2\frac{1}{3}$	$3\frac{5}{12}$	$1\frac{1}{6}$	$8\frac{5}{6}$
	$2\frac{7}{8}$	$3\frac{1}{2}$	$8\frac{3}{8}$	$2\frac{1}{2}$	$1\frac{5}{12}$	$5\frac{1}{3}$	$2\frac{5}{6}$
	$4\frac{1}{16}$	$1\frac{3}{10}$	$4\frac{1}{4}$	$5\frac{1}{3}$	$6\frac{5}{12}$	$4\frac{1}{4}$	$6\frac{1}{3}$
	$14\frac{1}{8}$	$6\frac{3}{5}$	$19\frac{1}{8}$	$10\frac{1}{6}$	$11\frac{1}{4}$	$10\frac{3}{4}$	18

Subtract. Check by going over the work:

5.	$9\frac{7}{10}$	$5\frac{5}{12}$	$8\frac{1}{2}$	$6\frac{7}{8}$	$7\frac{3}{8}$	$5\frac{3}{4}$	$4\frac{4}{5}$
	$4\frac{3}{10}$	$2\frac{7}{8}$	$1\frac{2}{3}$	$3\frac{3}{4}$	$2\frac{9}{16}$	$3\frac{1}{4}$	$2\frac{1}{2}$
	$5\frac{2}{5}$	$2\frac{13}{24}$	$6\frac{5}{6}$	$3\frac{1}{8}$	$4\frac{13}{16}$	$2\frac{1}{2}$	$2\frac{3}{10}$
6.	$2\frac{11}{12}$	$4\frac{9}{16}$	$6\frac{5}{6}$	$8\frac{1}{5}$	$3\frac{9}{10}$	$8\frac{1}{4}$	$9\frac{5}{6}$
	$1\frac{3}{4}$	$3\frac{3}{4}$	$1\frac{1}{2}$	$4\frac{1}{3}$	$2\frac{1}{5}$	$6\frac{5}{6}$	$3\frac{1}{6}$
	$1\frac{1}{6}$	$\frac{13}{16}$	$5\frac{1}{3}$	$3\frac{13}{15}$	$1\frac{7}{10}$	$1\frac{5}{12}$	$6\frac{2}{3}$

Multiply or divide. Check the work:

7. $5 \div \frac{2}{3}$ $7\frac{1}{2}$ $2\frac{1}{4} \times 3\frac{1}{3}$ $7\frac{1}{2}$ $6\frac{1}{4} \div 2\frac{1}{2}$ $2\frac{1}{2}$ $27 \times 1\frac{5}{6}$ $49\frac{1}{2}$

8. $\frac{5}{6} \div \frac{2}{3}$ $1\frac{1}{4}$ $7\frac{1}{2} \div 10$ $\frac{3}{4}$ $\frac{9}{10} \times 4\frac{1}{6}$ $3\frac{3}{4}$ $8\frac{1}{4} \div \frac{5}{12}$ $19\frac{4}{5}$

9. $\frac{5}{8} \div 2$ $\frac{5}{16}$ $2\frac{2}{5} \times \frac{5}{12}$ 1 $2\frac{1}{2} \times 2\frac{1}{2}$ $6\frac{1}{4}$ $20 \div 3\frac{3}{4}$ $5\frac{1}{3}$

10. $\frac{7}{8} \times \frac{4}{5}$ $\frac{7}{10}$ $15 \times \frac{4}{5}$ 12 $\frac{9}{16} \div 1\frac{7}{8}$ $\frac{3}{10}$ $3\frac{3}{4} \times 2\frac{2}{3}$ 10

Check papers carefully to determine kinds of errors made. Return them to pupils so they may find and correct errors. Plan for further specialized review or reteaching as needed.

Mixed Problems

1. Mary is trying to save an average of \$2.75 per month. Sometimes she saves more and sometimes less. For the first 5 mo. of the year she saved these amounts: \$2.75, \$3.00, \$2.25, \$2.10, \$2.95. How much should she save the sixth month to keep an average of \$2.75 per month? \$3.45

2. When driving on a new highway Miss Young timed herself and found that she went 29.4 mi. in $\frac{3}{4}$ hr. At this rate, how far would she go in 1 hr.? 39.2 mi.

3. A candy bar weighing $\frac{1}{8}$ lb. sells for 5¢. How much does 1 lb. of these candy bars cost? 40¢

4. The tank on Mr. Hatfield's car holds 16 gal. of gasoline. On the average, his car uses 1 gal. of gasoline to go 17.5 mi. How many miles can he expect to drive his car on a full tank of gasoline? 280

5. Last year Mr. Way picked only 30 bu. of apples in his orchard. This year he picked 195 bu. of apples. This year's crop is how many times as large as last year's? $6\frac{1}{2}$ This year's crop is what per cent of last year's? 650%

6. Find the cost of $\frac{3}{4}$ lb. of meat at 50¢ a pound. 38¢

7. Sally's school year lasted 185 days. What part of the whole year (365 da.) was the school year? $\frac{37}{73}$ What per cent of the year, to the nearest whole per cent, is this? 51%

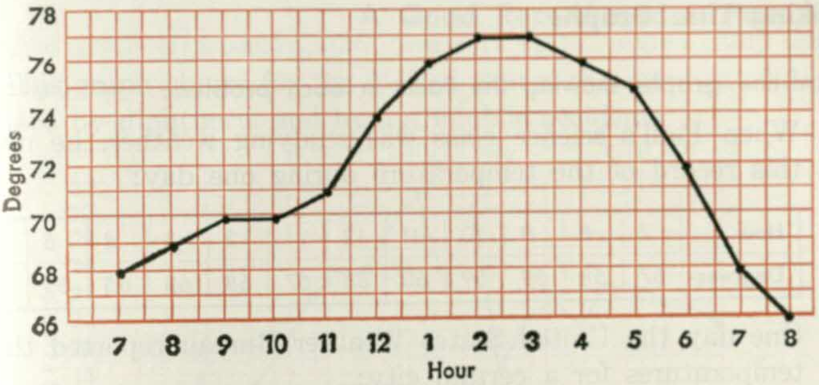
8. Bill went on a hike of $3\frac{1}{2}$ mi. He walked that distance in 70 min. How long did it take him, on the average, to walk 1 mi.? 20 min.

9. From Hudson to Mayfield is 72 mi. and from Hudson to Lakeside is 378 mi. Lakeside is how many times as far from Hudson as Mayfield? $5\frac{1}{4}$

10. Susan addressed 500 envelopes for her father to use for advertising material. She found that she could average 5 envelopes in 2 min. About how many hours and minutes did she spend addressing envelopes? 3 hr. 20 min.

Have pupils who had errors explain their solutions to the problems. This will enable you to determine if language, problem situations, computations, and so on, cause difficulty. Set up individual remedial programs as needed.

Review reading of simple line graphs.



Temperatures from 7 AM to 8 PM

Line Graphs

1. The picture above, which is called a **line graph,** shows the temperature for each hour of the day from 7 A.M. to 8 P.M. The numbers at the bottom of the graph show hours of the day; these numbers are called the **horizontal scale.** The numbers at the left side of the graph show degrees of temperature; these numbers are called the **vertical scale.** Be sure all pupils locate these scales.

2. When you draw a line graph, you first locate each of the heavy dots, like those shown on the graph, and then connect these dots with segments. Each dot represents **two things;** for example, the first heavy dot in the lower left corner of the graph above shows that the temperature was 68° at 7 A.M.; this heavy dot shows 68° because it is on the horizontal line representing 68°; this dot also shows 7 A.M. because the dot is also on the vertical line marked 7. The second heavy dot shows that the temperature was 69° at 8 A.M. What two things are shown by the 3rd dot? 70° at 9 a.m. the 4th dot? 70° at 10 a.m. the 5th dot? 71° at 11 a.m. by each of the other dots? See Guide.

3. At what time during the day was the temperature the highest? From 2 to 3 the lowest?

4. During what hour did the temperature rise most rapidly? From 11 to 12 During what hour did it fall most rapidly? From 6 to 7

5. Can you tell from the graph what the temperature was at 9:30 A.M.? 70° at 12:30 P.M.? 75° at 4:30 P.M.? $75\frac{1}{2}$° at 7:30 P.M.? 67°

Emphasize that each point on graph represents two values. Do ex. 1-5 with pupils and check to see that all are able to locate information correctly.

Give practice in drawing line graphs. Point out to pupils the three ways of presenting data: tables, ex. 1-2; paragraph form, ex. 5-6; graphs.

Making Line Graphs

Draw line graphs showing the facts in each problem: See Guide.

1. When Paul's science class was studying weather, he made this record of the temperature during one day:

Hour	7	8	9	10	11	12	1	2	3	4	5	6
Degrees	57	58	59	59	60	63	67	68	68	65	63	61

2. One day the United States Weather Bureau reported these temperatures for a certain city:

Hour	7	8	9	10	11	12	1	2	3	4	5	6
Degrees	60	62	65	68	70	72	73	74	75	72	70	68

3. During a recent year the average monthly temperature in one of our large cities was as follows:

Jan. 31°
Feb. 34°
Mar. 40°
Apr. 45°
May 61°
June 74°
July 76°
Aug. 75°
Sept. 60°
Oct. 55°
Nov. 45°
Dec. 33°

4. Ask your teacher to give you the figures showing the daily attendance of your class last month. Then draw a line graph to show the daily attendance.

5. During a recent year Farmington had the following number of cloudy days per month: January, 26; February, 16; March, 16; April, 19; May, 9; June, 8; July, 2; August, 8; September, 13; October, 16; November, 18; December, 18.

6. The population of Buck Hills for 5-year periods was as follows: 1920, 2800; 1925, 2900; 1930, 3000; 1935, 3200; 1940, 3300; 1945, 3500; 1950, 3700; 1955, 3900; 1960, 4200.

▶ Let each space on the vertical scale stand for 100; let the numbers start with 20 (which represents 2000) and end with 45. The horizontal scale will show the years 1920, 1925, and so on; allow 2 or 3 spaces for each 5-year period.

7. Find some line graphs in newspapers or magazines and bring them to class. Tell what each graph is trying to show. See if you can read each graph. Use these to increase skill in reading line graphs.

Have pupils draw some of these graphs under your direction to be sure all construct them correctly. Ask questions about graphs for ex. 1-2, similar to those on page 251 (ex. 2-5).

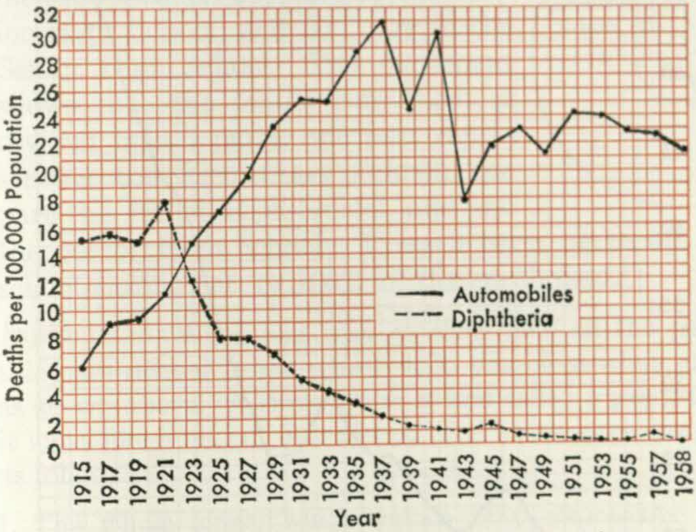
Give practice in reading line graphs representing statistics (pages 253-254).

A Good Record and a Poor One

Discuss graph with pupils. Have them note title, scales used, and so on.

This graph shows for the United States the number of deaths caused by diphtheria and by automobile accidents.

Then have them do ex. 1-5. Some pupils may need help for ex. 5.



Deaths Caused by Diphtheria and by Automobile Accidents

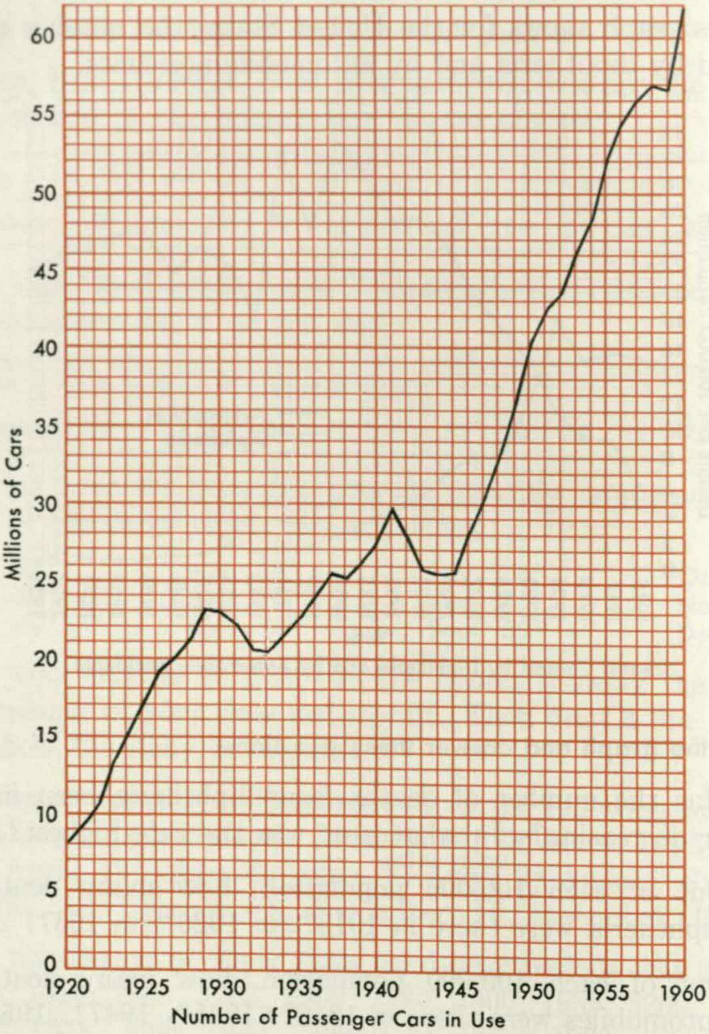
Study the graph and answer these questions:

1. Has the number of deaths from diphtheria been increasing or decreasing? In what year was the rate highest? lowest? 1955
Decreasing 1921
2. Out of each 100,000 population, how many deaths from diphtheria were there in 1915? 15 in 1923? 12 in 1937? 2 in 1941? 1
3. Out of each 100,000 population, how many deaths from automobiles were there in 1915? 6 1935? 1947? 23 1955? 23
About 29
4. In what year was there the greatest number of deaths due to automobile accidents? Why were there so many fewer deaths from automobile accidents in 1943? Because of less driving during gas rationing.
1937
5. In 1953 the population of the United States was 160,000,000. That year automobile accidents caused about 24 deaths out of every 100,000 people. How many deaths were there in 1953 due to automobile accidents? 38,400

Point out that vertical scale represents number of deaths out of each 100,000 population and not total number of deaths in whole United States. Have students check statistics for years not shown after 1958 and bring graphs up to date.

In connection with this graph, point out again that only approximations of exact values can be determined from graphs.

The Growth of American Motoring



Number of Passenger Cars in Use

1. On the vertical scale of this graph, what does 1 space represent? On the horizontal scale, what does 1 space represent? 1 year
1 million cars

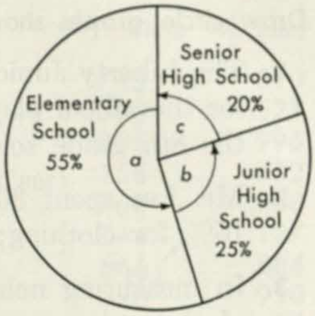
2. How many passenger cars were in use in the United States in 1920? in 1927? in 1930? in 1935? in 1940? in 1942? in 1945? in 1950? in 1955? in 1960? 61,700,000
8,000,000 20,000,000 23,000,000 22,500,000 27,500,000 27,500,000
25,500,000 40,000,000 52,000,000

3. Between what years did the number of cars decrease? (1) How do you account for these decreases? (2) (1) 1929-1933, (2) because of depression; (1) 1941-1944, (2) because of war.

Note that answers can be given to nearest million cars. You should also accept answers given to tenths of a million (7.2 million or 7,200,000) if they appear to be correct.

Teach construction of circle graphs when parts of a circle are used to represent per cents (pages 255-256). First review measuring and drawing angles with protractor.

Circle Graphs



1. **Problem** In the schools of Newtown, 55% of all the pupils are in the Elementary School, 25% are in the Junior High School, and 20% are in the Senior High School. This statement shows that the school enrollment is divided into 3 parts, one part representing the Elementary School, one part the Junior High School, and one the Senior High School. These 3 parts make up the **whole** school enrollment, or 100% of the enrollment.

Explanation When you wish to show how all the parts of anything are related to the **whole** of it, you often use a **circle** graph, as shown above. A circle graph is also called a **pie** graph because it looks like a pie cut into pieces. The above circle graph is drawn as follows:

1. The whole circle, which equals 360°, represents the **whole** school enrollment. The enrollment in the Elementary School is shown by drawing a part equal to 55% of the circle. Since 55% of 360° = 198°, draw an angle of 198° with a protractor. This angle has its vertex at the center of the circle, as shown at *a*.

2. The Junior High School has 25% of the total enrollment. 25% of 360° = 90°. Draw angle *b* equal to 90°, as shown in the figure. Angle *b* is drawn next to angle *a*.

3. The Senior High School has 20% of the total enrollment. 20% of 360° = 72°. You do not have to draw this angle because the rest of the circle must represent the Senior High School. But to be sure the work is right, measure angle *c* to see if it equals 72°. This is a check on your work.

2. The town of Bedford has an Elementary School, a Junior High School, and a Senior High School. In this town 60% (216°) of all the pupils are in the Elementary School, 25% (90°) are in the Junior High School, and the rest are in the Senior High School. What per cent are in the Senior High School? 15% (54°) Draw a circle graph showing these facts. See Guide.

Follow development as given in ex. 1 and have pupils draw the graph. Emphasize that circle graph shows the whole (100%), and relation of each part to all other parts and to the whole.

Emphasize importance of measuring and drawing angles accurately. Extend work by having pupils make comparisons of parts with parts and with whole.

Circle Graphs

Draw circle graphs showing the facts in each exercise: See Guide.

1. The Liberty Junior High School had a contest selling tickets for the school play. The 7th grade sold 35% of the tickets, (126°) the 8th grade sold 25% (90°), and the 9th grade sold 40%. (144°)

2. Mr. Lee spent 30% (108°) of his income for food; 25% (90°) for rent; 15% (54°) for clothing; 25% (90°) for other expenses; 5% (18°) for savings.

3. In measuring noise in a large city, it was found that 45% (162°) of the noise came from busses and street cars, 30% (108°) from trucks, 20% (72°) from cars and taxis, and 5% (18°) from airplanes.

4. During a recent year a survey of America's homes showed (72°) that 20% of the buildings were in poor condition, 45% in fair condition, and 35% (126°) in good condition.

5. In Fairfield last year the accidents that happened to eighth grade pupils were grouped as follows: 29% (104.4° or 104°) of all accidents occurred in school buildings, 15% (54°) occurred on the school grounds, 7% (25.2° or 25°) occurred on the way to and from school, 18% (64.8°, or 65°) occurred at home, 31% (111.6° or 112°) occurred in other places.

▶ When you find 29% of 360°, you get 104.4°; round off 104.4° to the nearest whole degree, which gives 104°. Then draw an angle of 104°. When you find 18% of 360°, you get 64.8°, which rounds off to 65°.

6. In one large city and its surrounding area people go to their work in the following ways: 82% go in private automobiles; (295°) 8% (29°) go in a bus or streetcar; 5% (18°) walk; 3% (11°) do their work at home; and 2% use other means of transportation. (70°)

7. Make 4 circle graphs showing how the population of the United States has gradually moved from the country to the cities. In the first graph show that in 1790, 95% (342°) of the people lived in the country and 5% (18°) in the city. In the other 3 graphs show these facts: in 1840, 90% (324°) lived in the country and 10% (36°) in the city; in 1890, 65% (234°) lived in the country (126°) and 35% in the city; in 1940, 45% (162°) lived in the country and (198°) 55% lived in the city.

Do ex. 1 with class. Ask them how they could check their work (sum should be 360°). Have them complete ex. 2-4. Ex. 5 should be discussed with class. Ex. 6-7 may be done independently.

Present last set of improvement tests in addition.

Improving by Practice

Addition Test 6a.

Time: $4\frac{1}{2}$ min.

1. 921	449	518	167	743	890
497	805	845	952	680	521
604	496	465	810	805	996
588	703	359	517	158	332
320	696	152	305	760	208
537	476	670	381	744	137
699	939	337	320	646	884
122	148	287	227	528	910
212	342	302	488	956	486 ⑥
4500	5054	3935	4167	6020	5364

Addition Test 6b.

Time: $4\frac{1}{2}$ min.

2. 177	702	649	620	892	654
466	973	722	273	762	875
556	369	408	946	154	185
808	101	344	514	552	378
795	320	347	643	395	849
737	168	130	803	131	449
952	649	815	950	760	589
265	905	424	986	886	721
709	237	102	706	871	923 ⑥
5465	4424	3941	6441	5403	5623

Addition Test 6c.

Time: $4\frac{1}{2}$ min.

3. 191	167	674	817	593	839
304	552	880	456	726	383
712	462	105	475	985	684
270	334	965	697	750	426
935	908	973	720	496	935
660	992	841	307	407	453
121	284	900	864	513	270
143	258	256	331	135	846
821	967	837	577	291	141 ⑥
4157	4924	6431	5244	4896	4977

To the Teacher. If individual pupils get low scores on Improvement Tests, such pupils may be assigned extra practice given on pages 325–328.

Urge pupils to work quickly but carefully in completing these tests. Since this is last set, note improvements and/or weaknesses on progress cards. Some pupils may still need help with facts or with adding by endings.

Decimal Review

Copy in columns and add or subtract:

1. 2.107 + 9.025 + 1.667 12.799 4829.2 – 3180.8 1648.4

2. \$4.95 + \$.81 + \$1.89 + \$.17 \$7.82 \$7000 – \$48.77 \$6951.23

Multiply. Check by going over the work:

3.	.437	15	6.28	.003	39.8	17.32
	29	.03	9.4	.03	.04	.125
	12.673	.45	59.032	.00009	1.592	2.16500

In ex. 4 and 5, divide until there is no remainder. In ex. 6 and 7, find quotients to the nearest tenth:

4.	.028 $8\overline{).224}$	17.9 $9\overline{)161.1}$	.053 $12\overline{).636}$	114 $.28\overline{)31.92}$	10.3 $22\overline{)226.6}$
5.	79 $.6\overline{)47.4}$	97.25 $8\overline{)778.00}$	5600 $.09\overline{)504^{00}}$	4.75 $36\overline{)171.00}$	24.625 $32\overline{)788.000}$
6.	2.6 $9\overline{)23.8}$	27.9 $7\overline{)195}$	22.5 $19\overline{)428}$	.9 $31\overline{)27.4}$	8.0 $83\overline{)661.8}$
7.	5.8 $.3\overline{)1.73}$	.5 $.9\overline{).428}$	2.4 $6.7\overline{)15.91}$	13.8 $.59\overline{)8.162}$	130.1 $.34\overline{)44.25}$

8. Arrange these decimals in order of size, beginning with the largest: .31, .07, 2.15, 4.10, .48, .99, .01. 4.10, 2.15, .99, .48, .31, .07, .01

9. Since 25 × 68 = 1700, tell the answer for 2.5 × 68; 170 for 2.5 × 6.8; 17 for .25 × .68; .17 for .025 × .68. .017

10. Find, to the nearest hundredth of an inch, the average monthly rainfall in Old Harbor for a recent year when the following amounts of rain were recorded: Jan., 2.27 in.; Feb., 2.14 in.; Mar., 2.94 in.; Apr., 3.50 in.; May, 1.83 in.; June, 4.24 in.; July, 5.73 in.; Aug., 4.77 in.; Sept., 5.08 in.; Oct., 4.31 in.; Nov., 2.70 in.; Dec., 2.06 in. 3.46

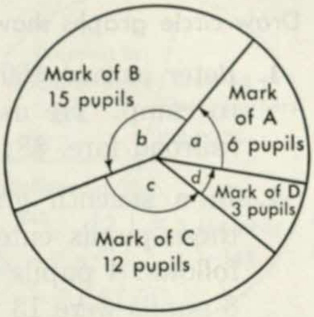
11. The same year the total rainfall in Lincoln was 14.09 in. How many more inches fell in Old Harbor that year than in Lincoln? See ex. 10. 27.48

12. The average monthly rainfall in Johnstown last year was 2.53 in. Find the total rainfall for the year. 30.36 in.

Check papers carefully and analyze errors. If many pupils seem weak in particular area, reteaching of topic may be necessary. Have volunteers explain their work at board and hold class discussion of examples which seemed to give difficulty.

Teach construction of circle graphs where parts of a circle are used to represent numbers, instead of per cents (pages 259-260).

Another Kind of Circle Graph



1. **Problem** There are 36 pupils in the seventh grade. On an arithmetic test, 6 pupils got a mark of A, 15 pupils got B, 12 got C, and 3 got D. Draw a circle graph showing how the class marks were divided.

Explanation In this problem **numbers** instead of **per cents** are used to tell how many pupils got each mark. In cases like this, the graph is drawn as follows:

1. Add the numbers showing how many got each mark. Does the total equal 36 pupils? Stress that whole circle represents 36.

2. Find what part of the whole class received a mark of A. Since there were 6 who received A, $\frac{6}{36}$, or $\frac{1}{6}$, of the class got A. Since the whole class is represented by a circle of 360°, $\frac{1}{6}$ of the class is represented by $\frac{1}{6}$ of 360°, or 60°. Draw angle *a* equal to 60°, to represent the number of pupils that got a mark of A, as shown above in the figure.

3. Similarly, $\frac{15}{36}$, or $\frac{5}{12}$, of the class got B on the test. Since $\frac{5}{12}$ of 360° is 150°, draw angle *b* equal to 150° to show the number that got B on the test.

4. What part of the class got a mark of C on the test? $\frac{1}{3}$ How many degrees were used in angle *c* to represent the number of pupils that got C? 120°

5. What does the rest of the circle represent? $\frac{1}{12}$ How many degrees should there be in this part? 30° While it is not necessary to draw angle *d*, you should measure this angle with a protractor to see if it equals $\frac{1}{12}$ of 360°, which is 30°. An additional check upon the accuracy of the work can be made as follows: Angles *a*, *b*, *c* together equal 60° + 150° + 120°, or 330°. Therefore angle *d* should equal 360° − 330°, or 30°.

2. In John's class there are 32 pupils. ($112\frac{1}{2}$°) On a spelling test 8 pupils (90°) got A, 12 pupils (135°) got B, 10 pupils got C, and 2 pupils ($22\frac{1}{2}$°) got D. Make a circle graph showing the class marks. See Guide.

3. On an arithmetic test 10 pupils got (90°) A, 16 got (144°) B, 12 got (108°) C, 2 (18°) got D. Make a circle graph using these facts. See Guide.

Follow explanation as given in ex. 1. Through questions, lead pupils to discover what part of class received A, B, and so on, and how many degrees represent each part. Note in ex. 3 that the total number of pupils is not given but must be found.

Emphasize importance of correct computations. Have pupils check sum of degrees before drawing each graph.

Making Circle Graphs

Draw circle graphs showing the facts in each exercise: See Guide.

1. Peter earned $60 in July and early August. Then he went to camp. He used the $60 as follows: camp expenses, $36; (216°) railroad fare, $8; clothes, $9; saved, $7. (42°)

(48°) (54°)

2. In a seventh grade there are 40 pupils. Last fall when these pupils entered the seventh grade their ages were as follows: 4 pupils (36°) were 11 yr. old; 25 (225°) pupils were 12 yr. old; 8 pupils (72°) were 13 yr. old; 3 pupils (27°) were 14 yr. old.

3. Last year the High School Athletic Association raised $400, which was used as follows: football, $100; (90°) baseball, $150; (135°) basketball, $80; track, $70. (63°)

(72°)

4. Mr. Brown and his family spent $270 last year for medical expenses. The money was used in the following ways: doctor, $150; dentist, $90; medicines, $30. (40°)

(200°) (120°)

5. The pupils of the Bay View School made $240 by giving a school play. They decided to divide the money as follows: $80 (120°) to the music club; $60 (90°) to the athletic club; $50 (75°) to the dramatic club, and $50 for motion picture films.

(75°)

6. When money is limited, the best food values can be obtained if each dollar spent for food is divided as follows: milk and (90°) cheese, 25¢; vegetables and fruits, (108°) 30¢; breads and cereals, (54°) 15¢; fats and sweets, (18°) 5¢; meat, fish, and eggs, 25¢. (90°)

7. Mr. Smith keeps a clothing store. Out of every dollar that he receives in sales, he says that (216°) 60¢ goes to pay the cost of the clothing, 35¢ (126°) is used to pay expenses, and 5¢ is (18°) profit.

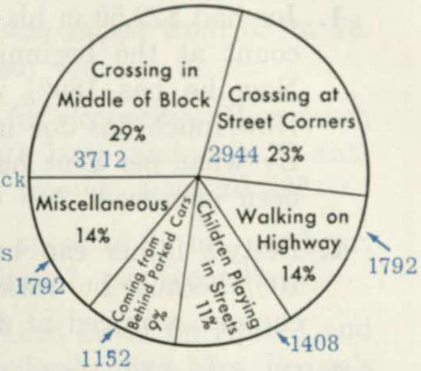
8. At the election of class officers, 120 votes were cast for class president. Fred got 72 (216°) votes, Tom got 28, (84°) Bert got 20. (60°) There were also 120 votes cast for class secretary. Amy got 64 votes, (192°) Mary got 32, (96°) and Judy got 24. (72°) (Make 2 graphs)

9. Of the 840 pupils in the Harris Junior High School, 350 (150°) are in the seventh grade, 280 (120°) are in the eighth grade, and 210 (90°) are in the ninth grade.

Have pupils check own graphs (or partner's) by reading information on it to see if it is the same as that given in problem. Ask pupils to bring to class and interpret circle graphs found in newspapers.

Present review of bar, line, and circle graphs. Note that in ex. 3 and 6 the number of degrees is given to tenths of a degree, so that sums equal 360°.

Review of Graphs



1. In a recent year 12,800 pedestrians were killed by automobiles. This graph shows what the pedestrians were doing when the accidents occurred. What act caused the largest number of deaths? Crossing in middle of block the next largest number? Crossing at street corners

2. Using the per cents given in the graph, find the number of deaths due to each cause. See ex. 1.
See answers on graph in ex. 1.

3. Check the graph to see whether it is drawn correctly. Find how many degrees are needed to represent 29% 104.4°; to represent each of the other per cents. 23%, 82.8°; 14%, 50.4°; 11%, 39.6°; 9%, 32.4°; 14%, 50.4° Then measure each angle in the graph to see if it is the right size.

4. Make a bar graph to compare the number of calories in different kinds of meat. The number of calories in a 4-ounce serving of each is: pork, 402; beef, 369; lamb, 367; chicken, 269; veal, 186; fish, 177. See Guide.

5. Make a line graph to show the change in the size of United States farms if the average acreage per farm is: See Guide.

Year	1870	1880	1890	1900	1910	1920	1930	1940	1950
Acres	153	134	137	146	138	148	157	174	215

6. Make a circle graph to show the educational background of World War II draftees: 11% (39.6°) were college men, 30% (108°) were high school graduates, 28% (100.8°) were high school nongraduates, 31% (111.6°) had gone to elementary school or had had no schooling. See Guide.

7. One year there were about 60,000 forest fires. The causes of these fires were: 15,000 (90°) were set by incendiaries; 14,000 (84°) were caused by smokers; 12,000 (72°) were caused by railroad sparks; 10,000 (60°) were caused by campers; 9000 (54°) were due to other causes. Make a circle graph showing these facts. See Guide.

8. Make up hourly temperatures for a day, from 7 A.M. to 7 P.M. and draw a line graph showing the changes.

Have pupils do this work independently. When checking, note if choosing scales, finding fractional parts, determining number of degrees, constructions, and so on, cause difficulty. Plan for reteaching and remedial work as needed.

Present review of all types of percentage problems.

Percentage Problems

1. Joe had $22.50 in his savings bank account at the beginning of this year. Now he has 180% of that amount. How much has Joe in the bank now? $40.50 By what per cent have Joe's savings been increased? 80%

2. Betty's father can buy her a new piano at a discount of 10% because he works in a music store. If the piano Betty chooses is priced at $750, how much is the discount? $75

3. On a problem test in arithmetic, 15% (54°) of the pupils had a mark of "excellent," 35% (126°) had a mark of "good," 40% (144°) had a mark of "fair," and the rest were marked "poor." What per cent of the pupils were marked "poor"? 10% (36°) Make a circle graph showing these facts. See Guide.

4. In ex. 3, there were 40 pupils who tried the test. Find the number of pupils in each group. 6, excellent; 14, good; 16, fair; 4, poor

5. William has $100. Tell how much he will have if he increases it 40%; $140 if he decreases it 40%; $60 if he has 240% as much. $240

6. Mr. Johnson's commission for selling a house was $962.50. The house was sold for $17,500. What per cent was he paid for selling the house? $5\frac{1}{2}\%$

7. Jack has earned $18.60 toward a bicycle that costs $49.60. What per cent of the money has he earned? $37\frac{1}{2}\%$

8. In the last 10 yr., the population of Newcastle has grown from 9260 to 10,649. The present population is what per cent of the population 10 yr. ago? 115%

9. What is the per cent of increase in the population of Newcastle in the last 10 yr.? See ex. 8. 15%

10. Mr. Mills bought some radios to sell in his store. He paid $15.50 for each one. He figured his expenses were $6.50 per radio. He sold each one for $25.00. How much profit was he getting? $3.00 What per cent of the selling price was his profit? 12%

After correcting papers and noting kinds of errors, return papers to pupils and conduct class discussion of solutions to problems. Let volunteers explain work at board. If some pupils still show lack of understanding, group them for remedial activities.

A Problem Test

1. The price of an ice cream cone was raised from 5¢ to 7¢. What was the per cent of increase? 40%

2. In Bayridge it costs 4¢ per kw-hr for electricity. How much does it cost to burn one 100-watt lamp and two 75-watt lamps every evening in the week from 5:30 to 10:30? 35¢

3. If an automobile is traveling at a speed of 50 mi. per hour, how many feet does it go per minute? 4400 per second? $73\frac{1}{3}$

4. Mrs. Brown's electric meter read 1396 kw-hr on April 1 and 1442 kw-hr on May 1. At $4\frac{1}{2}$¢ per kw-hr, find Mrs. Brown's electric bill for April. $2.07

5. Peggy has $75 (180°) in savings bonds, $60 (144°) in the school savings bank, and $15 (36°) in a coin bank. Make a circle graph to show what part of her savings is in each group. See Guide.

6. Last week Bill's allowance was $2.50. He spent 25¢ for a battery, 40¢ for school supplies, 10¢ for ice cream, 25¢ for the movies, and he saved 35¢. He spent the rest for 5 lunches at school. Find the average amount he paid per lunch. $.23

7. Mary's mother made Mary a new dress. She needed 8 buttons for the waist and skirt and 2 buttons for each sleeve. Find the cost of these buttons at 3 buttons for 25¢. $1.00

8. Mr. Wood's speedometer went from 3892.8 mi. to 4171.2 mi. on a trip where he used 16 gal. of gasoline. Find the average number of miles he drove per gallon of gasoline. 17.4

9. Last year the school enrollment in Jamestown was 22,825. Now the enrollment has increased 4%. How many more pupils are enrolled in Jamestown now? 913

10. In a recent month a dealer sold 528 automobiles and 176 of them were trucks. What per cent were trucks? $33\frac{1}{3}$%

Have pupils record scores on graphs.

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Pupils should compare scores with previous ones to see if they still need help. Have pupils explain their solutions so you may determine causes of errors and need for review or reteaching of special topic.

Present diagnostic test of skills taught thus far with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Add and check the work:

					Practice Pages
1.	$6\frac{3}{8}$ $5\frac{1}{2}$ $\underline{2\frac{3}{4}}$ $14\frac{5}{8}$	$3\frac{1}{2}$ $3\frac{9}{10}$ $\underline{2\frac{1}{5}}$ $9\frac{3}{5}$	$1\frac{3}{4}$ $6\frac{5}{6}$ $\underline{3\frac{1}{2}}$ $12\frac{1}{12}$	$5\frac{2}{3}$ $2\frac{1}{2}$ $\underline{8\frac{3}{5}}$ $16\frac{23}{30}$	54, 55 62

Subtract and check the work:

					Practice Pages
2.	8 $\underline{4\frac{2}{3}}$ $3\frac{1}{3}$	$6\frac{7}{16}$ $\underline{2\frac{1}{16}}$ $4\frac{3}{8}$	$4\frac{1}{2}$ $\underline{1\frac{4}{5}}$ $2\frac{7}{10}$	$5\frac{3}{4}$ $\underline{4\frac{15}{16}}$ $\frac{13}{16}$	58–61
3.	.75 $\underline{.25}$ .50	6.14 $\underline{5.87}$ .27	.013 $\underline{.005}$ .008	2.174 $\underline{.975}$ 1.199	91

Multiply and check the work:

				Practice Pages
4.	$4\frac{1}{2} \times 15$ $67\frac{1}{2}$	$\frac{5}{8} \times 12$ $7\frac{1}{2}$	$\frac{9}{16} \times \frac{2}{3}$ $\frac{3}{8}$	63–67
5.	$2\frac{1}{4} \times 2\frac{2}{3}$ 6	$\frac{2}{3} \times 5\frac{1}{4}$ $3\frac{1}{2}$	$1\frac{1}{2} \times \frac{7}{8}$ $1\frac{5}{16}$	63–67
6.	$9 \times .04$ .36	$.12 \times 3.5$ .420	$.08 \times .12$ .0096	96–100

Divide. Give quotients to the nearest hundredth:

					Practice Pages
7.	$\begin{array}{r}2.37\\2.4\overline{)5.688}\end{array}$	$\begin{array}{r}12.25\\48\overline{)588}\end{array}$	$\begin{array}{r}2.94\\25\overline{)73.5}\end{array}$	$\begin{array}{r}1.17\\43\overline{)50.5}\end{array}$	101–105
8.	$\begin{array}{r}.92\\.7\overline{).644}\end{array}$	$\begin{array}{r}.03\\.9\overline{).028}\end{array}$	$\begin{array}{r}.74\\5.9\overline{)4.36}\end{array}$	$\begin{array}{r}15.05\\.019\overline{).286}\end{array}$	111–113

Find the answers to the nearest cent:

				Practice Pages
9.	15% of \$36 \$5.40	25% of \$35 ↙ \$8.75	$37\frac{1}{2}$% of \$48 \$18	128–136
10.	$6\frac{1}{2}$% of \$94 \$6.11	135% of \$60 \$81	400% of \$55 \$220	142–146
11.	2% of \$7.75 \$.16	$4\frac{1}{4}$% of \$25 ↙ \$1.06	$12\frac{1}{2}$% of \$15 \$1.88	138

Emphasize importance of checking all work. Results should be noted on progress cards and compared with previous ones. Through conferences with pupils determine need for reteaching or review. Clear up difficulties before assigning remedial work.

See Guide for specific aims of Chapter 9.

Chapter 9

What Banks Do for Us

Acquaint pupils with services offered by commercial banks.

1. A bank provides its depositors with a safe place to keep their money and renders many other valuable services. If a depositor has a checking account at a bank, the money deposited in such an account may be paid out by means of checks. A bank also cashes the checks its depositors receive from other individuals. Money that is to be saved may be deposited in a savings account. The bank pays money, called **interest,** to a depositor for the use of the money in his savings account. Many people rent a safe deposit box at a bank in order to have a safe place to keep their valuable papers.

2. The services that a bank renders its depositors and the interest it pays on its savings accounts all cost the bank money. To earn money banks lend, at interest, a large part of the money deposited in them.

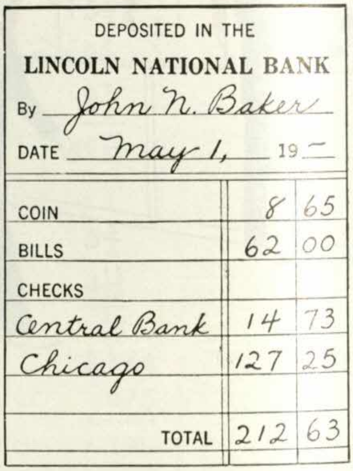
3. A bank does not pay interest on the money deposited in a checking account. Why not?

Conduct oral discussion of services of commercial banks, using ex. 1–3 to initiate discussion. Be sure pupils understand meanings of any new words or terms.

Teach procedure for opening a checking account and making out deposit slips.

Opening a Checking Account

1. Mr. Baker decided to open a checking account with the Lincoln National Bank of Detroit. He was required to give references and to leave a copy of his signature. Each time that Mr. Baker makes a deposit in his checking account he fills out a deposit slip like the one at the right. On this slip he tells how much of the money is in coin, how much is in bills, and how much is in checks. Each check is listed separately. If a check is drawn on a bank in Detroit, Mr. Baker gives the amount of the check and the name of the bank, as "Central Bank"; if a check is drawn on a bank in another city, he writes the amount of the check and the name of the city, as "Chicago."



DEPOSITED IN THE
LINCOLN NATIONAL BANK
By John N. Baker
DATE May 1, 19—

COIN	8	65
BILLS	62	00
CHECKS		
Central Bank	14	73
Chicago	127	25
TOTAL	212	63

Make some deposit slips similar to the one above. Then fill out slips for the deposits below. Date the slip and find the total amount of each deposit:

2. Henry L. Jones of Chicago deposits: coin, $15.75; bills, $214; check on New York bank, $56.39; check on Boston bank, $47.32; check on Flint bank, $115.87. Total: $449.33

3. James A. Smith of Seattle deposits: coin, $17.50; bills, $179; check on City Bank, $72.85; check on Chicago bank, $42.79; check on Cleveland bank, $326.48. Total: $638.62

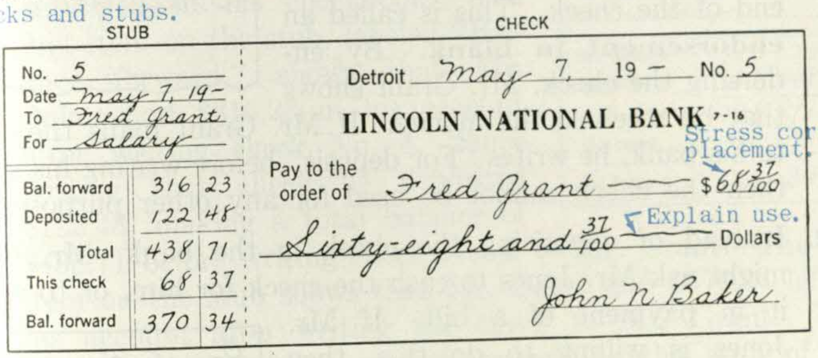
4. Frank P. Martin of Omaha deposits: bills, $68; check on Denver bank, $89.26; check on Baltimore bank, $37.18; check on Springfield bank, $172.57; check on First National Bank, $346.94; check on Atlanta bank, $29.64. Total: $743.59

5. Mary J. Adams of St. Louis deposits: coin, $5.65; bills, $45; check on San Francisco bank, $98.75. Total: $149.40

Have pupils study deposit slip shown and then discuss ex. 1. Be sure pupils understand all entries. Try to have deposit slips available for pupils to use for ex. 2-5. It would be well to do ex. 2 with class to be sure all make entries correctly.

Writing a Check

When Mr. Baker opened his checking account with the Lincoln National Bank, he received a checkbook which contains blank checks like the one below. Mr. Baker owed Mr. Grant $68.37 for salary, which he paid by writing this check and giving it to Mr. Grant: Try to have checkbook on hand so pupils may see actual checks and stubs.



STUB

No. 5
Date May 7, 19–
To Fred Grant
For Salary

Bal. forward	316	23
Deposited	122	48
Total	438	71
This check	68	37
Bal. forward	370	34

CHECK

Detroit May 7, 19 – No. 5

LINCOLN NATIONAL BANK 7-16

Pay to the order of Fred Grant 68\frac{37}{100}$

Sixty-eight and $\frac{37}{100}$ Dollars

John N. Baker

Stress correct placement.

Explain use.

To the left of the dotted line is the **stub** of the check, which remains in the checkbook; to the right of the dotted line is the check Mr. Baker gave to Mr. Grant.

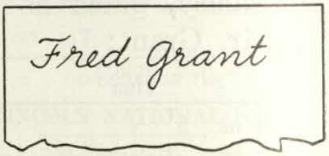
1. Why does Mr. Baker give a check rather than cash? It serves as a receipt since the bank returns it marked Paid to Mr. Baker at the end of the month.
2. Why is the amount written both in words and in figures? Then the check cannot be altered easily.
3. Why should no space be left between the sign $ and 68$\frac{37}{100}$, or before the amount written in words? To prevent alterations and the insertion of a figure following the $ sign.
4. Why is the amount written 68\frac{37}{100}$ instead of $68.37? A decimal point is easily erased to alter the check.
5. Would you accept a check from a stranger in payment for his purchases? No, because the check might not be good.
6. You may take a check you have written to your bank to be **certified.** When a bank stamps "Certified" across a check, the bank guarantees that the depositor has the money in his account and that the check will be paid. The bank immediately deducts the amount of the certified check from the depositor's account. Why? Under what circumstances would you need to have a check certified? See Guide.
7. Write 5 checks, using different amounts and names.

Have pupils study illustrated check. Explain and discuss purpose of each blank on both check and stub, and correct way of writing amounts (see Guide). Then do ex. 1-6 orally. Also point out that checks should never be written with pencil.

If possible, have canceled checks available so that pupils may see how and where to endorse a check. Have them also note date stamped on each and explain that this is date on which bank honored each check.

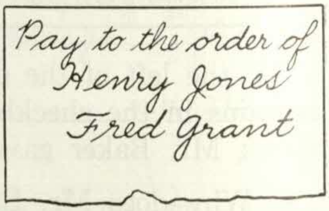
Endorsing a Check

1. After Mr. Grant has received the check from Mr. Baker, he can present it at Lincoln National Bank for payment or deposit it in his account at his own bank. He must first endorse the check by writing his name across the back of the left end of the check. This is called an **endorsement in blank.** By endorsing the check, Mr. Grant shows that he received the money. If Mr. Grant mails the check to his bank, he writes "For deposit" before writing his name; then the check cannot be used for any other purpose.



Fred Grant

2. Instead of depositing the check in the bank, Mr. Grant might ask Mr. Jones to cash the check for him, or to accept it in payment of a bill. If Mr. Jones is willing to do this, then Mr. Grant endorses the check **in full,** making it payable only to Mr. Jones or to his order, as shown at the right.



Pay to the order of
Henry Jones
Fred Grant

3. The check finally goes back to the Lincoln National Bank, where $68.37 is deducted from Mr. Baker's account and the check is stamped "Paid." At intervals Mr. Baker receives from his bank all of his canceled checks together with a statement showing his bank balance. If possible, let pupils examine real bank statement.

4. Explain why a canceled check serves as a receipt for money that has been paid. See Guide.

5. Should a check be endorsed as soon as it is received? Give reasons for your answer. Why is an endorsement in full safer than an endorsement in blank? See Guide.

6. Mr. Baker wished to withdraw $50.00 from his account for his own use. He wrote a check to "Cash" and presented it at his bank. Did he have to endorse this check? See Guide.

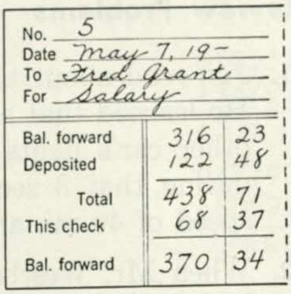
7. James H. Lee wrote a check to Henry Porter for $72.25. Write this check and endorse it in full payable to J. B. Fox.

Discuss ex. 1-7 with class. Be sure pupils understand types of endorsement in ex. 1-2. In ex. 3 make clear that bank balance is the difference between deposits and withdrawals.

Teach how to fill out check stub and to balance the account.

Filling Out Check Stubs

1. Every time Mr. Baker writes a check he fills out the stub of the check so that he knows where the money has gone and also what his balance is at the bank. After he sent Mr. Grant a check for $68.37, the stub, which is shown at the right, remained in his checkbook. The first item on the stub, labeled "Balance forward," shows that Mr. Baker had $316.23 in his account after writing check No. 4. Since writing that check he deposited $122.48, making a total balance of $438.71 before writing check No. 5 to Mr. Grant. The last item on the stub shows that Mr. Baker had $370.34 left in his account after writing the check to Mr. Grant. This balance of $370.34 is carried forward to the stub of the next check. If Mr. Baker writes a check that is larger than his balance, he overdraws his account; when such a check reaches the bank upon which it is drawn, that bank may refuse to pay it.



No. 5
Date May 7, 19–
To Fred Grant
For Salary

Bal. forward	316	23
Deposited	122	48
Total	438	71
This check	68	37
Bal. forward	370	34

2. When should the stub of a check be filled out? When the check is written; otherwise you may forget that you wrote the check.

3. Why is it important for Mr. Baker to keep a record of his bank balance on the stubs of his checks? So that he will know how much money he has left in the bank.

4. Why should checks be numbered? See Guide.

5. On May 9, Mr. Baker wrote check No. 6 to Carl A. Miller for $69.58 in payment for house repairs. Since writing check No. 5 he had made a deposit of $37.81 in his account. Fill out the stub of this check. What was Mr. Baker's balance to be carried forward to the next stub? $338.57

6. On May 10, Mr. Baker wrote his next check to George White for $96.75 in payment for furniture. He had made no additional deposits in his account. Write the stub for this check and find the balance in Mr. Baker's account at the end of the day. $241.82

Discuss ex. 1 thoroughly with class. Be sure purposes of each blank are understood and meaning of overdrawn is clear. Do ex. 2-4 orally also. Give pupils more practice like that in ex. 5-6.

Review Problems

1. Jim read about the danger of glaring automobile headlights. He learned that a driver may be blinded by the glare of another car's lights for 3 sec. How many feet will a car go during that 3 sec. of blindness if the car is traveling at a speed of 40 mi. an hour? 176 at a speed of 60 mi. an hour? 264

2. When Mr. Westbrook left on a business trip of 756 mi., he had 14 gal. of gasoline in the tank of his car. On the trip he bought 35 gal. and had 4 gal. left in the tank when he returned. Find the average number of miles Mr. Westbrook drove per gallon of gasoline. 16.8

3. Mr. Barker's weekly salary was increased from \$80 to \$92. Find the per cent of increase in his salary. 15%

4. Mr. Roy must make a trip of 210 mi. He usually averages 35 mi. an hour in his car. If he allows 45 min. to stop for lunch, at what time should he leave home in order to finish the trip by 4 P.M.? 9:15 A.M.

5. Ann bought some meat weighing $5\frac{1}{4}$ lb. at \$.48 a pound. Her change from \$5 was 2 one-dollar bills, 1 quarter, 2 nickels, and 3 pennies. Did she receive the correct change? No; it was short \$.10.

6. Miss Best bought 10 gal. of gasoline at 25.9¢ per gallon, 5 gal. at 26.6¢ per gallon, and 10 gal. at 27.8¢ per gallon. Find the average price per gallon she paid for the 25 gal. 26.8¢

7. In our junior high school this year there are 1155 pupils. Of this number 310 pupils will enter senior high school in the fall. What per cent will enter the senior high school? Find the result to the nearest whole per cent. 27%

8. In April Henry bought a ski jacket for only 50% of its marked price of \$12.50. How much did Henry pay for it? \$6.25

When discussing problems with pupils try to determine if they are using all problem aids they know, if they are computing correctly, if they recognize which process to use, and so on. Plan remedial work as needed.

Present last set of improvement tests in subtraction.

Improving by Practice

Subtraction Test 6a.

Time: 4 min.

1.	7311792 1420853 5,890,939	8501533 1579986 6.921,547	8300048 382069 7,917,979	
2.	5961716 2084967 3,876,749	6457217 3402428 3,054,789	6135539 1347925 4,787,614	
3.	6914852 2376494 4,538,358	3839274 2998603 840,671	9471907 6809925 2,661,982	⑨

Subtraction Test 6b.

Time: 4 min.

4.	7562540 3679405 3,883,135	5232680 668234 4,564,446	4001589 1586306 2,415,283	
5.	8302859 5904717 2,398,142	3485237 2508156 977,081	7183834 309876 6,873,958	
6.	6818561 4782996 2,035,565	7246745 1872955 5,373,790	7939223 269424 7,669,799	⑨

Subtraction Test 6c.

Time: 4 min.

7.	7527813 4658916 2,868,897	3400660 957793 2,442,867	9452065 1759479 7,692,586	
8.	9421250 5479812 3,941,438	8000111 3639864 4,360,247	7086678 932036 6,154,642	
9.	8668596 2154751 6,513,845	4664094 1429739 3,234,355	1164989 516788 648,201	⑨

To the Pupil. This is the last set of Improvement Tests in subtraction you will have this year. Try to get a score of 10 on each test.

Compare results of tests with previous ones and note improvements and/or further weaknesses on progress cards. Help pupils discover own weaknesses and causes of errors, and set up remedial programs as needed.

Teach how to find interest for 1 year and for $\frac{1}{2}$ year.

Borrowing Money

1. Mr. Smith needed some money to use in his business. He borrowed \$3000 from Mr. Miller for 1 yr. at 5% interest. This means that Mr. Smith agreed to pay back to Mr. Miller the \$3000 at the end of 1 yr., and, in addition, to pay Mr. Miller 5% of \$3000, or \$150, for the use of the money for 1 yr. The \$3000 that Mr. Smith borrowed is called the **principal.** The **rate of interest** is 5%; this is the per cent of the principal that is charged for the use of the money for 1 yr. \$150 is the **interest.** At the end of 1 yr. Mr. Smith will pay Mr. Miller a total of \$3150. This is called the **amount;** the amount is the sum of the principal and the interest. Give other examples and have pupils name the principal, rate of interest, and so on.

2. Mr. Chase borrowed \$500 from Mr. Hill for 1 yr. at 6% interest. How much was the interest? \$30 How much in all did Mr. Chase pay Mr. Hill at the end of 1 yr.? \$530

Find the interest for 1 yr. on the following. If necessary, give the result correct to the nearest cent: Stress.

3. \$200 at 4% \$8 — \$800 at 6% \$48 — \$3500 at 5% \$175
4. \$5000 at 6% \$300 — \$355 at 4% \$14.20 — \$4000 at $3\frac{1}{2}$% \$140
5. \$2735 at 5% \$136.75 — \$1800 at $6\frac{1}{2}$% \$117.00 — \$6184 at 3% \$185.52
6. \$127 at $4\frac{1}{2}$% \$5.72 — \$8750 at 7% \$612.50 — \$5379 at $3\frac{1}{2}$% \$188.27

7. Mr. Brown borrowed \$1200 from Mr. Lee for $\frac{1}{2}$ yr. at $3\frac{1}{2}$%. The interest for $\frac{1}{2}$ yr. is half as much as the interest for 1 yr. What is the interest for 1 yr.? \$42 for $\frac{1}{2}$ yr.? \$21 At the end of $\frac{1}{2}$ yr., when Mr. Brown pays back the \$1200 with interest, how much in all does he pay Mr. Lee? \$1221

Ex. 7 should be done orally. Then assign ex. 8–11 if there are no apparent difficulties.

Find the interest for $\frac{1}{2}$ yr. on the following:

8. \$400 at 5% \$10 — \$3600 at 4% \$72 — \$975 at 6% \$29.25
9. \$2500 at $3\frac{1}{2}$% \$43.75 — \$4650 at 5% \$116.25 — \$3925 at 6% \$117.75
10. \$1237 at $4\frac{1}{2}$% \$27.83 — \$4740 at $3\frac{1}{4}$% \$77.03 — \$1900 at $6\frac{1}{2}$% \$61.75
11. \$1400 at $2\frac{3}{4}$% \$19.25 — \$2000 at 7% \$70.00 — \$1600 at $2\frac{1}{4}$% \$18

Discuss ex. 1, emphasizing meanings of new terms. Stress that interest is money paid for use of borrowed money. Emphasize also that rate of interest means rate for 1 year, unless stated otherwise. Then do ex. 2 with class.

Using a Formula for Interest

1. Mr. Carter borrowed $800 from Mr. Lee for 5 mo. at 6% interest. How much interest must Mr. Carter pay? $20

▶The interest for 1 yr. is 6% of $800, or $48. Since 5 mo. is $\frac{5}{12}$ of a year, the interest for 5 mo. is $\frac{5}{12}$ of $48, or $20.

2. When solving interest problems, you use the rule: Use more examples to lead pupils to formulate rule in own words, before presenting it.

Interest = Principal × Rate × Time

This rule can be written as a formula, like this:

$$i = p \times r \times t$$

Stress. In this formula, *i* stands for interest, *p* for the principal, and *t* for the time **in years.** When writing formulas, the multiplication signs are usually omitted. Therefore the interest formula can be written:

$$i = prt$$

Always remember that *prt* means $p \times r \times t$.

3. Solve the problem in ex. 1, using the interest formula.

▶In this case $p = 800$, $r = \frac{6}{100}$, and $t = \frac{5}{12}$ since the time is expressed in years or in parts of a year. The work will be shorter if you cancel when possible:

$$i = \overset{\overset{4}{\cancel{8}}}{\cancel{800}} \times \frac{\overset{1}{\cancel{6}}}{\underset{1}{\cancel{100}}} \times \frac{5}{\underset{\underset{1}{\cancel{2}}}{\cancel{12}}} = 20$$

The result, $20, is the same as that found in ex. 1.

Pupils should do work as shown in ex. 3.

Using the formula, find the interest correct to the nearest cent:

4. $300, 6%, 2 mo. $3 — $600, 6%, 7 mo. $21 — $900, 4%, 4 mo. $12

5. $600, 4%, 11 mo. $22 — $800, 7%, 9 mo. $42 — $300, 4%, 8 mo. $8

6. $240, 3%, 7 mo. $4.20 — $480, $6\frac{1}{2}$%, 10 mo. $26 — $420, 5%, 6 mo. $10.50

More Practice. See (50) on page 341.

Ex. 1-3 should be discussed carefully with pupils. Be sure they understand that time must be expressed in years and why 5 mo. is $\frac{5}{12}$ yr. Do ex. 4 with class also. Then assign 5-6 as independent work.

Finding Interest for Days

1. **Problem** Mr. Parker borrowed $1200 at 5% interest. If he keeps the money 45 da., how much interest must he pay? $7.50

Explanation In computing interest, a month is usually counted as 30 da. and a year as 360 da. So 45 da. is $\frac{45}{360}$, or $\frac{1}{8}$, of a year. Using the formula $i = prt$, $p = 1200$, $r = \frac{5}{100}$, and $t = \frac{1}{8}$. So

$$i = \overset{\overset{3}{\cancel{12}}}{\cancel{1200}} \times \frac{5}{\underset{1}{\cancel{100}}} \times \frac{1}{\underset{2}{\cancel{8}}} = \frac{15}{2} = \$7.50$$

Mr. Parker must pay $7.50 interest. It avoids extra cancellations if the fraction representing the time is simplified in advance; Stress. this explains why $\frac{45}{360}$ was first changed to $\frac{1}{8}$. It is not necessary to use cancellation. The interest may be found for 1 yr., which is $60. Then the interest for $\frac{1}{8}$ yr. is $\frac{1}{8}$ of $60, or $7.50.

2. Mr. Gates borrowed $1000 from the bank at 6% for 24 da. How much interest did Mr. Gates pay the bank? $4

3. Mr. Brooks bought some goods for his store for $900. If he pays cash for the goods, he will get a discount of 5%. He does not have enough money now to pay cash, but he will have it a month later when his customers pay their bills. So he borrows $900 for 30 da. at 6%. How much interest does he pay? $4.50 How much does he save in this way? $40.50

4. Miss Lewis bought a cottage at Clear Lake. She saved $100 by paying cash for it, but she had to borrow $600 at the bank for 75 da. at 6% in order to have enough to pay cash. How much did she really save by borrowing the money? $92.50

Find the interest on the following: Work should be done as in ex. 1.

5. $600 at 5% for 60 da. $5 — $1500 at 3% for 24 da. $3

6. $800 at 6% for 45 da. $6 — $1200 at 4% for 27 da. $3.60

7. $700 at 7% for 90 da. $12.25 — $8000 at 3% for 66 da. $44

More Practice. See (51) on page 341.

When discussing ex. 1, emphasize that 360 da. is usually counted as a year when computing interest. Give pupils more practice in expressing days as fractional part of year. Ex. 2-4 should be done with class so that difficulties may be cleared up.

Present variety of problems involving interest. Instruct pupils to use interest formula.

Paying Interest

1. George saved $85 of the money he earned during the summer. He lent this money to his father for 1 yr. at 4% interest. How much interest will George's money earn? $3.40

2. Mr. Swift needed some money in order to buy new machinery for his farm. He borrowed $1500 from Mr. Wright for 3 yr. at 5% interest. When money is borrowed for more than a year, the interest is usually paid at the end of each year or each 6 mo. Mr. Swift agreed to pay the interest each year. What was the annual interest? $75 How much interest did Mr. Swift pay Mr. Wright in all? $225

3. James borrowed $400 from Mr. Parker to use for part of his college expenses. Mr. Parker said that James could pay the interest when he repaid the loan. Five years later James repaid the $400 with interest at 4%. How much interest in all did he pay? $80 What was the total amount that he paid? $480

4. Mary borrowed $400 from her uncle so that she could take a business course. She was able to repay her uncle $1\frac{1}{2}$ yr. later with interest at 3%. What was the interest? $18 What was the amount? $418

5. Mr. White bought a building lot for $2000 but he did not build a house on it. If he had not bought the lot, he could have invested the money at 5% interest. How much interest does he lose on this money each year that he keeps the lot? $100

6. Mr. Gates owns a house that cost $15,000. Considering that he could invest money at 4% interest, how much interest does he lose each year on the money invested in his house? $600 You see that lost interest is one of the expenses that one has when he owns a house, just like repairs and taxes.

7. John saw an advertisement in the newspaper which said that one could borrow any amount up to $300 at "2% monthly." What rate of interest is that per year? 24% Do you think this is a high rate of interest? Yes

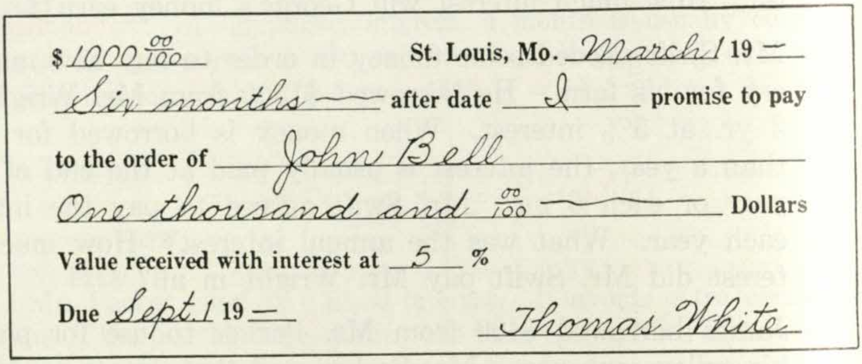
8. What is the interest on $425 for 1 yr. at 3%? $12.75

It may be well to do some of these problems with the class since the time is more than 1 year in some of the problems. Assign remaining problems as independent work. Be sure to discuss all solutions with class.

Explain promissory notes and how they are used.

Promissory Notes

1. When Mr. White borrowed $1000 from Mr. Bell, he gave Mr. Bell the promissory note shown below.



$ 1000 $\frac{00}{100}$ St. Louis, Mo. March 1 19 —

Six months — after date I promise to pay

to the order of John Bell

One thousand and $\frac{00}{100}$ — Dollars

Value received with interest at 5 %

Due Sept. 1 19 — Thomas White

A **promissory note** is a written promise to pay a given sum of money at a given time. The amount of money that is borrowed is called the **face** of the note, and the date on which the money is to be paid is the **date of maturity.**

2. Study the above note. What are the face and the date of maturity of the note? Who borrowed the money? Is interest to be paid upon the loan? If so, how much is it? $25

$1000 Sept. 1 Thomas White Yes

3. Write a promissory note for ex. 2 on page 275. Observe pupils to make sure that all do this correctly.

Find the interest (1) and the date of maturity for each note: (2)

	Date	Face	Rate	Time	
(1) (2) **4.**	April 1, 1963	$700	5%	6 mo.	$17.50 Oct. 1, 1963
5.	Sept. 10, 1964	$1200	$4\frac{1}{2}$%	1 yr.	$54 Sept. 10, 1965
6.	July 1, 1964	$2350	6%	3 mo.	$35.25 Oct. 1, 1964

7. On April 1 Mr. John Barker borrowed $1000 from Mr. Henry North for 1 yr. at 6%, agreeing to pay the interest quarterly, that is, at the end of each 3 mo. Write the promissory note, using the words "with interest at 6%, payable quarterly." How much interest does Mr. Barker pay at the end of each quarter and on what dates are these payments made? How much interest does he pay in all? $60

$15 on July 1, Oct. 1, Jan. 1, Apr. 1

When discussing note in ex. 1, point out and explain purpose of all spaces on it. Be sure pupils understand meanings of new terms. Some pupils may need help on ex. 7.

Opening a Savings Account

1. Susan had saved \$15. Her father advised her to open a savings account at the bank so that her money would be in a safe place and would also earn interest. When a savings account is opened at a bank, the depositor receives a passbook. In this book a record is kept of all money deposited and withdrawn. Interest on the money in a savings account is paid at regular intervals. Some banks do this every 3 mo. (quarterly); some every 6 mo. (semiannually); and others every year (annually). When Susan opened her savings account, she was told that the bank would pay interest on her money at 4% per year and that the interest would be computed and credited to her account as of January 1 and July 1.

In ex. 2 ask pupils how the bank paid interest (semiannually).

2. Susan opened her account with a deposit of \$15 on July 1, 1962. She made no more deposits before January 1, 1963. At 4%, how much interest did the bank add to her account as of January 1? \$.30

3. Early in January, 1963, Susan deposited \$17 in her savings account. What was the total amount in her account then if she had not drawn out any money? \$32.30

Emphasize.⟶

4. Banks do not pay interest on a fractional part of a dollar. If Susan made no more deposits between January 1 and July 1, what was the total amount in her account on July 1, 1963, after the bank had added the interest at 4% then due? \$32.94

5. Frank has a savings account at the same bank where Susan has her account. On January 1, 1963, there was \$137.83 in his account. If he makes no additional deposits, how much will be in his account on July 1, 1963, including interest at 4%? \$140.57

6. Try to find the rate of interest that banks in your city pay on savings accounts. Do they compute and credit interest quarterly, semiannually, or annually? Have pupils make up and solve problems based on this new information.

7. What is the difference between the interest at $3\frac{1}{2}\%$ and at 4% on \$100 for 1 year payable annually? \$.50

Emphasize meanings of terms "quarterly," "semiannually," and "annually." Do problems with class, letting different pupils explain their solutions. Ex. 6 may be used to give further practice in this work.

Give information about postal money orders and how to buy them.

Postal Money Orders

1. If you wish to send money by mail, a safe and convenient way to do so is by means of a **postal money order**, which you can purchase at the post office.

2. The fees for postal money orders depend upon the amount to be sent, and are shown on the right. Find the total cost, including the fee, for money orders for these amounts: \$3.80; \$8.75; \$24.85; \$62.50; \$87.98.

Amount of Order	Fee
\$0.01 to \$10.00	20¢
\$10.01 to \$50.00	30¢
\$50.01 to \$100.00	35¢

\$4.00 \$8.95 \$25.15 \$62.85 \$88.33

3. Postal money orders were first issued in 1865. During that year there were issued 74,277 money orders having a total value of \$1,360,122.52. Was the average value per money order nearer \$18 or \$19? \$18

4. In a recent year about 264,000,000 postal money orders having a total value of about \$4,958,000,000 were issued for payment in the United States. What was the average amount sent by these money orders? \$18.78 In this year about 1,630,000 postal money orders having a total value of about \$35,300,000 were issued for payment in other countries. What was the average size of these money orders? \$21.66 The fee for a money order which is to be sent to another country is, for most countries, twice the fee given in ex. 2.

Have pupils investigate current rates for money orders at the post office. If the rates have changed, use them to rework problems, and to compare former cost with the new cost.

Have pupils check rates for registered letters. If rates have changed, use them to rework problems.

5. When you receive a money order, you can cash it at a post office or a bank. When you receive the money, you must sign your name on the reverse side of the money order. Emphasize.

6. The largest amount that you can send by a single money order is $100. If you wish to send a larger amount, such as $110, you must buy two money orders, one for $100 and one for $10. Fees are charged for both money orders. What are the fees in this case? 35¢ and 20¢ How much in all would you pay the post office if you wish to send $110? $110.55

7. Tom says you can send $110 by buying money orders for $50 and $60. Is Tom's plan better than the one in ex. 6? His plan costs 15¢ more than the plan in ex. 6.

8. Find the cost, including fees, of the postal money orders for these amounts: $7.50; $7.70 $1.89; $2.09 $86.50; $86.85 $21.98; $22.28 $103.50; $104.05 $150.00; $150.65 $185.00; $185.70 $215.25; $216.25 $360.00 $361.40

9. When a letter contains money in the form of bills, it should be sent by **registered mail.** A registered letter is handled with great care so that it will not be lost. The extra fee for registering a letter can be found on page 169. John wishes to send $3 in payment of a year's subscription to *Boys' Life*. Should he send a postal money order, or three $1 bills in a registered letter? Explain. The money order costs 20¢; the registration fee for $3 is 60¢. The money order is cheaper.

10. If you make a purchase at a store in a distant city and do not wish to pay in advance, you may have your purchase sent by parcel post **collect on delivery.** The postman will collect from you the cost of your purchase and send the money to the store, using a postal money order. You will also have to pay a C.O.D. collection fee and the fee for this money order. The C.O.D. collection fee for a package having a value of $37.50 is 80¢. What is the money order fee? 30¢ What is the total amount that must be paid when this C.O.D. package is received? $38.60

11. Jean's mother ordered a blouse priced at $6.50. She asked that it be sent to her by parcel post C.O.D. The C.O.D. collection fee for this amount is 50¢. What is the total amount that must be paid when the blouse is received? $7.20

Ex. 5-7 should be done orally. Have pupils complete ex. 8. Check work and then hold class discussion of ex. 9-10, including reasons for collection fee. Have pupils do ex. 11 and some similar problems independently.

Present miscellaneous review of preceding work.

Mixed Practice

1. Round off 2.99 to the nearest tenth. 3.0

2. Multiply $2\frac{2}{3}$ by 7; $18\frac{2}{3}$ multiply $17\frac{3}{8}$ by 4. $69\frac{1}{2}$

3. Find the average of $3\frac{5}{16}$, $9\frac{1}{2}$, $7\frac{3}{4}$, and $5\frac{3}{16}$. $6\frac{7}{16}$

4. Subtract 47,693 from 60,000. 12,307

5. Which is larger, $\frac{5}{8}$ or .627? .627

6. What per cent of 465 is 930? 200%

7. Divide 15 by $\frac{2}{3}$; $22\frac{1}{2}$ divide 15 by $1\frac{2}{3}$. 9

8. What part of a bushel is 8 qt.? $\frac{1}{4}$ 12 qt.? $\frac{3}{8}$ 24 qt.? $\frac{3}{4}$

9. Find $4\frac{1}{2}\%$ of \$39.35 correct to the nearest cent. \$1.77

10. Find the quotient when 4567.2 is divided by .88. 5190

11. Find the product of 16, $12\frac{1}{3}$, and $10\frac{7}{8}$. 2146

12. Find $3\frac{1}{2}\%$ of \$12,555 correct to the nearest cent. \$439.43

13. Find the interest on \$1650 for 1 yr. at 5%. \$82.50

14. Multiply 31.8 by .002 and divide the product by .03. 2.12

15. Find the average of 9.68, 4.42, 6.12, 8.07, and 1.51. 5.96

16. Add 62, 312, and 76. Then multiply the sum by 12. 5400

17. When a car that cost \$2400 sells for \$1350, what is the per cent of decrease in value to the nearest whole per cent? 44%

18. Find the difference between \$28,247.58 and \$17,959.50. \$10,288.08 Then divide the difference by 12. \$857.34

19. Find the per cent of increase if \$1550 increases to \$1674. 8%

20. What part of a mile does Jack run in going 4 times around a 660-foot circular track? $\frac{1}{2}$

21. What is the per cent of discount when a sweater marked \$7.95 is sold for \$6.36? 20%

22. Change the fraction $\frac{241}{716}$ to a per cent correct to the nearest whole per cent. 34%

This review may indicate need for further specialized review or reteaching of certain topic. Try to clear up difficulties through class discussion of solutions. Plan remedial activities as needed.

Sending Money by Telegraph

Amount of Order	Fee
$20.00 or less	$.50
$20.01 to $25.00	.55
25.01 to 30.00	.60
30.01 to 35.00	.65
35.01 to 40.00	.70
40.01 to 45.00	.75
45.01 to 50.00	.80
50.01 to 60.00	.90
60.01 to 70.00	1.00
70.01 to 80.00	1.10
80.01 to 90.00	1.20
90.01 to 100.00	1.30

1. The quickest way you can send money to a person in another place is to use a telegraphic money order. You can purchase these money orders at any telegraph office. The money order fees depend upon the amount of money to be sent and are given in the table at the right. In addition to the money order fee, you must pay for a 15-word telegram to the place where the money is being sent; this charge depends upon the distance the message is sent. You must also pay a Federal tax of 10% on the cost of the telegram. The figures given below show the total cost of sending $75 by telegraph to a person in a place where the rate for a 15-word telegram is $1.30. Show how the Federal tax of 13¢ was found.

Amount sent	$75.00
Money order fee	1.10
Telegram charge	1.30
Federal tax	.13
Total	$77.53

2. Find the total cost of sending $37.50 by telegraph to a person in a place where the rate for a 15-word telegram is $1.20; $1.30; $1.45; $1.60. In finding the tax of 10%, consider any fraction of a cent as another cent. Stress.

$39.52 $39.63 $39.80 $39.96

3. What is the total cost of sending $51.75 by telegraph to a place where the 15-word rate is $1.75? $1.90? $54.58 $54.74

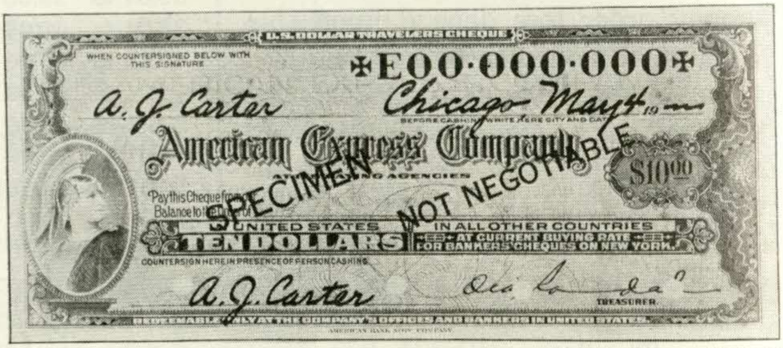
4. Find the total cost of sending $63 by telegraph to a person in a city where the rate for a 15-word telegram is $1.10; $1.30; $1.45; $1.60. $65.76 $65.43 $65.60

5. How much more does it cost to send the money in ex. 1 by telegraphic money order than by postal money order? See Guide. See page 278. Under what circumstances would you consider it important to pay the extra cost to send the money by telegraph? Give several answers. See Guide.

Follow development and discussion as given in ex. 1. Do ex. 2 as class activity and have pupils complete ex. 3-4. It might be well to do ex. 5 with class also.

How Travelers Carry Money

1. When Mr. Carter went on his vacation trip, he decided that he would need to take $250 with him to pay his expenses. Instead of taking $250 in cash, he took only $50 in cash and bought **travelers checks** amounting to $200. Why did he do this? A travelers check is shown below.



U.S. DOLLAR TRAVELERS CHEQUE

WHEN COUNTERSIGNED BELOW WITH THIS SIGNATURE

✠E00·000·000✠

A. J. Carter

Chicago May 4 19—

BEFORE CASHING WRITE HERE CITY AND DATE

American Express Company

AGENCIES

$10.00

Pay this Cheque from Balance to the Order of

UNITED STATES TEN DOLLARS

IN ALL OTHER COUNTRIES AT CURRENT BUYING RATE FOR BANKERS' CHEQUES ON NEW YORK.

COUNTERSIGN HERE IN PRESENCE OF PERSON CASHING

A. J. Carter

TREASURER

SPECIMEN NOT NEGOTIABLE

American Express Company

2. Travelers checks are sold by banks and express offices and are issued in denominations of $10, $20, $50, and $100. When Mr. Carter bought the checks, he had to write his name in the upper left corner of each check. When he cashes a check he signs his name again in the lower left corner in the presence of the person cashing the check. Unless the signatures agree, the check will not be cashed. Hotels, stores, and banks will usually cash travelers checks. Is it wise for Mr. Carter to sign a check in the lower left corner before he is ready to cash it? Why not?

3. When Mr. Carter bought checks amounting to $200, he was charged a fee equal to 1% of $200. What fee did he pay? $2

4. Mr. Walker bought eight 10-dollar checks, three 20-dollar checks, and two 50-dollar checks. Find the total amount bought. $240 Find the fee at 1% of the total amount. $2.40

5. Find the service fee of 1% (1) and the total cost of (2) travelers checks for $150; for $340; $500; $875; $1000.
(1)$1.50 (2)$343.40 (1)$8.75, (2)$883.75
(1) $1.50, (2)$151.50 (1)$5, (2)$505 (1)$10, (2)$1010

6. What advantage have these checks over personal checks? Give as many advantages as you can. See Guide.

Use ex. 1-2 to initiate discussion of advantages of using travelers checks, reasons for fee, how they are cashed, and so on. Have pupils complete ex. 3-5. Ex. 6 should also be used for class discussion.

Give quick oral check and review language of arithmetic.

Oral Review

1. Name the next two leap years.

2. Which is more, 46 oz. or $2\frac{3}{4}$ lb.? 46 oz.

3. Tell what fraction added to $\frac{1}{2}$ makes $\frac{9}{10}$. $\frac{2}{5}$

4. How many minutes is it from 7:19 P.M. to 8:08 P.M.? 49

$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2$; $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, 1\frac{1}{8}, 1\frac{3}{8}, 1\frac{5}{8}, 1\frac{7}{8}$

5. Count by $\frac{1}{4}$'s from $\frac{1}{4}$ to 2; from $\frac{1}{8}$ to $1\frac{7}{8}$.

6. How much is 1450 ÷ 100? 14.5 236 ÷ 100? 2.36 3.87 ÷ 100? .0387

7. Read these numbers: MCCCX, 1310 DXI, 511 DCI, 601 XLIV, 44 DCCCCIX, 909 MCMLXIX. 1969

8. How much is 200% of \$200? \$400 100% of \$200? \$200

Tell the number to put in each space:

9. 45 min. = $\frac{3}{4}$ . . hr. $\frac{1}{2}$ bu. = 16 . . . qt. 10 oz. = $\frac{5}{8}$ . . . lb.

10. 12 gal. = 96 . . . pt. $\frac{5}{8}$ yd. = $22\frac{1}{2}$ . . . in. $2\frac{1}{2}$ pk. = 20 . . . qt.

11. 10 tons = 20,000 . . . lb. $\frac{3}{4}$ lb. = 12 . . . oz. $3\frac{1}{3}$ yd. = 120 . . . in.

Review in ex. 1-11 may be conducted as quiz program, with different pupils acting as quiz master.

The Language of Arithmetic

In ex. 12 to 29, show that you know what each word or group of words means by using it in a sentence or by giving an example:

12. protractor
13. gas meter
14. pictograph
15. parcel post
16. rounding off
17. size of angle
18. parallel lines
19. formula
20. diameter
21. travelers check
22. deposit slip
23. circle graph
24. postal money order
25. rate of interest
26. promissory note
27. per cent of increase
28. installment price
29. kilowatt-hour

Instruct pupils to use words in ex. 12-29 in sentences and examples which are sensible. If pupils show lack of understanding, redevelop meanings of terms.

Chapter Review

1. How do banks earn money so that they can pay interest on savings accounts? By lending money to reliable people; see Guide.

2. Tell two things a person must do to open a checking account in a bank. Give references; leave a copy of his signature at the bank. See text page 266.

3. Make out a deposit slip for these deposits: bills, \$15.00; check on Newton bank, \$34.25; check on Dayton bank, \$10.00; check on Johnstown bank, \$18.50.

4. Why is it important to fill out the stub each time you write a check? You might forget that you wrote the check.

5. What is a "certified" check? When would you need to have a check certified? See text page 267.

6. Write a check for \$35.16 on your home bank. Make the check payable to George Jones.

7. Suppose you are George Jones. Endorse the check in blank. Which is safer, an endorsement in blank or an endorsement in full?

8. Why is it important to keep your canceled checks? They serve as receipts for payments made.

9. What does it mean to overdraw your account? You draw a check for an amount that is larger than your balance in the bank; see Guide.

10. What is the interest on \$275 for 1 yr. at 4%? \$11 at 5%? \$13.75

11. What is the interest on \$400 for 6 mo. at 4%? \$8 at 6%? \$12

12. Use the interest formula to find the interest on \$500 at 4% for 3 mo.; \$5 on \$850 at 6% for 4 mo. \$17

13. If you pay interest at the rate of "$1\frac{1}{2}$% monthly," what yearly rate of interest are you paying? 18%

14. Write a promissory note due in 3 mo. for \$200 with interest at 4% dated today. Find the interest \$2 and the date of maturity.

15. How does a savings account differ from a checking account?

16. If you wish to send \$8 to a store in Chicago, which is cheaper, to send a postal money order or bills in a registered letter? Money order How much cheaper? 40¢ See pages 169 and 278.

Chapter review may be considered as readiness test for new work. Pupils' answers should be checked carefully and re-explanation of topic given if needed. Assign remedial work after difficulties are cleared up.

A Problem Test

1. From camp Bill sent a 21-word telegram to his mother. The charge was \$1.20 for the first 15 words and $5\frac{1}{2}$¢ for each extra word. There was also a 10% tax on the total cost of the telegram. How much did Bill pay to send the telegram? \$1.68
2. The tank of Mr. Brown's car holds 16 gal. of gasoline. On a trip in Canada the tank was only $\frac{1}{4}$ full when Mr. Brown had it filled up. Since a Canadian gallon is about 1.2 times as large as a United States gallon, how many Canadian gallons did Mr. Brown buy? 10
3. A jet plane flew 175 mi. in 21 min. 48 sec. Find, to the nearest mile per hour, the speed of the plane. 481 mph
4. Mr. Day earned \$3500 last year. 14% He spent \$700 for rent, 20% 23% \$800 for food, and \$490 for clothing. What per cent of \$3500, to the nearest whole per cent, was spent for each item?
5. Mr. Thomas raised 560 bu. of potatoes last year. This year he raised only 420 bu. This year's crop was what per cent less than last year's crop? 25%
6. Find the interest on \$1500 for $1\frac{1}{2}$ yr. at 4%. \$90
7. Find the cost of sending a 14-pound box by parcel post at 58¢ for the first two pounds, and 16¢ for each additional pound. \$2.50
8. Mr. Beach has made a down payment of \$982 on a car and is now paying \$98.50 a month for 12 mo. Find the total amount Mr. Beach will pay for the car. \$2164
9. Tom wants to save \$40 in the next 10 wk. He earns \$3.50 a week and gets an allowance of \$1.25 a week. If he saves $\frac{4}{5}$ of this money, how much short of his goal will he be? \$2
10. At an average speed of 275 mi. per hour, how far can an airplane travel in 8 hr.? 2200 mi.

Have pupils record scores.

SCORE	0–5	5–6	8–9	10
	You need help	Fair	Good	Excellent

Instruct pupils to read problems carefully. Confer with pupils who had mistakes to try to determine causes. Their explanations will give you insight into the way they approach problems.

Present diagnostic test with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Add and check the work:

					Practice Pages
1.	$\begin{array}{r} 4\frac{5}{6} \\ 2\frac{1}{4} \\ 2\frac{2}{3} \\ \hline 9\frac{3}{4} \end{array}$	$\begin{array}{r} 1\frac{3}{4} \\ 4\frac{4}{5} \\ 2\frac{1}{2} \\ \hline 9\frac{1}{20} \end{array}$	$\begin{array}{r} 7\frac{1}{3} \\ 3\frac{1}{5} \\ 4\frac{1}{2} \\ \hline 15\frac{1}{30} \end{array}$	$\begin{array}{r} 1\frac{3}{8} \\ 1\frac{1}{6} \\ 2\frac{2}{3} \\ \hline 5\frac{5}{24} \end{array}$	62

Subtract and check the work:

2.	$\begin{array}{r} 6 \\ 3\frac{1}{8} \\ \hline 2\frac{7}{8} \end{array}$	$\begin{array}{r} 4\frac{1}{2} \\ 2\frac{11}{16} \\ \hline 1\frac{13}{16} \end{array}$	$\begin{array}{r} 9\frac{3}{10} \\ 8\frac{1}{2} \\ \hline \frac{4}{5} \end{array}$	$\begin{array}{r} 7\frac{1}{8} \\ 4\frac{5}{8} \\ \hline 2\frac{1}{2} \end{array}$	58, 59
3.	$\begin{array}{r} 5\frac{1}{4} \\ 2\frac{5}{6} \\ \hline 2\frac{5}{12} \end{array}$	$\begin{array}{r} 7\frac{11}{12} \\ 4\frac{1}{8} \\ \hline 3\frac{19}{24} \end{array}$	$\begin{array}{r} 9\frac{3}{10} \\ 5\frac{3}{4} \\ \hline 3\frac{11}{20} \end{array}$	$\begin{array}{r} 3\frac{1}{6} \\ 2\frac{3}{10} \\ \hline \frac{13}{15} \end{array}$	61

Multiply and check the work:

4.	$\begin{array}{r} .008 \\ 7 \\ \hline .056 \end{array}$	$\begin{array}{r} 9.37 \\ 24 \\ \hline 224.88 \end{array}$	$\begin{array}{r} 438.4 \\ .129 \\ \hline 56.5536 \end{array}$	$\begin{array}{r} 67.5 \\ .24 \\ \hline 16.200 \end{array}$	96, 98

Divide and check the work:

5.	$21 \div \frac{7}{8}$ 24	$\frac{5}{6} \div \frac{1}{5}$ $4\frac{1}{6}$	$\frac{15}{16} \div \frac{9}{10}$ $1\frac{1}{24}$	72, 73
6.	$2\frac{1}{2} \div \frac{3}{8}$ $6\frac{2}{3}$	$1\frac{1}{8} \div \frac{2}{3}$ $1\frac{11}{16}$	$5\frac{1}{4} \div \frac{3}{4}$ 7	74
7.	$3\frac{3}{4} \div 10$ $\frac{3}{8}$	$9\frac{1}{3} \div 1\frac{1}{6}$ 8	$1\frac{1}{2} \div 1\frac{3}{5}$ $\frac{15}{16}$	75, 76

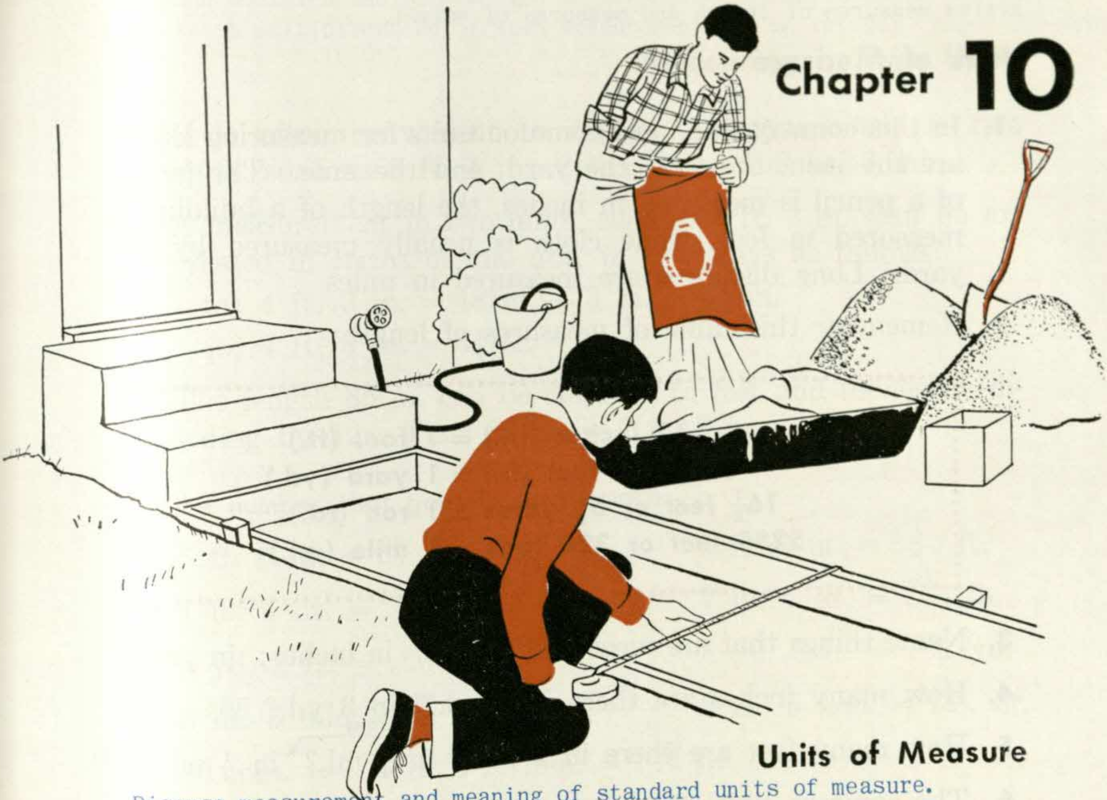
Find the answers to the nearest cent:

8.	4% of $35 $1.40	75% of $52 $39	$12\frac{1}{2}$% of $32 $4	128–136
9.	$1\frac{1}{2}$% of $20 $.30	$5\frac{1}{4}$% of $48 $2.52	$15\frac{2}{3}$% of $60 $9.40	144
10.	100% of $13 $13	250% of $92 $230	173% of $50 $86.50	146
11.	3% of $6.25 $.19	125% of $4.33 $5.41	$62\frac{1}{2}$% of $15 $9.38	138

Note that skills tested are: three processes with fractions, multiplication of decimals, finding a per cent of a number. Reteach as needed, using developmental material in text. Then assign remedial work.

See Guide for specific aims of Chapter 10.

Chapter 10



Units of Measure

Discuss measurement and meaning of standard units of measure.

1. In everyday life, numbers are used in **counting** and in **measurement.** You find the number of pupils in a room by counting, but you find the length of a room by measuring. Measurement is very important in everyday life; without it automobiles, trains, bridges, planes, houses could not be built.

2. Whenever you measure anything, you must have a **unit of measure,** such as the foot, the mile, the pound, or the gallon. These units are called **standard units** because they represent the same amount wherever they are used. For example, if a box weighs 5 lb. in Boston it will also weigh 5 lb. in San Francisco. Have pupils investigate how units of measure were standardized and how they are preserved.

3. When you measure something, you also need a measuring instrument. To measure length you can use a ruler, a yardstick, or a 100-foot steel tape. To measure weight you will need some kind of scale.

See Guide for audio-visual aids pertinent to this topic. Use tables of measures on pages 342-343 to review the various units of measure.

Review measures of length and measures of weight.

Units of Measure

1. In this country, the most common units for measuring length are the inch, the foot, the yard, and the mile. The length of a pencil is measured in inches, the length of a building is measured in feet, while cloth is usually measured by the yard. Long distances are measured in miles.

2. Remember this table of measures of length:

12 inches (in.) = 1 foot (ft.)
3 feet (ft.) = 1 yard (yd.)
$16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards = 1 rod (rd.)
5280 feet or 320 rods = 1 mile (mi.)

3. Name things that are measured in feet; in inches; in yards.

4. How many inches are there in 1 yd.? 36 in 3 yd.? 108

5. How many feet are there in 1 mi.? 5280 in $\frac{1}{4}$ mi.? 1320 in $\frac{1}{2}$ mi.? 2640

6. The common units for measuring weight are the ounce, the pound, and the ton. The weight of canned food is measured in ounces, the weight of a man is measured in pounds, while coal is measured in tons. Freight rates are usually given by the **hundredweight.** Cattle are bought and sold by the hundredweight.

7. Remember this table of measures of weight:

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

8. How many ounces are there in 3 lb.? 48 8 lb.? 128 $2\frac{1}{2}$ lb.? 40

9. How many hundredweight equal 350 lb.? $3\frac{1}{2}$ 4700 lb.? 47

Give more practice as in ex. 9.

10. How many tons equal 10,000 lb.? 5 7000 lb.? $3\frac{1}{2}$

Emphasize that size of object to be measured determines unit to be used. Give pupils opportunity to select appropriate units for various things. Make up problems such as finding cost of shipping 225 lb. at $1.70 a hundredweight.

Reteach addition and subtraction of measurements expressed in two units, and teach multiplication of such measurements by an integer (pages 289–290).

Measuring in Two Units

1. It is often convenient to express a measurement in two units, such as 5 ft. 4 in., 2 lb. 5 oz., or 3 hr. 7 min.

2. A measurement in two units, such as 4 ft. 3 in., can be expressed in terms of one unit in two ways as follows:

(a) 4 ft. 3 in. = 48 in. + 3 in. = 51 in.

(b) 4 ft. 3 in. = $4\frac{1}{4}$ ft.

3. The length 86 in. can be changed to feet and inches by dividing by 12. Thus 86 in. = 7 ft. 2 in.

Tell the number that goes in each space:

4. 5 ft. 6 in. = $\overset{66}{\ldots}$ in. 5 ft. 6 in. = $\overset{5\frac{1}{2}}{\ldots}$ ft.

5. 7 lb. 4 oz. = $\overset{116}{\ldots}$ oz. 7 lb. 4 oz. = $\overset{7\frac{1}{4}}{\ldots}$ lb.

6. 8 yd. 2 ft. = $\overset{26}{\ldots}$ ft. 8 yd. 2 ft. = $\overset{8\frac{2}{3}}{\ldots}$ yd.

7. 3 hr. 5 min. = $\overset{185}{\ldots}$ min. 3 hr. 5 min. = $\overset{3\frac{1}{12}}{\ldots}$ hr.

8. 80 min. = $\overset{1}{\ldots}$ hr. $\overset{20}{\ldots}$ min. = $\overset{1\frac{1}{3}}{\ldots}$ hr.

9. 92 in. = $\overset{7}{\ldots}$ ft. $\overset{8}{\ldots}$ in. = $\overset{7\frac{2}{3}}{\ldots}$ ft.

10. **Problem** Sally plans to practice her piano lessons a total of at least 5 hr. every week. During one week her record was: Monday, 1 hr. 55 min.; Wednesday, 1 hr. 20 min.; Friday, 1 hr. 5 min.; Saturday, 50 min. Find the total time Sally practiced that week. 5 hr. 10 min.

Explanation Add the minutes, which gives 130 min. Then add the hours, which gives 3 hr. The total is 3 hr. 130 min. But 130 min. can be changed to 2 hr. 10 min. Thus 3 hr. 130 min. equals 5 hr. 10 min. Sally practiced 5 hr. 10 min. that week.

1 hr.	55 min.
1 hr.	20 min.
1 hr.	5 min.
	50 min.
3 hr.	130 min.
or 5 hr.	10 min.

Add:

11.

6 ft. 5 in.	7 ft. 9 in.	3 hr. 35 min.	8 lb. 5 oz.
9 ft. 8 in.	3 ft. 4 in.	7 hr. 48 min.	6 lb. 14 oz.
4 ft. 9 in.	9 ft. 5 in.	2 hr. 26 min.	5 lb. 9 oz.
20 ft. 10 in.	20 ft. 6 in.	13 hr. 49 min	20 lb. 12 oz.

Discuss ex. 1–3 thoroughly with class. Have pupils explain work in ex. 4–9. In ex. 10 emphasize that minutes are added to minutes and hours to hours.

Measuring in Two Units

1. **Problem** Frank has a board 8 ft. 6 in. long. If he cuts off a piece 2 ft. 9 in. long, how long will the remaining piece be? 5 ft. 9 in.

Explanation You cannot subtract 9 in. from 6 in., so change 8 ft. 6 in. to 7 ft. 18 in. Then you can subtract, and the result is 5 ft. 9 in. The remaining piece will be 5 ft. 9 in. long.

8 ft. 6 in. = 7 ft. 18 in.
2 ft. 9 in. = 2 ft. 9 in.
5 ft. 9 in.

Subtract:

2. 9 lb. 12 oz.
4 lb. 9 oz.
5 lb. 3 oz.

7 ft.
3 ft. 4 in.
3 ft. 8 in.

4 hr. 15 min.
1 hr. 40 min.
2 hr. 35 min.

3. 8 yd. 7 in.
3 yd. 14 in.
4 yd. 29 in.

23 lb. 5 oz.
19 lb. 13 oz.
3 lb. 8 oz.

12 hr.
3 hr. 37 min.
8 hr. 23 min.

4. **Problem** In a sewing class, four girls have decided to make their aprons just alike. The amount of material needed for each apron is 1 yd. 24 in. How much material will be needed for all four aprons? 6 yd. 24 in.

Explanation Multiply 24 in. by 4, which gives 96 in. Also multiply 1 yd. by 4, which gives 4 yd. The result is 4 yd. 96 in. But 96 in. can be changed to 2 yd. 24 in. So 4 yd. 96 in. equals 6 yd. 24 in. The four aprons will require 6 yd. 24 in. of material in all.

1 yd. 24 in.
4
4 yd. 96 in.
or 6 yd. 24 in.

5. In ex. 4, it requires 4 yd. 16 in. of braid to trim each apron. How many yards of braid, to the nearest whole yard, will be needed for the 4 aprons? 18

6. Multiply 3 yd. 13 in. by 6; by 8; by 10. 20 yd. 6 in. 26 yd. 32 in. 33 yd. 22 in.

Multiply:

7. 4 ft. 7 in.
3
13 ft. 9 in.

8 lb. 9 oz.
8
68 lb. 8 oz.

6 lb. 14 oz.
5
34 lb. 6 oz.

4 hr. 27 min.
4
17 hr. 48 min.

More Practice. See (52) on page 341.

When discussing ex. 1 have pupils explain why and how 8 ft. 6 in. is changed to 7 ft. 18 in. Ex. 4 should be explained thoroughly. Emphasize that smaller units should be changed to larger ones in answer when possible.

Have pupils make several measurements and give their results as they wish. This will produce different approximations and show the need for stating the precise measurement.

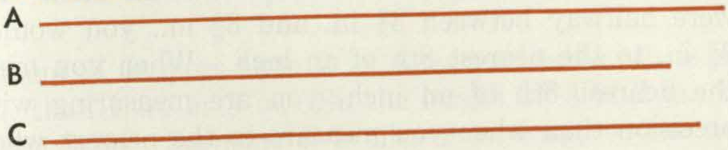
The Nature of Measurement

Stress.

1. When you measure anything, the results are **approximate;** but some measurements are made more precisely than others. Suppose you wish to get the length of the segment A in terms of **whole inches.** A is more than 3 in. long and less than 4 in. long, but you see it is closer to 4 in. than to 3 in. So you say that A has a length of 4 in., correct to the **nearest whole inch.** Before pupils measure lines, make sure they all have found mark on rulers where first inch begins.

Stress meaning.

Stress.



The segment B is over 3 in. long, but it is closer to 3 in. than to 4 in. In this case you say that B is 3 in. long, to the nearest whole inch. The length of the segment C is halfway between 3 in. and 4 in., or $3\frac{1}{2}$ in. But if you are measuring to the **nearest whole inch,** you call the length 4 in. In this case a half inch is called another whole inch, just as $3\frac{1}{2}$¢ is called 4¢ to the nearest whole cent. Emphasize. Have pupils do all activities in ex. 2–7, under your direction.

2. Measure, correct to the nearest inch, the length and width of your desk; the lengths and widths of several books; the lengths of several pencils.
3. Measure, to the nearest foot, the length and width of your classroom; the length and width of the blackboard.
4. Measure, to the nearest inch, the heights of several of your classmates.
5. What is your weight to the nearest pound?
6. Joe found the length of his garden to be about 28 ft. 10 in. and its width about 15 ft. 4 in. Give each dimension, correct to the nearest foot. 29 ft.; 15 ft.
7. Express each of the following measurements, correct to the nearest foot:

16 ft. 9 in. 17 ft 43 ft. 2 in. 43 ft. 89 ft. 11 in. 90 ft. 23 ft. 6 in. 24 ft.

To develop a sense of distance, you can accompany the activities of this page with the making of estimates prior to measurement.

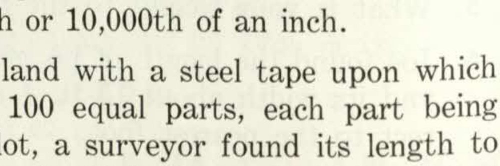
Measuring with More Precision

1. In ex. 1 on page 291, you measured the length of each line to the nearest whole inch. If you wish to get the lengths more precisely, you can measure each line to the nearest 8th of an inch. In the figure on page 291, the length of A is closer to $3\frac{5}{8}$ in. than to $3\frac{6}{8}$ in., so you say that A is $3\frac{5}{8}$ in. long, to the nearest 8th of an inch.

On page 291, line B is closer to $3\frac{3}{8}$ in. than to $3\frac{2}{8}$ in., so you call it $3\frac{3}{8}$ in. to the nearest 8th of an inch. If line B were halfway between $3\frac{2}{8}$ in. and $3\frac{3}{8}$ in., you would call it $3\frac{3}{8}$ in. to the nearest 8th of an inch. When you measure to the nearest 8th of an inch, you are measuring with more precision than when you measure to the nearest whole inch. But measuring to the nearest 8th of an inch is **approximate** measurement. Stress.

2. Measure, to the nearest 8th of an inch, the length and width of a sheet of paper; the length, width, and height of a small box; the length and width of a post card; the length and width of this book.

3. In science and industry, more precise measurements are usually made by dividing the inch into 100, 1000, or 10,000 equal parts. The micrometer shown on the right is an instrument that will make measurements correct to the nearest 1000th of an inch. Some parts of an automobile are measured to the nearest 1000th or 10,000th of an inch.



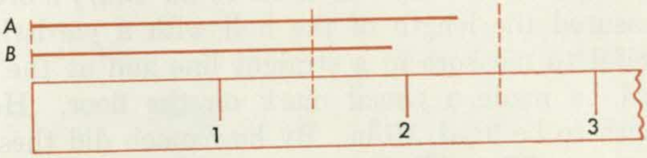
4. Surveyors often measure land with a steel tape upon which the foot is divided into 100 equal parts, each part being .01 ft. In measuring a lot, a surveyor found its length to be closer to 62.45 ft. than to 62.44 ft. or 62.46 ft. What did he call the length of the lot, to the nearest 100th of a foot? 62.45 ft.

As ex. 1 is discussed, have pupils remeasure lines on page 291. Then have pupils do ex. 2. Bring out fact that all measurement is approximate. Give pupils other opportunities to measure lines and objects to reinforce this concept.

Review reading the fractional parts of the inch on pupils' rulers. Give practice in reading a marking on the ruler, and the marking on each side of it as well.

More about Measurement

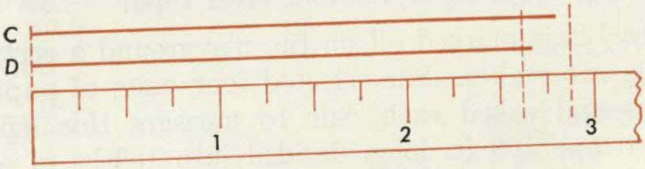
1. The diagram below shows a portion of a ruler marked in inches and two line segments whose left endpoints are in line with the "0" point of the ruler.



Segment *A* is more than 2 in. long but its length is nearer 2 in. than 1 in. or 3 in. So the length of *A* is 2 in., to the nearest inch. Segment *B* is also 2 in. long, to the nearest inch. Why? You can see from the diagram that **any** segment whose real length is between $1\frac{1}{2}$ in. and $2\frac{1}{2}$ in. is called 2 in. long, to the nearest inch.

2. When *A* is called 2 in. long, to the nearest inch, you can also see from the diagram that the greatest possible difference between its real length and 2 in. is $\frac{1}{2}$ in. So the **greatest possible error** in this measurement of *A* is $\frac{1}{2}$ in.

3. The diagram below shows a portion of a ruler marked in inches and fourths of an inch, and two segments whose left endpoints are in line with the "0" point of the ruler.



Segment *D* is less than $2\frac{3}{4}$ in. long but its length is nearer $2\frac{3}{4}$ in. than $2\frac{1}{2}$ in. or 3 in. So the length of *D* is $2\frac{3}{4}$ in., to the nearest $\frac{1}{4}$ in. Explain why *C* is also $2\frac{3}{4}$ in. long, to the nearest $\frac{1}{4}$ in. You can see from the diagram that any segment whose real length is between $2\frac{5}{8}$ in. and $2\frac{7}{8}$ in. is called $2\frac{3}{4}$ in. long, to the nearest $\frac{1}{4}$ in. What is the greatest possible error when the length of *D* is called $2\frac{3}{4}$ in., to the nearest $\frac{1}{4}$ in.? $\frac{1}{8}$ in.

Develop the generalization that the greatest possible error is always one half of the smallest subdivision of the unit of measure that is used in expressing the measurement.

Show how easily mistakes can be made when measuring.

Measurement Difficulties

1. Mary measured the length of the hall for her mother. She used a foot ruler and marked the end of each foot with her finger as she moved the ruler along the hall. Her result for the length of the hall was 28 ft. 10 in. Mary's brother George measured the length of the hall with a yardstick. He was careful to measure in a straight line and at the end of each yard he made a pencil mark on the floor. He found the length to be 9 yd. 15 in. By how much did these two measurements differ? 7 in. Can you give some reasons for this? Which measurement do you think was more accurate? Have pupils also explain why one measurement was more accurate.

2. When Mary's father came home, he measured the length of the hall with a 100-foot steel tape and found the length to be 28 ft. 2 in. By how much and by what per cent did Mary's result differ from the result obtained by her father? 8 in; 2% By how much and by what per cent did George's result differ from his father's result? Give your answers to the nearest whole per cent. 1 in.; 0%

3. Measure the length of your classroom with a foot ruler, with a yardstick, and with a steel tape. Find out by what per cent the results obtained with the ruler and the yardstick differ from that obtained with the steel tape. Find per cents to the nearest whole per cent. What difficulties are avoided by using a 100-foot steel tape?

4. Miss Lewis marked off on the playground a certain distance with two stakes. She selected four pairs of pupils from her class and asked each pair to measure this length with a steel tape 100 ft. long, divided into 100ths of a foot. The results obtained for the length of the distance were:

73.85 73.86 73.87 73.90

Why were these results different? What is the average of these four measurements? 73.87 ft.

Ex. 1–2 should be discussed with class. If a 100-foot steel tape is available, have pupils follow directions in ex. 3. Have several sets of 3 pupils measure with ruler, yardstick, and tape, and compare results.

Present last sets of improvement tests in multiplication and division.

Improving by Practice

Multiplication Test 6a. Time: 5 min. after copying.

1. $\begin{array}{r}7538\\ \underline{4612}\\ 34,765,256\end{array}$ $\begin{array}{r}2019\\ \underline{3875}\\ 7,823,625\end{array}$ $\begin{array}{r}4672\\ \underline{3906}\\ 18,248,832\end{array}$ $\begin{array}{r}5843\\ \underline{2700}\\ 15,776,100\end{array}$ ④

Multiplication Test 6b. Time: 5 min. after copying.

2. $\begin{array}{r}3650\\ \underline{2761}\\ 10,077,650\end{array}$ $\begin{array}{r}3548\\ \underline{2497}\\ 8,859,356\end{array}$ $\begin{array}{r}9632\\ \underline{5174}\\ 49,835,968\end{array}$ $\begin{array}{r}2617\\ \underline{6900}\\ 18,057,300\end{array}$ ④

Multiplication Test 6c. Time: 5 min. after copying.

3. $\begin{array}{r}4083\\ \underline{9714}\\ 39,662,262\end{array}$ $\begin{array}{r}2569\\ \underline{3852}\\ 9,895,788\end{array}$ $\begin{array}{r}7148\\ \underline{6307}\\ 45,082,436\end{array}$ $\begin{array}{r}3765\\ \underline{8500}\\ 32,002,500\end{array}$ ④

Division Test 6a. Time: 6 min. after copying.

4. $\begin{array}{r}60.4\\ 654\overline{)39525}\end{array}$ $\begin{array}{r}34.5\\ 894\overline{)30843}\end{array}$

5. $\begin{array}{r}78.3\\ 535\overline{)41875}\end{array}$ $\begin{array}{r}77.4\\ 765\overline{)59211}\end{array}$ ④

Division Test 6b. Time: 6 min. after copying.

6. $\begin{array}{r}40.0\\ 737\overline{)29463}\end{array}$ $\begin{array}{r}91.8\\ 515\overline{)47277}\end{array}$

7. $\begin{array}{r}80.5\\ 876\overline{)70518}\end{array}$ $\begin{array}{r}14.2\\ 963\overline{)13678}\end{array}$ ④

Division Test 6c. Time: 6 min. after copying.

8. $\begin{array}{r}82.2\\ 465\overline{)38223}\end{array}$ $\begin{array}{r}37.7\\ 636\overline{)23975}\end{array}$

9. $\begin{array}{r}23.5\\ 874\overline{)20548}\end{array}$ $\begin{array}{r}87.5\\ 542\overline{)47425}\end{array}$ ④

To the Teacher. In ex. 4–9, instruct pupils to give quotients to nearest tenth. Pupils should complete graphs begun on page 50, and compare scores with previous ones to note further needs. Confer with pupils who had errors to determine causes. Assign remedial work after difficulties are cleared up.

Teach pupils how to measure distances roughly by pacing. Have each pupil find average length of his step as explained in ex. 1. Emphasize importance of walking at natural pace.

Measuring Distances by Pacing

It is convenient to be able to measure a distance roughly by **pacing** or stepping it off. Boy Scouts frequently measure distances in this way. In order to measure by pacing, you must know the average length of your step or **pace.**

1. Henry wanted to find the average length of his pace. He drew a starting line and stood with his heels on this line. Then, at his natural pace, he took 30 steps in a straight line and marked the position of his foot, at the heel, on the last step. He measured the distance that he had walked and found it to be 75 ft. Thus the length of 30 of Henry's steps is 75 ft. Dividing 75 ft. by 30, Henry found the average length of his step to be $2\frac{1}{2}$ ft.

2. Henry paced off the length of a piece of land, taking 71 steps. What is the length of this piece of land? What does $71 \times 2\frac{1}{2}$ ft. equal? $177\frac{1}{2}$ ft.

3. Mary measured the distance she walked in 40 paces and found it to be 90 ft. What is the average length of her
$2\frac{1}{4}$ ft. step? How many feet does she walk when she takes 60 steps? 80 steps? 120 steps? 37 steps? $83\frac{1}{4}$
135 180 270

4. A pedometer is a small instrument, somewhat like a watch, which will count the number of steps taken by the person carrying it. George's average step is $2\frac{1}{2}$ ft. long. His pedometer shows he has taken 314 steps. How far has he walked? 785 ft. How many steps would George take in walking a mile? 2112

5. George is going to take Ann's picture and he wants her to stand 15 ft. from the camera. How many steps must he take to pace off 15 ft.? 6 40 ft.? 16

6. Find the average length of your step and measure the length of the playground by pacing.

Let pupils practice measuring distances by pacing, as in ex. 2. Limit distances to those not over 250 ft. long. Check measurements by using measuring instruments.

Show how to estimate short distances by the eye.

Estimating Lengths

1. It is often desirable to be able to estimate lengths by the eye. If you have to plant seeds 1 in. deep, you should be able to make a furrow of about the right depth without using a ruler. If you have witnessed an automobile accident, you may be asked how far one of the cars was from the sidewalk just before the accident.

2. On a foot ruler, study carefully the distance represented by 1 in. Then try to draw a 1-in. length without a ruler. Measure it to see if it is close to 1 in. Repeat this several times until you can draw a line segment very nearly 1 in. long without a ruler.

3. Try to draw segments of the following lengths without using the marked side of a ruler. Check each by measuring:
   2 in. 5 in. 10 in. 7 in. 4 in. 12 in.

4. Boy and Girl Scouts are expected to be able to estimate short distances so that the error is not more than 25% of the true distance. For example, in estimating the length of a blackboard whose true length is 12 ft., the error must not be over 25% of 12 ft., which is 3 ft. So the estimate is satisfactory if it is anywhere between 9 ft. (12 ft. − 3 ft.) and 15 ft. (12 ft. + 3 ft.). Estimates such as 10 ft. or 13 ft. would be acceptable because the error in each case is less than 3 ft.

5. Jack estimated a distance to be 19 ft. When he measured it, he found it to be 16 ft. Was his estimate within 25% of the true distance? Yes What is the longest and the shortest estimate that would be satisfactory in this case? 20 ft. and 12 ft.

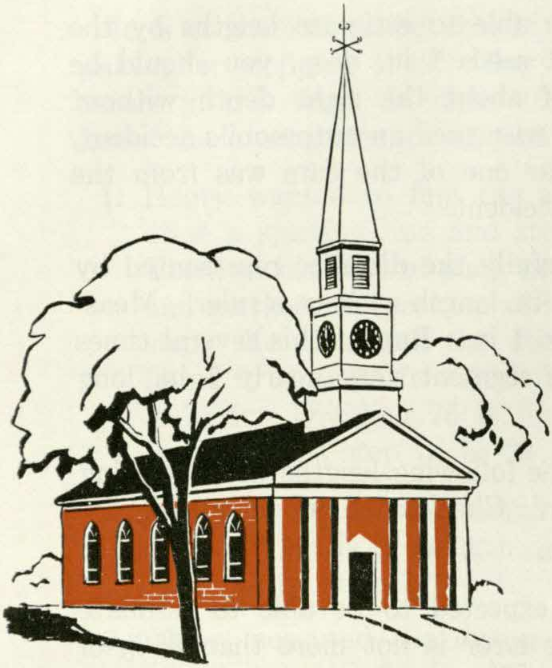
6. From one wall of your classroom, mark a point on the floor which you estimate to be 10 ft. away; 12 ft. away; 20 ft. away. Check each distance by measuring. Was each estimate within 25% of the true distance?

7. Estimate the height and the width of the door of your classroom. Are your estimates correct within 25%?

Pupils will need much practice in this work to achieve any degree of skill. Have them do suggested activities in ex. 2-3, 6, 7. Ex. 4 should be explained thoroughly so pupils understand meaning of "correct within 25%."

Show how to estimate height of tall object by comparing its height with some known height.

Estimating by Comparison



1. In the picture at the left, the white columns in front of the church are about 16 ft. high. You can use this fact to help you estimate the height of the church from the ground to the tip of the weathervane on the steeple. To do this, mark off the height of the column on the edge of a piece of paper and then see how many times this height is contained in the height of the church. About how many times is it? Is the church about 4 times, 5 times, $5\frac{1}{2}$ times, or 6 times as high as the column? $5\frac{1}{2}$ About how high is the church? 88 ft.

2. John wanted to estimate the height of a tree. He asked his father, who is 6 ft. tall, to stand near the tree. By eye, John estimated the tree to be about 4 times as high as his father. About how many feet high is the tree? 24

3. In ex. 1, if you had estimated by eye the number of times the height of the column is contained in the height of the church, would the result have been as accurate as the one you got by measuring with a piece of paper? No

4. Measure the height of one of your classmates. Then have him stand near one of the walls of the classroom and estimate the height of the room. In the same way, estimate the height of a tree; of the school flagpole.

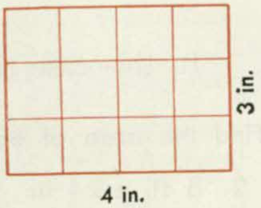
5. Measure the length of the teacher's desk or of a table in your classroom. Then, by comparison, estimate the length of the wall of the room behind the desk or table.

Pupils will need to develop ability to estimate by eye the number of times a known height is contained in a greater height. Give much practice, as in ex. 4-5, to develop this skill.

Review finding areas of rectangles and develop formulas for finding these areas (pages 299–300).

Area of a Rectangle

1. You know that a four-sided figure containing four right angles is called a **rectangle.** A **square** is a rectangle having all four sides equal. To measure the **length** and **width** of a rectangle, you use linear measure. To measure the **area** of a rectangle, you use square measure and find out how many square units its interior contains. A **square inch** is a square having each side 1 in. long. A **square foot** is a square having each side 1 ft. long. What is a square yard? Using a ruler, draw a square inch.



2. This figure represents a rectangle 4 in. long and 3 in. wide. Its area has been divided into square inches. Each row contains 4 sq. in. and there are 3 rows, so all together there are 3 × 4 sq. in., or 12 sq. in. Therefore the area of this rectangle is 12 sq. in.

3. To find the area of any rectangle, you multiply the length by the width:

   Area of rectangle = length × width

   If you let A stand for area, l for length, and w for width, you can write a **formula** for the area like this: Emphasize meaning of lw.

   $A = l \times w$ or $A = lw$

4. Draw a rectangle 6 in. long and 5 in. wide. Use your drawing to show that the area of this rectangle is 30 sq. in.

Find the areas of these rectangles, using the formula:

13.68 sq. in.

5. 9 ft. × 5 ft. 45 sq. ft. 7 in. × 4 in. 28 sq. in. 5.7 in. × 2.4 in.
6. 6 ft. × $4\frac{1}{2}$ ft. 27 sq. ft. 12 yd. × 8 yd. 96 sq. yd. 8.3 ft. × 7.8 ft.

64.74 sq. ft.

7. A rectangle has an area of 60 sq. in. Its width is 4 in. So $60 = l \times 4$. What is the unknown factor l, which stands for the length of the rectangle? 15 in.
8. A rectangle has an area of 140 sq. in. What is its length if its width is 10 in.? 4 in.? What is its width if its length is 20 in.? 7 in. 14 in. 35 in.

Discuss ex. 1 with class and use diagram to develop concepts of sq. in., sq. ft., sq. yd. Before assigning ex. 7 and 8, review how to find unknown factor of a product.

Area of a Rectangle

1. In finding the area of a rectangle, the length and the width of the rectangle must **both** be given in inches, or in feet, or in yards. If a rectangle is 7 yd. long and 12 ft. wide, first change 12 ft. to 4 yd. Then the area is

$$A = 7 \times 4 = 28$$

The area of the rectangle is 28 sq. yd.

You could also work this problem by changing 7 yd. to 21 ft. Then the area would be

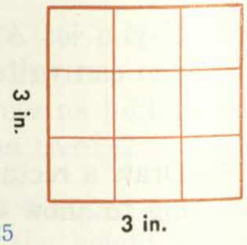
$$A = 21 \times 12 = 252$$

In this case the area of the rectangle is 252 sq. ft.

Find the area of each rectangle in two different ways:

16 sq. ft., or 2304 sq. in. 4 sq. ft., or 576 sq. in. 18 sq. yd., or 162 sq. ft.
2. 8 ft. × 24 in. 8 ft. × 6 in. 9 yd. × 6 ft.
3 sq. ft., or 432 sq in. 4 sq. ft., or 576 sq. in. $3\frac{2}{3}$ sq. yd., or 33 sq. ft.
3. 4 ft. × 9 in. 3 ft. × 16 in. 3 yd. 2 ft. × 3 ft.

Area of a Square



4. To find the area of a square having each side 3 in. long, multiply 3 by 3; that is, multiply 3 by itself. The area is 9 sq. in.

5. When you multiply a number by itself, you **square the number.** The square of 3 is 9. What is the square of 4? 16 of 5? 25

6. You know that you can use an exponent to write 3×3 as 3^2. This can be read "3 squared" as well as "3 to the second power." What is the value of 6^2? 36 of 9^2? 81 of 12^2? 144

7. If you represent each side of a square by s, the formula for its area is written

$$A = s^2$$

Using the formula, find the areas of squares having these sides:

36 sq. in. 25 sq. ft. 4 sq. mi. $\frac{9}{16}$ sq. mi. 841 sq. in. 151.29 sq. in.
8. 6 in. 5 ft. 2 mi. $\frac{3}{4}$ mi. 29 in. 12.3 in.

Emphasize that both dimensions of rectangle must be in same unit of measure. In developing formula for area of a square, remind pupils that a square is a rectangle. Then discuss ex. 4-7. Give further work as in ex. 6 and 8.

Review table of square measure and use it in problems (pages 301-303).

Square Measure

1. On a large sheet of paper, draw a square foot and divide each side of the square into inches. Then draw lines dividing the square foot into square inches. How many square inches are there in 1 sq. ft.? 144 Find the answer another way.

2. On the board, draw a square yard and divide it into square feet. How many square feet are there in 1 sq. yd.? 9

3. A large piece of land is measured by the **acre.** An acre contains 43,560 sq. ft. Find the area of a square having each side 208 ft.; 43,264 sq. ft. then find the area of a square having each side 209 ft. 43,681 sq. ft. Which square has an area closer to an acre? Second one On the playground, measure or pace off a square 209 ft. on a side. This will show you about how large an acre is.

4. Mr. Fox has a piece of land 360 ft. long and 242 ft. wide. (87,120 sq. ft.) He says it contains 2 acres. (2 x 43,560 sq. ft. = 87,120 sq. ft.) Is he right? Yes.

5. Learn this table of square measure: Pupils should memorize first two lines.

144 square inches = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
43,560 square feet = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

6. Mr. Lewis is building a concrete sidewalk 3 ft. wide and 75 ft. long. How many square yards of concrete will the walk contain? 25 Find the cost at $4.75 per square yard. $118.75

7. A rectangular piece of land is 2 mi. long and 1.5 mi. wide. Find the value of this piece of land at $50 an acre. $96,000

Tell what number goes in each space:

8. 5 sq. ft. = .720. sq. in. 432 sq. in. = . 3 . sq. ft.
9. 2 sq. mi. = 1280. A. 135 sq. ft. = . 15. sq. yd.
10. $\frac{1}{4}$ A. = 10,890 sq. ft. 1280 A. = . 2. sq. mi.

Let some pupils do ex. 1-2 at board while others work at desks. Have them compare their results with table. If possible, follow suggestion in ex. 3 to give pupils idea of size of acre.

Problems about Areas

1. Draw to scale a rectangular garden that measures 24 ft. by 48 ft. Divide the garden into small squares, each 3 ft. on a side. If you plant a tomato plant in the middle of each small square, you will need as many tomato plants as there are squares. How many plants will that be? 128 Put a dot in the middle of each square to represent the tomato plant. By putting each plant in a small square, show that you have allowed 9 sq. ft. for each plant. Also show that the tomato plants are in rows that are 3 ft. apart and that the plants in each row are 3 ft. apart. In this case, you say that the tomato plants are planted 3 ft. × 3 ft.

2. Suppose you wish to have the rows of tomato plants 4 ft. apart while the plants in each row are to be 3 ft. apart. Then you will divide the garden in ex. 1 into small rectangles each 4 ft. × 3 ft. How many small rectangles will there be? 96 If you plant a tomato plant in the middle of each small rectangle, how many tomato plants will you need? 96 These tomato plants are planted 4 ft. × 3 ft.

3. In ex. 2, a shorter way to find the number of tomato plants you need is as follows: Find the area of the garden in square feet; then divide this area by the area of each small rectangle. Do you get the same answer? Yes

4. How many tomato plants will it take per acre of ground if they are planted 4 ft. × 3 ft.? Find the answer as shown in ex. 3. 1 acre = 43,560 sq. ft. 3630

5. How many strawberry plants per acre will be needed if they are planted 3 ft. × 2 ft.? if they are planted 4 ft. × 1 ft.? 10,890 if they are planted 4 ft. × $1\frac{1}{2}$ ft.? 7260

6. Peter's garden is 200 ft. by 70 ft. If he allows 50 lb. of fertilizer and 75 lb. of powdered limestone for each 1000 sq. ft. of the garden, how many pounds of fertilizer 700 and how many pounds of limestone 1050 must he buy? If he buys both the fertilizer and the limestone in 100-pound bags, how many bags of each does he need? 7 of fertilizer, 11 of limestone.

Have pupils make scale drawings (1 ft = $\frac{1}{4}$ in) for ex. 1–2, under your direction. After pupils study their drawings, discuss ex. 3. Do not rush this work. Be sure all have good understanding before proceeding.

7. John has a garden 100 ft. long and 88 ft. wide. How many bags of fertilizer does he need to buy to fertilize this garden if it takes 900 lb. of the fertilizer per acre and if the fertilizer is sold in 100-pound bags? 2

▶ First find the area of the garden and what fractional part of an acre this area is. Change the fraction to a decimal, correct to the nearest hundredth, and multiply 900 lb. by that decimal.

8. Find, to the nearest hundredth, what decimal part of an acre each of these gardens is: 30 ft. × 40 ft.; 60 ft. × 75 ft.; .10 acre .03 acre 60 ft. × 100 ft.; 80 ft. × 125 ft. .23 acre .14 acre

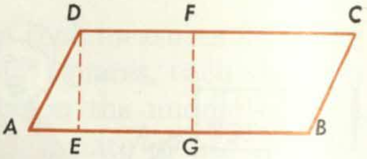
9. Mr. Johnson is going to insulate his house by covering the floor of the unused attic with rock wool. The floor measures 30 ft. by 24 ft. If one bag of rock wool covers 20 sq. ft., how many bags of it will Mr. Johnson need? 36 If 1 bag of rock wool costs $1.29, what will be the cost of the rock wool? $46.44

10. Mr. Wells is going to paint the floors of the living room and the dining room in his house. The living room is 13 ft. by 21 ft. and the dining room is 12 ft. by 14 ft. If 1 qt. of paint will cover 150 sq. ft., how many quarts should Mr. Wells buy? 3

Ex. 7 should be done orally with class. Be sure all know how to express area as fractional part of acre. Remind pupils to simplify this fraction before changing it to a decimal. Have pupils do ex. 8 and check work before they complete ex. 9–10.

Teach how to find area of parallelogram and develop formula for its area (pages 304–305).

Parallelograms



1. A parallelogram is a four-sided figure having its opposite sides parallel. Stress. Figure $ABCD$ is a parallelogram; sides AB and DC are parallel, and sides AD and BC are parallel. The side AB, on which the parallelogram stands, is its **base;** and segment DE, which is perpendicular to the base at E, is its **height.** Illustrate that all 4-sided figures do not have opposite sides parallel (trapezoid).

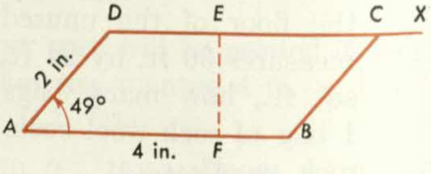
2. The height of a parallelogram is always measured on a line Emphasize. which is **perpendicular** to the base. In the figure above, DE represents the height. The slanting side AD is **not** the Stress. height. Any other segment perpendicular to the base, such as FG, also represents the height. FG equals DE since parallel lines are the same distance apart at all points. Place parallelograms of different shapes on board and have pupils draw height of each.

3. In the figure in ex. 1, AD and BC are called **opposite sides.** Measure the lengths of AD and BC. Are they equal? AB and DC are another pair of **opposite sides.** Are they equal? The opposite sides of a parallelogram are equal. Stress.

4. In the parallelogram in ex. 1, any two sides that meet in a point, such as sides AD and AB, are called **adjacent sides.** Name some other adjacent sides in the above figure.

5. **Problem** Construct a parallelogram if the adjacent sides are 4 in. and 2 in. and if the angle between these sides is 49°. Then draw the height of this parallelogram and measure it. Height is $1\frac{1}{2}$ in.

Explanation Draw $AB = 4$ in. At A, draw an angle of 49° with your protractor and make $AD = 2$ in. Through point D, draw DX parallel to AB; on DX, mark off $DC = 4$ in. Join points B and C. $ABCD$ is the desired parallelogram. In this parallelogram, are the opposite sides equal? Draw EF as the height of the parallelogram and measure it.



6. Draw a parallelogram with adjacent sides of 5 in. and $2\frac{1}{2}$ in. and with an angle of 53° between the sides. Then draw the height of the parallelogram and measure it. Height is 2 in.

Have pupils illustrate terms "base," "height," "perpendicular," "opposite sides," "adjacent sides" in other parallelograms to increase understanding of terms. Have pupils perform construction in ex. 5–6, under your supervision.

Area of a Parallelogram

1. On a sheet of paper, draw a parallelogram *ABCD* as shown below in Fig. 1. Then draw *CX* perpendicular to *AB*; *CX* is the **height** of the parallelogram. Cut out the parallelogram *ABCD*; then cut off triangle *CXB* and place it at the left of *AD* as shown in Fig. 2 at *M*. You see that Fig. 2 is a rectangle that has been built from the parallelogram. This rectangle has the **same base,** the **same height,** and the **same area** as the parallelogram. The area of a rectangle equals its base times its height, so you have:

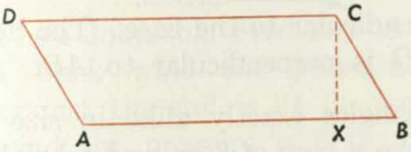


Fig. 1

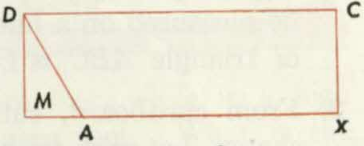


Fig. 2

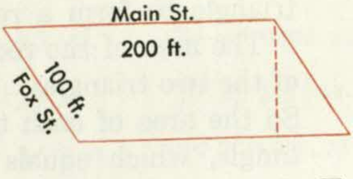
Area of parallelogram = base × height

or $A = bh$

What does b represent? h? What does bh mean?

2. In Fig. 1, the height of *ABCD* is **not** *AD*. Why?

3. Find the area of a parallelogram whose base is 30 ft. and whose height is 15 ft. Use the formula $A = bh$. 450 sq. ft.



4. Mr. Swift owns a lot at the corner of Main St. and Fox St.; it measures 200 ft. along Main St. and 100 ft. along Fox St. What is the shape of this lot? The dotted segment is 85 ft. long; what does it represent? To find the area of this lot, what segment will represent the base? Find the area. 17,000 sq. ft. Why is it wrong to multiply 200 by 100 to find the area? Discuss this point thoroughly.

Find the areas of these parallelograms. The first number is the base and the second number is the height:

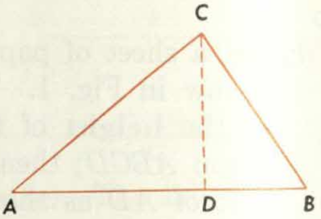
5. 8 in.; 7 in. 56 sq. in. 9½ ft.; 6 ft. 57 sq. ft. 150 ft.; 95 ft. 14,250 sq. ft.

6. 6 in.; 2½ in. 15 sq. in. 1½ ft.; 4 ft. 6 sq. ft. 160 ft.; 43½ ft. 6 960 sq. ft.

Have pupils perform experiment in ex. 1, and lead them to discover rule before stating it. Rule and formula should be memorized after understanding is assured.

Develop formula for finding area of triangle. Use this formula in solving problems (pages 306-307).

Area of a Triangle



1. Triangles are three-sided figures. The **base** of a triangle is the side on which it stands. Thus, the base of triangle *ABC* is *AB*. The **height** of a triangle is the distance from its highest point to its base and must be measured on a line perpendicular to the base. The height of triangle *ABC* is *CD*; *CD* is perpendicular to *AB*.

Stress.

2. From cardboard, cut 2 triangles exactly alike in size and shape. Let each triangle have a base of 3 in. and a height of 2 in. Color one triangle, as shown below in Fig. 1. On the other triangle, draw a dotted line to represent its height, as shown in Fig. 2; then divide this triangle into 2 pieces by cutting along the dotted line, as shown in Fig. 3; mark these pieces I and II. Place these pieces around the colored triangle to form a rectangle, as shown in Fig. 4.

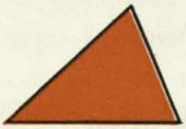


Fig. 1

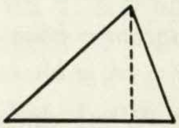


Fig. 2

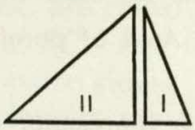


Fig. 3

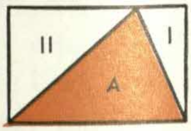


Fig. 4

The area of the rectangle is equal to the sum of the areas of the two triangles. But the areas of the triangles are equal. So the area of each triangle is one half the area of the rectangle, which equals its base times its height. Since the rectangle and the triangles have the same base and height:

Rule and formula should be memorized after understanding is assured.

Area of triangle $= \frac{1}{2} \times$ *base* $\times$ *height*

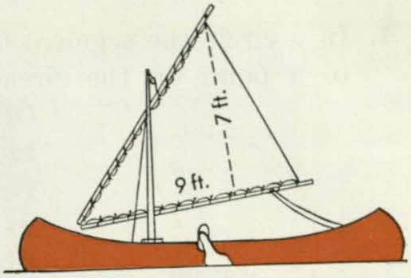
or $A = \frac{1}{2} bh$

3. **Problem** Mary's garden has the shape of a triangle with a base of 20 ft. and a height of 15 ft. Find its area.

Explanation Using $A = \frac{1}{2} bh$, you get $A = \frac{1}{2} \times 20 \times 15 = 150$. The area is 150 sq. ft.

In ex. 1 emphasize meaning of base and height and the fact that height is measured on line perpendicular to base. Have pupils perform experiment in ex. 2 and lead better students to formulate rule before presenting it.

Sail Boats



1. Fred's canoe has a triangular sail which has a base of 9 ft. and a height of 7 ft. How many square feet of sail area has the canoe? $31\frac{1}{2}$

2. Uncle John told Fred that in a brisk wind the pressure against the sail is $2\frac{1}{2}$ lb. per square foot, while in a high wind the pressure is 5 lb. per square foot. What is the force on Fred's sail in a brisk wind? in a high wind? See ex. 1. (Force = Pressure × Area) $78\frac{3}{4}$ lb. $157\frac{1}{2}$ lb.

3. Uncle John says that on an open canoe it is not safe to use a sail with a larger area than Fred's (see ex. 1). Find the areas$^{(1)}$ of sails with the following bases and heights and tell which would be safe for Fred to use.$^{(2)}$

(1) 30 sq. ft. $31\frac{1}{4}$ sq. ft. $40\frac{3}{8}$ sq. ft.
(2) (a) $7\frac{1}{2}$ ft.; 8 ft. safe (c) 10 ft.; $6\frac{1}{4}$ ft. safe (e) $9\frac{1}{2}$ ft.; $8\frac{1}{2}$ ft. unsafe
34 sq. ft. $33\frac{3}{4}$ sq. ft. $42\frac{1}{2}$ sq. ft.
(b) $8\frac{1}{2}$ ft.; 8 ft. unsafe (d) 9 ft.; $7\frac{1}{2}$ ft. unsafe (f) 10 ft.; $8\frac{1}{2}$ ft. unsafe

Find the areas of the following triangles:

	Base	Height		Base	Height	
4.	14 yd.	5 yd.	35 sq. yd.	65 yd.	34 yd.	1105 sq. yd.
5.	15 in.	8 in.	60 sq. in.	29 in.	17 in.	$246\frac{1}{2}$ sq. in.
6.	7.4 ft.	3.7 ft.	13.69 sq. ft.	46.8 ft.	29.2 ft.	683.28 sq. ft.
7.	$6\frac{1}{2}$ in.	5 in.	$16\frac{1}{4}$ sq. in.	$8\frac{1}{2}$ in.	$4\frac{1}{2}$ in.	$19\frac{1}{8}$ sq. in.
8.	18.5 in.	9.6 in.	88.80 sq. in., 88.8 sq. in.	$10\frac{1}{3}$ ft.	9 ft.	$46\frac{1}{2}$ sq. ft.

9. A triangle has an area of 80 sq. in. Its base is 16 in. So $80 = \frac{1}{2} \times 16 \times h$. What is the unknown factor h, which represents the height of this triangle? 10 in.

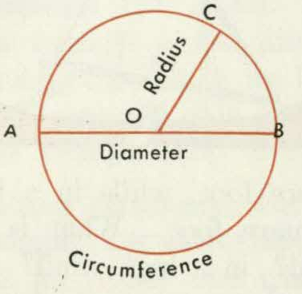
10. A triangle has an area of 60 sq. ft. What is its height if its base is 8 ft.? What is its base if its height is 10 ft.? 12 ft. 15 ft.

Do ex. 1-2 and 3(a) with class. Discuss ex. 9 carefully. Pupils should first find $\frac{1}{2}$ x 16 and write the formula as $80 = 8 \times h$; then find the unknown factor h.

Show how to find experimentally the formula for circumference of a circle. Use this formula in solving problems (pages 308–311).

Circumference of a Circle

1. In a circle the segment OC, which is drawn from the center to a point on the circle, is called a **radius** of the circle. OB and OA are also **radii** (plural of radius). The segment AB, which passes through the center, with both its ends on the circle, is called a **diameter.** You see that a diameter is twice as long as a radius. If you let d stand for the length of the diameter and r for the length of the radius, then $d = 2r$. If r is 3 in. long, how long is d? If d is 16 in. long, how long is r? 8 in.
6 in.



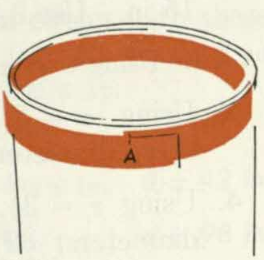
2. The distance around a circle is called its **circumference.** It is not easy to measure the length of the circumference because it is curved, so it is fortunate to be able to find the circumference of a circle **without measuring it.** It is easier to measure the length of the diameter of a circle because it is a line segment. The circumference of any circle is about $3\frac{1}{7}$ times as long as the diameter of the same circle. If you have a small circle whose diameter is 7 in., the circumference of that circle is about $3\frac{1}{7} \times 7$ in., or 22 in. long. If you have a large circle whose diameter is 14 ft., the circumference of that circle is $3\frac{1}{7} \times 14$ ft., or 44 ft. How long is the circumference of a circle if its diameter is 21 ft.? 66 ft. if its diameter is 35 in.? 110 in. 49 in.? 154 in.

3. Mathematicians have proved the following relationship between the diameter and the circumference of a circle. If the length of the circumference of **any** circle is divided by the length of its diameter, the quotient is the **same number.** This number is very important in mathematics and is represented by the Greek letter π. This letter is spelled **pi** and pronounced "pie." π is approximately equal to $3\frac{1}{7}$. As a decimal π is equal to 3.14, correct to the nearest hundredth. A more exact value of π is 3.14159, but for most purposes you can use $3\frac{1}{7}$ or 3.14 for the value of π.

Discuss ex. 1–3 with class. Be sure pupils understand meanings of terms "radius", "radii," "diameter," and "circumference." Have pupils locate these on other circles. Emphasize that diameter is longest distance across circle.

4. Jim wanted to find out for himself if the circumference of a circle is about 3.14 times as long as its diameter. As an experiment, he measured the top of a drinking glass, which is a circle. First he measured the diameter of the top of the glass and found it to be $2\frac{11}{16}$ in. to the nearest $\frac{1}{16}$ in. In measuring the diameter, he could not measure on a line passing through the center because the center is not given. So he took the greatest width across the top of the glass as the diameter, measuring to the outside edge of the glass.

Next Jim measured the circumference of the glass like this: He cut a narrow strip of paper about 10 in. long and $\frac{1}{2}$ in. wide and wrapped it tightly around the glass, close to the upper edge, letting the ends of the paper overlap as shown in the picture. He held the paper firmly in this position with one hand while, with the other hand, he made a pencil mark on the paper at *A*, opposite the end of the paper underneath it. Then he unwrapped the paper and measured the distance from mark *A* to the other end of the paper. This distance, which was $8\frac{7}{16}$ in., represented the circumference of the circle.



Then Jim divided $8\frac{7}{16}$ by $2\frac{11}{16}$, as shown at the right. How did he get $\frac{135}{16}$? $\frac{16}{43}$? When he divided 135 by 43, he carried the quotient to 3 decimal places, which gave 3.139. Rounding off 3.139 to the nearest hundredth, he got 3.14. So Jim's experiment showed the circumference to be about 3.14 times as long as the diameter.

$$\frac{135}{\underset{1}{\cancel{16}}} \times \frac{\overset{1}{\cancel{16}}}{43} = \frac{135}{43} = 3.139$$

5. Try Jim's experiment with a drinking glass. Make your measurements to the nearest $\frac{1}{16}$ in. Do you get 3.14?

6. Measure the diameter and the circumference of the top of a circular waste basket and of a circular baking pan. Use a cloth tape or flexible steel tape. For the diameter be sure to find the greatest width across the circle. Divide and see how close to 3.14 your results are.

Discuss experiment in ex. 4 with class. Be sure pupils know why greatest width was used and why $8\frac{7}{16}$ was divided by $2\frac{11}{16}$. Have pupils perform experiments suggested in ex. 5-6.

A Formula for the Circumference

1. Since the length of the circumference divided by the length of the diameter equals π, for any circle:

Circumference = π × diameter

If you let C represent the circumference and d the diameter, a formula for the circumference is:

$$C = \pi d$$

In this formula you may use 3.14 or $3\frac{1}{7}$ for π.

2. Find the circumference of a circle that has a diameter of 15 in. Use 3.14 for π.

▶ Using $C = \pi d$, you get $C = 3.14 \times 15$ in., or 47.1 in.

3. Using $\pi = 3.14$, find the circumferences of circles with these diameters: 12 in.; 6.5 ft.; 25 in.; 10 ft.; 100 ft. 314 ft.
37.68 in. 20.41 ft. 78.5 in. 31.4 ft.

4. Using $\pi = 3\frac{1}{7}$, find the circumferences of circles with these diameters: 63 in.; 56 ft.; 70 in.; 3.5 ft. 11 ft.
198 in. 176 ft. 220 in.

5. If a circle has a circumference of 44 in., what is its diameter? Use $\pi = 3\frac{1}{7}$.

▶ In the formula $C = \pi d$ you know that $C = 44$. So $44 = 3\frac{1}{7} \times d$. To find the unknown factor d, you can divide 44 by $3\frac{1}{7}$. Thus, $d = 44 \div 3\frac{1}{7} = 44 \div \frac{22}{7} = 14$. The diameter is 14 in.

6. Using $\pi = 3\frac{1}{7}$, find the diameters of circles with these circumferences: 66 ft.; 33 in.; 88 ft.
21 ft. $10\frac{1}{2}$ in. 28 ft.

7. Using $\pi = 3.14$, find the diameters of circles with these circumferences: 62.8 ft.; 21.98 in.; 46.8 in.; 132.7 ft. Give answers to the nearest tenth.
20 ft. 7 in. 14.9 in. 42.3 ft.

8. The wheels on Tom's bicycle have a diameter of 28 in. Find the circumference of each wheel. 88 in. How many inches does Tom ride each time the wheels revolve once? 88 How many feet? $7\frac{1}{3}$ How many revolutions do the wheels make when Tom rides 1 mi.? 720 Use $\pi = 3\frac{1}{7}$.

9. Change $3\frac{1}{7}$ to a mixed decimal and round off the result to the nearest hundredth. 3.14

Have pupils draw two line segments side by side whose lengths are equal to the diameter and the circumference of a circle, to show visually the relation of these two lengths.

Problems about Circles

1. You know that the diameter of a circle equals twice the radius; that is, $d = 2\,r$. In the formula $C = \pi d$, you can replace d by $2\,r$, which will give this new formula:

$$C = 2\,\pi r$$

This formula shows how to find the circumference of a circle when its radius is given.

2. Find the circumference of a circle if the radius is 3 ft. Using $C = 2\,\pi r$, you get $C = 2 \times 3.14 \times 3$. Finish the work. 18.84 ft.

3. Using $\pi = 3.14$, find the circumferences of circles with these radii: 5 ft., 30 ft., 10 in., 24 ft., 60 ft., 40 yd. 251.2 yd.
31.4 ft 188.4 ft. 62.8 in. 150.72 ft. 376.8 ft.

Using $\pi = 3\frac{1}{7}$, find the circumference of each circle:

4. $d = 7$ ft. 22 ft. $r = 10\frac{1}{2}$ ft. 66 ft. $r = 17\frac{1}{2}$ in. 110 in. $d = 42$ in. 132 in.

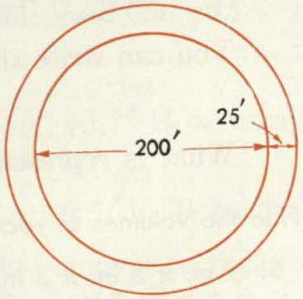
5. $r = 3$ ft. $18\frac{6}{7}$ ft. $d = 20$ yd. $62\frac{6}{7}$ yd. $d = 84$ ft. 264 ft. $d = 98$ in. 308 in.

6. A circular tin can has a diameter of $3\frac{1}{2}$ in. How long must the paper label for this can be if $\frac{3}{8}$ in. is allowed for overlapping? Use $\pi = 3\frac{1}{7}$. $11\frac{3}{8}$ in.

7. Betty wants to put a ruffle of lace around a circular pillow slip that is 15 in. in diameter. She must allow $1\frac{1}{2}$ times the length of the circumference in order to ruffle the lace. How many inches of lace does Betty need? About 71 How many yards of lace should she buy? 2 Use $\pi = 3.14$.

8. A circular plot of ground has a diameter of 200 ft. It is surrounded by a racetrack 25 ft. wide. Find the (1) inner and the (2) outer circumferences of the track. Use $\pi = 3.14$.

(1) 628 ft.; (2) 785 ft.



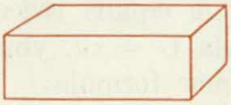
9. In ex. 8 above, how many feet less is it around the inside of the track than it is around the outside? 157

10. If you know that the circumference of a circle is 22 ft., what is the length of its radius? Let $\pi = 3\frac{1}{7}$. $3\frac{1}{2}$ ft.

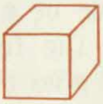
Present original problem and lead pupils to discover formula in ex. 1. Use this formula in ex. 10 to find the unknown factor r. You can also use the formula on page 310 and find the unknown diameter first, then the radius.

Reteach finding volume of rectangular prism and develop formula for finding these volumes (pages 312–315). Bring to class various rectangular solids (boxes, blocks, a brick).

Rectangular Prisms



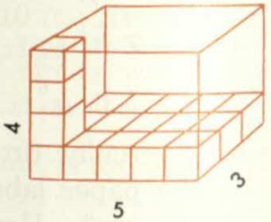
Rectangular Prism



Cube

1. A geometric figure shaped like a box is called a **rectangular prism** because its 6 sides are all rectangles. If the 6 sides are all squares, the **prism** is called a **cube.** The dimensions of a rectangular **prism** are its **length, width,** and **height**; these dimensions may all be different. If the **prism** is a cube, the dimensions are all the same. Demonstrate meanings of terms.

2. A cube that is 1 in. on each edge contains 1 **cubic inch;** its **volume** is 1 **cu. in.** What is the volume of a cube that is 1 ft. on each edge? 1 cu. ft. of a cube that is 1 yd. on each edge? 1 cu. yd.



3. To find the volume of a box that is 5 in. long, 3 in. wide, and 4 in. high, you must find how many 1-inch cubes it will hold. It will take 15 cubes to cover the bottom of the box and 4 layers like this, or 4 × 15 cubes, to fill the box. Each cube is 1 cu. in., so the volume of the box is 4 × 15 cu. in., or 60 cu. in. A short way to find 60 is to multiply the 3 dimensions together, like this: $4 \times 5 \times 3 = 60$. Try to perform similar experiment using shoe box and inch cubes.

4. Here is the way to find the volume of a rectangular prism:

Volume of rectangular prism = length × width × height

You can write this as a formula, as shown below:

$$V = lwh$$

What is represented by V? by l? by w? by h? by lwh?

Find the volumes of rectangular prisms with these dimensions:

5. 9 in. × 5 in. × 3 in. 135 cu. in. 12 in. × 8 in. × 9 in. 864 cu. in.

6. 7 in. × 4 in. × $2\frac{1}{2}$ in. 70 cu. in. $8\frac{3}{4}$ in. × 6 in. × 5 in. $262\frac{1}{2}$ cu. in.

7. 4 ft. × $\frac{1}{2}$ ft. × 3 ft. 6 cu. ft. $2\frac{1}{2}$ ft. × $\frac{3}{4}$ ft. × 6 ft. $11\frac{1}{4}$ cu. ft.

Emphasize that rectangular prism has square corners and that there are 6 sides which are rectangles. Stress that volume is amount a prism will hold, if filled with something. It is also called capacity.

Cubic Measure

1. John has a box that is 1 ft. long, 8 in. wide, and 9 in. high. Find the volume of the box.

▶ To find the volume, you need to have all dimensions given in the same unit. You can change 1 ft. to 12 in., and then all the dimensions will be in **inches.** Or you can change 8 in. to $\frac{2}{3}$ ft. and 9 in. to $\frac{3}{4}$ ft., and then all dimensions will be in **feet.** Find the volume of the box both in cubic inches and in cubic feet.
864 cu. in., $\frac{1}{2}$ cu. ft.

Find the volumes of boxes with these dimensions:

144 cu. in., $\frac{1}{12}$ cu. ft. 3 cu. ft., 5184 cu. in. 2 cu. ft., 3456 cu. in.
2. 1′ × 3″ × 4″ 6″ × $1\frac{1}{2}$′ × 4′ 2′ × $1\frac{1}{2}$′ × 8″
5 cu. ft., 8640 cu. in. 1 cu. ft., 1728 cu. in. 12,960 cu. in. $7\frac{1}{2}$ cu. ft.
3. 2′ × 10″ × 3′ 9″ × $1\frac{1}{3}$′ × 1′ 4′ × 15″ × 18″

4. If the three dimensions of a cube are each 1 ft., the volume is 1 cu. ft. Change 1 ft. to 12 in., and then the dimensions are 12 in. × 12 in. × 12 in. and the volume is 1728 cu. in. This shows that 1 cu. ft. equals 1728 cu. in.

5. If the three dimensions of a cube are each 1 yd., the volume is 1 cu. yd. Change 1 yd. to 3 ft., and then the dimensions are 3 ft. × 3 ft. × 3 ft. and the volume is 27 cu. ft. This shows that 1 cu. yd. equals 27 cu. ft.

6. Learn this table of **cubic measure:**

Emphasize.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet (cu. ft.) = 1 cubic yard (cu. yd.)

7. How many cubic feet are there in 6 cu. yd.? 162 How many are there in $2\frac{1}{3}$ cu. yd.? 63 in $\frac{2}{3}$ cu. yd.? 18

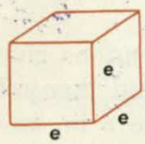
8. How many cubic inches are there in $\frac{1}{2}$ cu. ft.? 864 in $3\frac{1}{2}$ cu. ft.? 6048

9. How many cubic yards of earth must be removed in digging a cellar 24 ft. long, 18 ft. wide, and 9 ft. deep? 144 What is the cost of digging this cellar if the charge for doing the work is $3.25 per cubic yard of earth removed? $468

Do ex. 1 with class and emphasize that all dimensions must be given in, or changed to, same unit. In connection with ex. 4 have some pupils make a cubic inch and a cubic foot from stiff paper.

Volume of a Cube

1. When the three dimensions of a rectangular prism are equal, the prism is a **cube.** If e is the length of one edge of the cube, the formula for its volume is:



$$V = e \times e \times e$$
$$V = e^3$$

You know that a short way to write $e \times e \times e$ is e^3, which can be read "e cubed." Point out that 3 is an exponent.

Using the formula, find the volume of cubes with these edges:

64 cu. in. 1 cu. yd. 216 cu. ft. $\frac{1}{8}$ cu. in. 8 cu. yd. 343 cu. in.
2. 4 in. 1 yd. 6 ft. $\frac{1}{2}$ in. 2 yd. 7 in.
125 cu. yd. 8 cu. ft. 27 cu. in. $\frac{8}{27}$ cu. ft. $\frac{1}{64}$ cu. yd. 729 cu. ft.
3. 5 yd. 2 ft. 3 in. $\frac{2}{3}$ ft. $\frac{1}{4}$ yd. 9 ft.

4. Find the value of 7^3; 9^3; 8^3; 10^3; 11^3. 1331
343, 729, 512, 1000

5. Which box will hold more, one that is 5 ft. by 3 ft. by 4 ft. or one that is 4 ft. on each edge? 4 ft. on each edge

6. How many more boxes in the shape of a 1-foot cube can be packed in a large box 3 ft. on each edge than in a box 2 ft. by 3 ft. by 4 ft.? 3

7. A ton of hard coal occupies about 35 cu. ft. Mr. Bell has a coal bin 9 ft. long and 7 ft. wide. How many tons of hard coal will this bin hold when it is filled to a depth of 5 ft.? 9 to a depth of 4 ft.? 7.2 to a depth of 3 ft.? 5.4

8. The fish tank in the science room is 28 in. long, 18 in. wide, and 13 in. high. The water in it comes to within 2 in. of the top. How many gallons of water are in the tank if 1 gal. occupies 231 cu. in. of space? 24

9. A schoolroom is 35 ft. by 30 ft. by 12 ft. How many cubic feet of air space does it contain? 12,600 If 200 cu. ft. of air space are allowed for each pupil, this room is large enough for how many pupils? 63

Ex. 10 may be done as class project.

10. Get the dimensions of your schoolroom. It has enough air space for how many pupils? See ex. 9.

Through original problems and questions, lead pupils to discover formula in ex. 1. Do ex. 2 with class and have them complete ex. 3-9. Discuss these problems thoroughly and clear up any difficulties.

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Finding Volumes

1. The new house being built on Elm Street will have a cellar 39 ft. long, 24 ft. wide, and 9 ft. deep. The earth that is being removed is carried off in truck loads of 4 cu. yd. each. How many truck loads will there be in all? 78

2. A crane is used to lift blocks of marble from a quarry. This marble weighs 168 lb. per cubic foot. If a block of marble measures 7 ft. by 4 ft. by $4\frac{1}{2}$ ft., how many pounds does it weigh? 21, 168 About how many tons does it weigh? $10\frac{1}{2}$ Could a 5-ton truck carry it? No

3. Find the weight of a block of marble containing $6\frac{1}{4}$ cu. ft. 1050 lb.

4. Nan has a fish tank 18 in. long, 12 in. wide, and 11 in. deep. She keeps it filled with water to a depth of 8 in. Water weighs $62\frac{1}{2}$ lb. per cubic foot. The empty tank weighs 15 lb. What is the weight of the tank and the water together? $77\frac{1}{2}$ lb.

Pupils should complete ex. 1-4 independently. Discuss answers with class and let different pupils explain their solutions at board. Give pupils much practice in measuring boxes and computing volumes.

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Present review of problems and computations of year's work (pages 316-320). Individual conferences with pupils who had errors will still be helpful at this time in strengthening problem-solving skills.

Problems

1. Mrs. Clark wants to cover her kitchen floor with linoleum that is 6 ft. wide. The linoleum comes in large rolls and she can buy a piece any length she needs. Her kitchen floor is 18 ft. long and 14 ft. wide. How many running feet of this linoleum should she buy? 42 Draw a plan of the kitchen floor, showing how the linoleum can be laid in strips so that there will be no waste.

2. The linoleum in ex. 1 is sold at $2.15 a square yard. How much will Mrs. Clark have to pay for the linoleum? $60.20

3. Mary measured the circumference and the diameter of the the circular top of a waste basket. She got $34\frac{1}{8}$ in. for the circumference and $10\frac{1}{2}$ for the diameter. 3.25 What result did she get when she divided $34\frac{1}{8}$ by $10\frac{1}{2}$? Since her result was much more than 3.14, she made her measurements again and found that the diameter should have been $10\frac{7}{8}$ instead of $10\frac{1}{2}$. Divide again, using $10\frac{7}{8}$. Is this result closer to 3.14? 3.137; yes

4. A circular flower bed is $10\frac{1}{2}$ ft. in diameter. If tulip bulbs are to be planted 9 in. apart around the edge of this bed, how many bulbs are needed? 44 Use $\pi = 3\frac{1}{7}$.

5. Draw 3 triangles of different shapes. Make the base of each triangle 4 in. long and the height 3 in. 6 sq. in. What is the area of each triangle? Does the fact that the triangles have different shapes affect their area? No

6. The scale on a map shows that 1 in. on the map represents 2 mi. How many miles apart are two towns if they are $2\frac{1}{2}$ in. apart on the map? 5

7. A swimming pool is 60 ft. long and 30 ft. wide. How many cubic feet of water 8100 will it take to fill the pool to an average depth of $4\frac{1}{2}$ ft.? How much will this amount of water cost at 30¢ per 100 cu. ft.? $24.30

Review Problems

1. Mr. Martin needed \$1200 to use in his business. He borrowed that amount at 5% interest for 6 mo. How much interest did he pay? \$30

2. The sales in Miss King's bookshop amounted to \$15,000 last year and to \$13,800 this year. Find the per cent of decrease in her sales. 8%

3. For the past 5 yr. Miss King's annual sales have been \$15,500; \$17,400; \$16,000; \$15,000; and \$13,800. Find her average annual sales. \$15,540

4. Sam can walk 2 mi. in 36 min. At this rate, how many hours and minutes does it take him to walk 8 mi.? 2 hr. 24 min.

5. A lot shaped like a parallelogram has a base of 242 ft. and a height of 90 ft. What part of an acre is this lot? $\frac{1}{2}$

6. Jim spends an average school day as follows: $7\frac{3}{4}$ hr. at school and lessons, $3\frac{1}{4}$ hr. for recreation, $1\frac{1}{2}$ hr. for meals, $2\frac{3}{4}$ hr. for practicing his music lesson and for work. The rest of the time he is in bed. How many hours is Jim in bed? $8\frac{3}{4}$

7. Last month Jack sold 12 doz. eggs at 55¢ a dozen, 10 doz. at 62¢ a dozen, and 8 doz. at 65¢ a dozen. Find the average price per dozen Jack got for all these eggs. \$.60

8. On a map $\frac{3}{4}$ in. represents 10 mi. If the map distance between two cities is $3\frac{3}{4}$ in., how many miles is the airline distance between these two cities? 50

9. The gas company in Weston charges \$1.50 per 1000 cu. ft. of gas. Find the charge for 1600 cu. ft. of gas. \$2.40

10. Mr. Ward drove his car 250 mi. and used 14 gal. of gasoline. Find, to the nearest tenth of a mile, the average number of miles Mr. Ward drove per gallon of gasoline. 17.9

11. Draw a circle graph to represent Miss Lee's budget for her annual salary of \$3600 after taxes. She estimates \$1300 (130°) for room and board, \$1000 (100°) for savings, \$400 (40°) for clothing, \$500 for advancement, and \$400 (40°) for a reserve fund. See Guide.

Check papers carefully and analyze errors. Help pupils to see causes of errors. Conduct class discussion of problems to help clarify pupils' thinking. Plan specialized review of certain topics if needed.

Review

Add up. Check the work by adding down:

1. 4246	1890	$11\frac{9}{16}$	$15\frac{7}{8}$	.875	9.38
3388	7851	$2\frac{1}{2}$	$10\frac{1}{6}$	.025	5.20
9911	2770	10	$4\frac{2}{3}$	.375	4.15
4167	9037	$12\frac{5}{8}$	$11\frac{3}{4}$	.500	7.39
9063	8162	$7\frac{3}{4}$	9	.125	6.57
6131	9856	$15\frac{5}{16}$	$13\frac{7}{12}$	.750	5.00
36,906	39,566	$59\frac{3}{4}$	$65\frac{1}{24}$	2.650	37.69

Subtract. Check the work by adding:

2. 21473	$115\frac{7}{16}$	$425\frac{2}{5}$	9.250	\$900.00
19328	$109\frac{3}{16}$	$176\frac{9}{10}$	4.875	89.95
2145	$6\frac{1}{4}$	$248\frac{1}{2}$	4.375	\$810.05

Multiply and check the work:

3. 6749 × 8007 54,093,243   17.8 × 4593 81,755.4 $2\frac{2}{5} \times 9\frac{3}{8}$ $22\frac{1}{2}$

4. 1842 × 2500 4,605,000   .006 × 72.25 .43350 $17 \times 3\frac{3}{4}$ $63\frac{3}{4}$

5. 3520 × 4926 17,339,520   5.32 × 9.026 48.01832 $\frac{11}{12} \times 8\frac{2}{5}$ $7\frac{7}{10}$

Divide. Give answers to the nearest tenth:

6. 2,602,557 ÷ 369 7053   702 ÷ .72 975   24.6 ÷ 3.8 6.5

7. 1,961,244 ÷ 628 3123   46.7 ÷ .86 54.3   169 ÷ 208 .8

8. 359,748 ÷ 4817 74.7   18.6 ÷ .15 124   746 ÷ 317 2.4

Find the answers:

9. 15% of \$86 \$12.90 $12\frac{1}{2}$% of \$44 \$5.50   200% of \$557 \$1114

10. $4\frac{1}{4}$% of \$92 \$3.91   314% of \$90 \$282.60   125% of \$276 \$345

Put the correct number in each space:

11. 12 is $\overset{16}{..}$% of 75 $\overset{140}{..}$% of 120 is 168   5 is $\overset{12\frac{1}{2}}{..}$% of 40

12. 25 is $\overset{62\frac{1}{2}}{..}$% of 40 $\overset{56\frac{1}{4}}{..}$% of 192 is 108   9 is $\overset{20}{..}$% of 45

Emphasize importance of carefully checking each example. Check papers and return to pupils so they may find and correct mistakes.

Problems

1. Here is Mrs. Ford's recipe for vanilla ice cream:

4 cups milk 6
4 eggs 6
1 cup sugar $1\frac{1}{2}$
$\frac{1}{4}$ teaspoon salt $\frac{3}{8}$
1 cup cream, whipped $1\frac{1}{2}$
$1\frac{1}{2}$ teaspoon vanilla $2\frac{1}{4}$

Mrs. Ford wishes to make enough ice cream so she will have half as much again as this recipe will make. How much of each ingredient should she use? See answers above.

2. An airplane left New York at 3:15 P.M., traveling south. It covered 170 mi. in the first 40 min. At that rate, find the time it will arrive at an airport 850 mi. from New York. 6:35 P.M.

3. Draw a line graph showing hourly changes in temperature for a day, beginning at 8 A.M. and ending at 6 P.M. Make up the temperatures.

4. Mary's commission for selling Christmas cards was \$4.95. She had received \$27.50 from the sale of cards. Find the rate of commission Mary was allowed. 18%

5. Dick rode 14.5 mi. on his bicycle in 1 hr. 18 min. Find, to the nearest tenth of a mile, Dick's speed in miles per hour. (First change 1 hr. 18 min. to hours.) 11.2

6. John estimated the length of a fish pole to be 15 ft. When he measured the length, he found it to be 18 ft. Was his estimate of 15 ft. within 25% of the true length? Yes; see Guide.

7. On a map Fairview, Jamestown, and Lee are each situated at a corner of a triangle. The distance between Fairview and Jamestown is 36.8 mi., between Jamestown and Lee 17.5 mi., and between Lee and Fairview 29.0 mi. Find the distance around the triangle. 83.3 mi.

See suggestions on pages 316–317.

Mixed Practice

1. Which is more, $12\frac{1}{2}\%$ of \$52 or 15% of \$44? 15% of \$44

2. Find the average of 286,324; 307,190; and 292,254. 295, 256

3. What is the product of $2\frac{1}{2}$, $5\frac{1}{3}$, and $2\frac{1}{10}$? 28

4. What number is 60% more than 175? 280

5. How many times is $2\frac{1}{4}$ contained in 72? 32

6. What number is 8 less than twice 35? 62

7. Which is less, $3\frac{3}{4} \times 84$ or $12\frac{7}{8} \times 24$? $12\frac{7}{8} \times 24$

8. Find the average of $12\frac{3}{4}$, $10\frac{5}{8}$, $14\frac{7}{8}$, $11\frac{1}{2}$, $10\frac{7}{8}$. $12\frac{1}{8}$

9. Find the cost of 18 candy bars at 3 for 13¢. 78¢

10. Find the difference between 11.35 and 9.68 and multiply the difference by 2.5. 4. 175

11. Find the cost of 310 kw-hr of electricity if the rate is 6¢ per kw-hr for the first 200 kw-hr and 5¢ per kw-hr for all above 200 kw-hr. \$17. 50

12. 75 is $\frac{1}{2}$ of what number? 150 75 is $\frac{5}{8}$ of what number? 120

13. Which coin bank contains more money, one that has 27 quarters in it or one that has 77 nickels and 28 dimes in it? How much more? \$. 10 The one with the quarters

14. Find the product of 12,650 and 28. 354, 200 Then divide the product by 49 and find the quotient to the nearest tenth. 7228. 6

Multiply:

15.
$$\begin{array}{r} 6.275 \\ .36 \\ \hline 2.25900 \end{array}$$

Subtract:

16.
$$\begin{array}{r} 921678 \\ 299834 \\ \hline 621,844 \end{array}$$

Divide:

17. $3.6\overline{)11808}$ 3280

Add:

18.
$$\begin{array}{r} 4.125 \\ 6.500 \\ 3.750 \\ 2.000 \\ 4.625 \\ 7.400 \\ 5.250 \\ \hline 33.650 \end{array}$$

Subtract:

19.
$$\begin{array}{r} 9\frac{3}{4} \\ 2\frac{5}{6} \\ \hline 6\frac{11}{12} \end{array}$$

Add:

20.
$$\begin{array}{r} 3\frac{7}{8} \\ 4\frac{5}{8} \\ \hline 8\frac{1}{2} \end{array}$$

Divide:

21. $12\frac{1}{4} \div 2\frac{5}{8}$ $4\frac{2}{3}$

Pupils who had errors should do computations aloud so you may check their understanding of concepts and skills. Class discussion of solutions may help clarify thinking.

Present last problem test.

A Problem Test

1. The inside measurements of a rectangular can are $3\frac{1}{2}$ in. by $5\frac{1}{2}$ in. by 6 in. If a gallon of maple sirup fills a space of 231 cu. in., what part of a gallon will this can hold? $\frac{1}{2}$

2. Mr. Low earns \$4000 a year after taxes. He spends 25% of it for rent, 30% for food, 15% for clothing, and 18% for other things. He saves the rest. How much is saved? \$480

3. At the flower shop large red roses are \$6.00 a dozen. How much must Miss Fair pay for 3 roses? \$1.50

4. Henry can give correct answers to the 100 addition facts in $2\frac{1}{2}$ min. This is an average of how many seconds per fact? $1\frac{1}{2}$

5. Mrs. Gray uses her electric toaster an average of 15 min. a day. If it costs $3\frac{1}{5}$¢ per hour for electricity to run the toaster, what does it cost to use the toaster for 30 days? 24¢

6. How many yards of rug binding must you buy to sew around a circular rug that has a diameter of $3\frac{1}{2}$ ft.? Allow 6 in. of binding extra. The binding is sold only by the whole yard. 4yd. Use $\pi = 3\frac{1}{7}$.

7. A telegram cost \$2.95 without the tax of 10%. Find the tax \$.30 and the total cost of the telegram. Count any fraction of a cent as an extra cent. \$3.25

8. In Newtown the amount of rain that fell during the year was 41.74 in. Find, to the nearest hundredth of an inch, the average number of inches of rain that fell per month. 3.48

9. Last year Mr. Allen paid \$750 a year for rent for his house. This year the rent has been raised to \$840. What per cent has Mr. Allen's rent been increased? 12%

10. A roll of linen toweling contains $50\frac{1}{4}$ yd. How many towels each 27 in. long can be cut from this whole roll? 67

Have pupils complete bar graphs started on page 41.

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Record summary of results of these problem tests on progress cards. Discussion of problems will still be helpful at this time to reinforce pupils' skills in problem solving.

Present diagnostic tests (pages 322–323) of skills taught in Grade 7 with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Practice Pages

(1) (2)

Add. Then subtract. Check the work:

1. 3 hr. 25 min. / 1 hr. 40 min. — 8 yd. 11 in. / 4 yd. 25 in. — 289, 290

(1) 5hr. 5min.; (2) 1hr. 45min (1) 13yd.; (2) 3yd. 22in.

Find the areas of rectangles with these dimensions:

2. 10 ft. × 7 ft. 70 sq. ft. 5 yd. × $\frac{3}{4}$ yd. $3\frac{3}{4}$ sq. yd. 299

3. 2 ft. 6 in. × 2 ft. 5 sq. ft. 20 in. × $\frac{1}{2}$ yd. 360 sq. in., or $\frac{5}{18}$ sq. yd. 300

Find the areas of squares having these sides:

4. 3 in. 9 sq. in. 7 ft. 49 sq. ft. $3\frac{1}{2}$ yd. $12\frac{1}{4}$ sq. yd. 10.5 ft. 110.25 sq. ft. $\frac{1}{4}$ mi. $\frac{1}{16}$ sq. mi. 300

Find the number to put in each space:

5. 2 sq. ft. = $\overset{288}{\ldots}$ sq. in. 30 sq. ft. = $\overset{3\frac{1}{3}}{\ldots}$ sq. yd. 301

6. 81 cu. ft. = $\overset{3}{\ldots}$ cu. yd. 5 cu. yd. = $\overset{135}{\ldots}$ cu. ft. 313

Find the areas of these parallelograms:

	Base	Height		Base	Height		
7.	6 ft.	4 ft.	24 sq. ft.	$\frac{3}{4}$ yd.	$\frac{1}{2}$ yd.	$\frac{3}{8}$ sq. yd.	305

Find the areas of these triangles:

	Base	Height		Base	Height		
8.	10 ft.	13 ft.	65 sq. ft.	9 in.	7 in.	$31\frac{1}{2}$ sq. in.	306, 307

Find the circumference of circles with these diameters. Use $\pi = 3\frac{1}{7}$:

9. 14 in. 44 in. $10\frac{1}{2}$ in. 33 in. $1\frac{3}{4}$ ft. $5\frac{1}{2}$ ft. 12 ft. $37\frac{5}{7}$ ft. 311

Find the volumes of boxes with these dimensions:

10. $4' \times 5\frac{1}{2}' \times 6'$ 132 cu. ft. $4' \times 4' \times 4'$ 64 cu. ft. 312

These pages may be used as end-of-year tests. Check papers carefully and analyze errors. Through individual conferences try to clear up difficulties. Assign practice pages for remedial work as needed.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Practice Pages

Subtract. Check by going over the work:

1. $\begin{array}{r} 7\frac{3}{4} \\ \underline{2\frac{1}{8}} \\ 5\frac{5}{8} \end{array}$ $\begin{array}{r} 9 \\ \underline{6\frac{5}{6}} \\ 2\frac{1}{6} \end{array}$ $\begin{array}{r} 4\frac{1}{8} \\ \underline{2\frac{5}{8}} \\ 1\frac{1}{2} \end{array}$ $\begin{array}{r} 8\frac{1}{4} \\ \underline{5\frac{5}{6}} \\ 2\frac{5}{12} \end{array}$ $\begin{array}{r} 3\frac{1}{4} \\ \underline{2\frac{7}{12}} \\ \frac{2}{3} \end{array}$ 56, 58 59

Multiply or divide and check:

2. $\frac{5}{8} \times 12$ $7\frac{1}{2}$ $9 \times 3\frac{1}{2}$ $31\frac{1}{2}$ $\frac{15}{16} \times \frac{2}{5}$ $\frac{3}{8}$ 63–66

3. $1\frac{4}{5} \times 3\frac{1}{3}$ 6 $2\frac{1}{2} \times 2\frac{1}{2}$ $6\frac{1}{4}$ $\frac{5}{6} \times 1\frac{1}{5}$ 1 67

4. $1\frac{7}{8} \div \frac{9}{16}$ $3\frac{1}{3}$ $\frac{1}{6} \div \frac{5}{12}$ $\frac{2}{5}$ $4 \div \frac{3}{8}$ $10\frac{2}{3}$ 72–74

5. $2\frac{1}{4} \div 12$ $\frac{3}{16}$ $\frac{3}{4} \div 2\frac{5}{8}$ $\frac{2}{7}$ $6\frac{2}{3} \div 3\frac{1}{5}$ $2\frac{1}{12}$ 75, 76

Divide. If you continue to have a remainder, give the quotient correct to the nearest tenth:

6. $\begin{array}{r} 10.6 \\ .7\overline{)7.39} \end{array}$ $\begin{array}{r} 1.1 \\ 2.9\overline{)3.316} \end{array}$ $\begin{array}{r} 13.6 \\ .31\overline{)4.203} \end{array}$ $\begin{array}{r} 17.2 \\ .6\overline{)10.3} \end{array}$ 111–113

7. $\begin{array}{r} 540 \\ .9\overline{)486} \end{array}$ $\begin{array}{r} 528.4 \\ .19\overline{)100.4} \end{array}$ $\begin{array}{r} .2 \\ 8.2\overline{)1.267} \end{array}$ $\begin{array}{r} .09 \\ .8\overline{).072} \end{array}$ 116

Find the answers correct to the nearest cent:

8. 5% of \$3.75 \$.19 25% of \$9.25 \$2.31 $2\frac{1}{4}$% of \$73 \$1.64 138

9. $3\frac{1}{2}$% of \$120 \$4.20 $2\frac{1}{3}$% of \$294 \$6.86 $1\frac{3}{4}$% of \$68 \$1.19 142, 144

10. 116% of \$318 \$368.88 300% of \$738 \$2214 235% of \$52 \$122.20 145, 146

Find the number to put in each space:

11. 36 is . $37\frac{1}{2}$.% of 96 21 is . $26\frac{1}{4}$.% of 80 214–216

Find the per cent of increase or decrease that the second number shows over the first number:

12. \$60, \$81 35% increase \$250, \$185 26% decrease \$450, \$900 100% increase 227

Results of tests should be summarized on pupils' progress cards. If pupils need extra summer work, refer them to practice pages or "More Practice," pages 325–341.

Provide end-of-year test on year's work. Use to test readiness for Grade 8.

Are You Ready for the Next Grade?

If you can do the exercises on this page with few or no mistakes, you are ready for the next grade.

Find the answers:

1. $2\frac{3}{5}+\frac{1}{2}+1\frac{3}{10}$ $4\frac{2}{5}$ $37\frac{1}{2}\%$ of $\$224$ $\$84$ $\frac{7}{8}\times 13$ $11\frac{3}{8}$

2. $.375\times 8.34$ 3.12750   128% of $\$375$ $\$480$ $9\div 2\frac{1}{4}$ 4

3. $392.7\div .07$ 5610 $210.02-7.93$ 202.09 $3\frac{3}{4}\times 1\frac{1}{5}$ $4\frac{1}{2}$

4. 16% of $\$225$ $\$36$ $128{,}018\div 2783$ 46 $7\frac{1}{2}\div 10$ $\frac{3}{4}$

5. $2\frac{3}{4}\%$ of $\$150$ $\$4.13$ $6.251+9.588+5.375$ 21.214 $8\frac{1}{4}-3\frac{5}{6}$ $4\frac{5}{12}$

6. $4827+1209+6000+155+3276+983+7814$ 24,264

What is the correct number for each space?

7. 24 is $\overset{75}{\ldots}$ % of 32   9 ft. 8 in. = $\overset{9\frac{2}{3}}{\ldots}$ ft.

8. 45 is $\overset{150}{\ldots}$ % of 30   21 sq. ft. = $\overset{2\frac{1}{3}}{\ldots}$ sq. yd.

9. 12 is $\overset{12\frac{1}{2}}{\ldots}$ % of 96 $\frac{3}{4}$ sq. ft. = $\overset{108}{\ldots}$ sq. in.

10. When the cost of a pound of butter increases from 40¢ to 88¢, find the per cent of increase. 120%

11. Three boys ordered these supplies for their camping trip: groceries, $8.29; meat, $3.16; camp stove, $10.75; 15 yd. of mosquito netting at $.17 a yard. The boys shared the cost equally. How much did Bill have left from $10? $1.75

12. How much did Patty pay for a suit marked $30.00 that she bought at a discount of 25%? $22.50

13. Draw a hexagon in a circle. Then draw an equilateral triangle inside the hexagon with its corners on the circle.

14. The cost of sending a telegram to East Harbor is $1.45 for the first 15 words and $6\frac{1}{2}$¢ for each extra word. There is a tax of 10% on the cost of the telegram. Find the entire cost of sending a telegram of 27 words at this rate. $2.45

Check papers carefully and note kinds of errors. Record results on progress cards to show improvements or weaknesses. Class discussion of these examples may still help to increase understanding and clear up difficulties.

Pages 325-341 provide practice material in all skills reviewed or taught. Grade 7.

More Practice

①

1. 7 + 6 + 1 + 8 + 8 + 6 + 3 + 2 + 9 + 7 + 3 + 9 69
2. 8 + 2 + 1 + 9 + 5 + 6 + 2 + 0 + 9 + 1 + 2 + 5 50
3. 1 + 2 + 9 + 9 + 7 + 1 + 8 + 3 + 9 + 4 + 8 + 4 65
4. 7 + 3 + 4 + 3 + 6 + 8 + 2 + 9 + 5 + 7 + 4 + 6 64
5. 43 + 55 + 25 + 56 + 74 + 18 + 83 + 26 + 69 + 27 476
6. 94 + 77 + 17 + 36 + 19 + 65 + 50 + 67 + 13 + 56 494
7. 66 + 34 + 58 + 43 + 34 + 49 + 25 + 38 + 57 + 45 449
8. 15 + 76 + 66 + 14 + 58 + 36 + 23 + 74 + 35 + 42 439
9. 64 + 92 + 75 + 95 + 91 + 88 + 87 + 17 + 39 + 56 704
10. 81 + 26 + 43 + 15 + 63 + 56 + 65 + 34 + 63 + 29 475

②

11.	324	269	107	462	4010	2573	8100
	580	763	248	551	7223	3069	3175
	497	384	539	907	8118	1265	6618
	588	994	940	653	2501	3848	5177
	665	120	722	610	3941	8923	7510
	188	279	406	773	5030	3704	7086
	189	612	452	967	9173	6699	4272
	+453	+388	+567	+626	+2229	+2961	+4425
	3484	3809	3981	5549	42, 225	33, 042	46, 363

12.	200	620	518	120	3004	7000	5333
	937	599	493	752	7537	2449	8018
	828	868	480	937	2998	6361	8261
	606	235	275	716	1285	9181	3489
	972	119	731	447	8410	3990	7203
	930	156	738	154	5635	8634	6474
	839	128	537	425	1928	3307	4680
	778	604	666	101	9754	8260	5135
	+162	+236	+596	+246	+2509	+2471	+9638
	6252	3565	5034	3898	43,060	51,653	58,231

13.	$396.79	$726.35	$488.06	$109.97	$531.64
	888.27	238.91	45.62	460.45	758.80
	695.49	529.72	974.11	112.06	688.23
	736.95	7.80	701.44	55.26	67.11
	753.98	458.22	161.71	305.59	834.90
	3.17	408.56	432.80	922.74	312.22
	39.69	53.66	81.24	369.67	30.86
	+805.72	+444.53	+670.16	+ 33.77	+319.99
	$4320.06	$2867.75	$2555.14	$2369.51	$3543.75

These pages are correlated with ones throughout the test. Use for individual pupil assistance, review and reinforcement of processes, testing, and so on.

More Practice

③

1.	938 −566 372	1879 −1488 391	5000 −1252 3748	14635 −5369 9266	681331 −641455 39,876	$982.93 −783.63 $199.30
2.	596 −394 202	6007 −2999 3008	6194 −4208 1986	27567 −8079 19,488	489006 −111468 377,538	$885.91 −678.59 $207.32
3.	600 −168 432	5813 −3889 1924	3421 −1523 1898	21421 −6465 14,956	915943 −888274 27,669	$700.00 −499.85 $200.15
4.	375 −358 17	4000 −3864 136	4815 −1337 3478	50726 −8048 42,678	864470 −369591 494,879	$945.46 −269.91 $675.55
5.	900 −303 597	3284 −1408 1876	7000 −4415 2585	78319 −76954 1365	557168 −513843 43,325	$522.22 −153.69 $368.53

④

6.	74 × 86 6364	45 × 167 7515	262,548 429 × 612	895,752 936 × 957	86,832 432 × 201
7.	36 × 36 1296	94 × 208 19,552	45,441 187 × 243	419,475 425 × 987	120,540 980 × 123
8.	49 × 94 4606	28 × 260 7280	531,069 627 × 847	179,324 706 × 254	454,346 619 × 734
9.	82 × 65 5330	94 × 451 42,394	669,460 748 × 895	163,548 396 × 413	314,384 401 × 784
10.	56 × 95 5320	73 × 981 71,613	330,132 492 × 671	37,275 175 × 213	440,910 690 × 639
11.	98 × 98 9604	90 × 875 78,750	333,330 542 × 615	166,242 309 × 538	109,782 321 × 342
12.	67 × 89 5963	86 × 127 10,922	58,310 238 × 245	129,108 159 × 812	396,500 500 × 793
13.	29 × 39 1131	54 × 263 14,202	693,160 806 × 860	141,680 230 × 616	621,216 719 × 864

⑤

14.	390,260 65 × 6004	230,144 128 × 1798	1,639,624 536 × 3059	3,609,375 1375 × 2625
15.	174,640 37 × 4720	6,789,930 735 × 9238	1,056,450 150 × 7043	11,284,260 1794 × 6290
16.	238,659 53 × 4503	419,055 307 × 1365	488,940 290 × 1686	14,312,810 2006 × 7135
17.	40,223 19 × 2117	7,500,444 812 × 9237	1,125,978 357 × 3154	39,190,064 4798 × 8168
18.	252,685 97 × 2605	767,250 375 × 2046	1,951,750 370 × 5275	28,883,200 4513 × 6400
19.	455,058 81 × 5618	4,864,178 506 × 9613	4,994,260 794 × 6290	5,987,080 1237 × 4840
20.	28,080 26 × 1080	759,156 246 × 3086	832,200 600 × 1387	8,711,820 3060 × 2847
21.	249,676 74 × 3374	823,935 413 × 1995	2,843,328 502 × 5664	32,481,000 7218 × 4500

More Practice

⑥

1.	1095 ÷ 5 219	3645 ÷ 5 729	45,027 ÷ 9 5003	76,935 ÷ 4 19, 233 R3
2.	8127 ÷ 7 1161	5753 ÷ 6 958 R5	84,208 ÷ 8 10, 526	74,270 ÷ 7 10, 610
3.	3705 ÷ 3 1235	3645 ÷ 9 405	75,320 ÷ 5 15,064	26,568 ÷ 8 3321
4.	2539 ÷ 5 507 R4	5577 ÷ 7 796 R5	83,641 ÷ 9 9293 R4	28,242 ÷ 4 7060 R2

⑦

5.	861 ÷ 21 41	2418 ÷ 31 78	29,484 ÷ 52 567	435,260 ÷ 70 6218
6.	901 ÷ 55 16 R21	3237 ÷ 34 95 R7	18,488 ÷ 25 739 R13	137,277 ÷ 63 2179
7.	416 ÷ 32 13	1958 ÷ 22 89	83,942 ÷ 94 893	317,875 ÷ 75 4238 R25
8.	597 ÷ 71 8 R29	4067 ÷ 83 49	11,309 ÷ 43 263	139,132 ÷ 14 9938
9.	490 ÷ 35 14	1848 ÷ 24 77	27,615 ÷ 35 789	231,872 ÷ 32 7246
10.	726 ÷ 22 33	5067 ÷ 55 92 R7	13,811 ÷ 14 986 R7	736,185 ÷ 85 8661
11.	891 ÷ 81 11	5999 ÷ 95 63 R14	10,540 ÷ 12 878 R4	218,780 ÷ 24 9115 R20
12.	486 ÷ 54 9	1425 ÷ 15 95	16,318 ÷ 41 398	126,061 ÷ 13 9697
13.	996 ÷ 24 41 R12	5543 ÷ 92 60 R23	28,952 ÷ 32 904 R24	266,219 ÷ 53 5023
14.	504 ÷ 42 12	3524 ÷ 44 80 R4	13,635 ÷ 15 909	220,867 ÷ 54 4090 R7

⑧

15.	396 ÷ 36 11	2441 ÷ 38 64 R9	27,604 ÷ 67 412	253,827 ÷ 79 3213
16.	644 ÷ 28 23	1476 ÷ 16 92 R4	25,866 ÷ 36 718 R18	178,035 ÷ 66 2697 R33
17.	368 ÷ 46 8	3096 ÷ 86 36	37,914 ÷ 89 426	587,511 ÷ 87 6753
18.	399 ÷ 19 21	3038 ÷ 49 62	17,700 ÷ 18 983 R6	180,830 ÷ 26 6955
19.	976 ÷ 48 20 R16	1666 ÷ 17 98	20,300 ÷ 28 725	630,729 ÷ 69 9141
20.	333 ÷ 37 9	2832 ÷ 59 48	16,591 ÷ 39 425 R16	278,538 ÷ 78 3571
21.	413 ÷ 29 14 R7	4019 ÷ 47 85 R24	14,877 ÷ 58 256 R29	182,856 ÷ 19 9624
22.	621 ÷ 27 23	2167 ÷ 88 24 R55	16,796 ÷ 19 884	355,684 ÷ 77 4619 R21
23.	245 ÷ 46 5 R15	3927 ÷ 49 80 R7	50,414 ÷ 56 900 R14	277,213 ÷ 66 4200 R13
24.	232 ÷ 29 8	1350 ÷ 27 50	49,392 ÷ 98 504	476,204 ÷ 68 7003
25.	266 ÷ 38 7	1235 ÷ 19 65	13,041 ÷ 27 483	283,613 ÷ 59 4807

More Practice

⑨

1. $\begin{array}{r}31\frac{3}{7}\\252\overline{)7920}\end{array}$ $\begin{array}{r}26\\923\overline{)23998}\end{array}$ $\begin{array}{r}836\\438\overline{)366168}\end{array}$ $\begin{array}{r}4296\\619\overline{)2659224}\end{array}$

2. $\begin{array}{r}25\\338\overline{)8450}\end{array}$ $\begin{array}{r}29\\811\overline{)23519}\end{array}$ $\begin{array}{r}364\\754\overline{)274456}\end{array}$ $\begin{array}{r}6485\\226\overline{)1465610}\end{array}$

3. $\begin{array}{r}24\\412\overline{)9888}\end{array}$ $\begin{array}{r}46\frac{11}{532}\\532\overline{)24483}\end{array}$ $\begin{array}{r}880\\427\overline{)375760}\end{array}$ $\begin{array}{r}2505\frac{19}{640}\\640\overline{)1603219}\end{array}$

4. $\begin{array}{r}32\\253\overline{)8096}\end{array}$ $\begin{array}{r}74\frac{19}{256}\\256\overline{)18963}\end{array}$ $\begin{array}{r}405\frac{33}{736}\\736\overline{)298113}\end{array}$ $\begin{array}{r}7009\\241\overline{)1689169}\end{array}$

5. $\begin{array}{r}5\\342\overline{)1710}\end{array}$ $\begin{array}{r}31\frac{431}{708}\\708\overline{)22379}\end{array}$ $\begin{array}{r}719\\546\overline{)392574}\end{array}$ $\begin{array}{r}7630\\153\overline{)1167390}\end{array}$

⑩

6. $\begin{array}{r}14\\590\overline{)8260}\end{array}$ $\begin{array}{r}83\text{ R}89\\594\overline{)49391}\end{array}$ $\begin{array}{r}129\text{ R}120\\888\overline{)114672}\end{array}$ $\begin{array}{r}4216\\694\overline{)2925904}\end{array}$

7. $\begin{array}{r}15\\282\overline{)4230}\end{array}$ $\begin{array}{r}87\text{ R}120\\180\overline{)15780}\end{array}$ $\begin{array}{r}810\\396\overline{)320760}\end{array}$ $\begin{array}{r}5835\\383\overline{)2234805}\end{array}$

8. $\begin{array}{r}23\\291\overline{)6693}\end{array}$ $\begin{array}{r}21\text{ R}300\\974\overline{)20754}\end{array}$ $\begin{array}{r}627\\371\overline{)232617}\end{array}$ $\begin{array}{r}7637\\167\overline{)1275379}\end{array}$

9. $\begin{array}{r}12\\575\overline{)6900}\end{array}$ $\begin{array}{r}42\\661\overline{)27762}\end{array}$ $\begin{array}{r}924\text{ R}17\\165\overline{)152477}\end{array}$ $\begin{array}{r}9000\text{ R}219\\376\overline{)3384219}\end{array}$

10. $\begin{array}{r}9\text{ R}27\\864\overline{)7803}\end{array}$ $\begin{array}{r}58\text{ R}11\\194\overline{)11263}\end{array}$ $\begin{array}{r}320\text{ R}729\\972\overline{)311769}\end{array}$ $\begin{array}{r}3008\text{ R}121\\463\overline{)1392825}\end{array}$

⑪

11. $\begin{array}{r}7\frac{3}{4}\\+2\frac{3}{4}\\\hline 10\frac{1}{2}\end{array}$ $\begin{array}{r}4\frac{5}{6}\\+1\frac{5}{6}\\\hline 6\frac{2}{3}\end{array}$ $\begin{array}{r}2\frac{5}{8}\\+2\frac{5}{8}\\\hline 5\frac{1}{4}\end{array}$ $\begin{array}{r}7\frac{3}{8}\\+6\frac{7}{8}\\\hline 14\frac{1}{4}\end{array}$ $\begin{array}{r}1\frac{2}{3}\\+4\frac{1}{3}\\\hline 6\end{array}$ $\begin{array}{r}2\frac{5}{6}\\+8\frac{1}{6}\\\hline 11\end{array}$ $\begin{array}{r}8\frac{3}{10}\\+4\frac{3}{10}\\\hline 12\frac{3}{5}\end{array}$ $\begin{array}{r}2\frac{1}{12}\\+5\frac{1}{12}\\\hline 7\frac{1}{6}\end{array}$

12. $\begin{array}{r}4\frac{1}{4}\\+8\frac{1}{4}\\\hline 12\frac{1}{2}\end{array}$ $\begin{array}{r}5\frac{1}{8}\\+2\frac{5}{8}\\\hline 7\frac{3}{4}\end{array}$ $\begin{array}{r}6\frac{1}{4}\\+\ \frac{3}{4}\\\hline 7\end{array}$ $\begin{array}{r}1\frac{1}{6}\\+6\frac{1}{6}\\\hline 7\frac{1}{3}\end{array}$ $\begin{array}{r}9\frac{1}{2}\\+4\frac{1}{2}\\\hline 14\end{array}$ $\begin{array}{r}7\frac{2}{3}\\+3\frac{2}{3}\\\hline 11\frac{1}{3}\end{array}$ $\begin{array}{r}1\frac{5}{12}\\+7\frac{11}{12}\\\hline 9\frac{1}{3}\end{array}$ $\begin{array}{r}6\frac{7}{16}\\+6\frac{9}{16}\\\hline 13\end{array}$

13. $\begin{array}{r}6\frac{3}{5}\\+3\frac{4}{5}\\\hline 10\frac{2}{5}\end{array}$ $\begin{array}{r}3\frac{1}{3}\\+8\frac{1}{3}\\\hline 11\frac{2}{3}\end{array}$ $\begin{array}{r}2\frac{2}{5}\\+1\frac{2}{5}\\\hline 3\frac{4}{5}\end{array}$ $\begin{array}{r}9\frac{5}{8}\\+4\frac{5}{8}\\\hline 14\frac{1}{4}\end{array}$ $\begin{array}{r}3\frac{4}{5}\\+3\frac{4}{5}\\\hline 7\frac{3}{5}\end{array}$ $\begin{array}{r}4\frac{7}{8}\\+1\frac{7}{8}\\\hline 6\frac{3}{4}\end{array}$ $\begin{array}{r}8\frac{3}{16}\\+2\frac{15}{16}\\\hline 11\frac{1}{8}\end{array}$ $\begin{array}{r}3\frac{7}{10}\\+5\frac{9}{10}\\\hline 9\frac{3}{5}\end{array}$

14. $\begin{array}{r}4\frac{1}{3}\\1\frac{2}{3}\\+3\frac{2}{3}\\\hline 9\frac{2}{3}\end{array}$ $\begin{array}{r}6\frac{3}{4}\\2\frac{3}{4}\\+1\frac{3}{4}\\\hline 11\frac{1}{4}\end{array}$ $\begin{array}{r}3\frac{1}{5}\\2\frac{1}{5}\\+3\frac{3}{5}\\\hline 9\end{array}$ $\begin{array}{r}6\frac{1}{8}\\4\frac{3}{8}\\+1\frac{7}{8}\\\hline 12\frac{3}{8}\end{array}$ $\begin{array}{r}1\frac{5}{6}\\2\frac{5}{6}\\+4\frac{5}{6}\\\hline 9\frac{1}{2}\end{array}$ $\begin{array}{r}9\frac{2}{3}\\1\frac{2}{3}\\+4\frac{2}{3}\\\hline 16\end{array}$ $\begin{array}{r}2\frac{3}{10}\\\frac{9}{10}\\+5\frac{3}{10}\\\hline 8\frac{1}{2}\end{array}$ $\begin{array}{r}1\frac{5}{16}\\1\frac{7}{16}\\+1\frac{3}{16}\\\hline 3\frac{15}{16}\end{array}$

15. $\begin{array}{r}6\frac{1}{6}\\5\frac{5}{6}\\+2\frac{1}{6}\\\hline 14\frac{1}{6}\end{array}$ $\begin{array}{r}1\frac{2}{5}\\1\frac{2}{5}\\+1\frac{2}{5}\\\hline 4\frac{1}{5}\end{array}$ $\begin{array}{r}2\frac{5}{8}\\3\frac{7}{8}\\+1\frac{5}{8}\\\hline 8\frac{1}{8}\end{array}$ $\begin{array}{r}6\frac{1}{3}\\2\frac{1}{3}\\+4\frac{1}{3}\\\hline 13\end{array}$ $\begin{array}{r}8\frac{3}{4}\\1\frac{3}{4}\\+2\frac{3}{4}\\\hline 13\frac{1}{4}\end{array}$ $\begin{array}{r}6\frac{1}{2}\\4\frac{1}{2}\\+5\frac{1}{2}\\\hline 16\frac{1}{2}\end{array}$ $\begin{array}{r}3\frac{11}{12}\\2\frac{11}{12}\\+2\frac{5}{12}\\\hline 9\frac{1}{4}\end{array}$ $\begin{array}{r}1\frac{7}{10}\\7\frac{7}{10}\\+2\frac{9}{10}\\\hline 12\frac{3}{10}\end{array}$

⑫

1.	$7\frac{3}{8}$ $+2\frac{3}{4}$ $10\frac{1}{8}$	$1\frac{1}{4}$ $+\frac{7}{8}$ $2\frac{1}{8}$	$8\frac{7}{8}$ $+2\frac{9}{16}$ $11\frac{7}{16}$	$1\frac{3}{10}$ $+5\frac{4}{5}$ $7\frac{1}{10}$	$4\frac{5}{12}$ $+1\frac{5}{6}$ $6\frac{1}{4}$	$5\frac{1}{2}$ $+3\frac{5}{16}$ $8\frac{13}{16}$	$13\frac{5}{8}$ $+26\frac{3}{4}$ $40\frac{3}{8}$	$9\frac{2}{3}$ $+7\frac{1}{6}$ $16\frac{5}{6}$
2.	$3\frac{1}{6}$ $+3\frac{1}{3}$ $6\frac{1}{2}$	$4\frac{2}{3}$ $+2\frac{5}{6}$ $7\frac{1}{2}$	$1\frac{3}{4}$ $+5\frac{11}{16}$ $7\frac{7}{16}$	$5\frac{1}{2}$ $+8\frac{7}{10}$ $14\frac{1}{5}$	$\frac{5}{12}$ $+2\frac{3}{4}$ $3\frac{1}{6}$	$4\frac{1}{5}$ $+7\frac{3}{10}$ $11\frac{1}{2}$	$12\frac{3}{4}$ $+12\frac{5}{8}$ $25\frac{3}{8}$	$8\frac{1}{2}$ $+8\frac{5}{8}$ $17\frac{1}{8}$
3.	$6\frac{3}{4}$ $+3\frac{1}{2}$ $10\frac{1}{4}$	$8\frac{2}{3}$ $+7\frac{3}{4}$ $16\frac{5}{12}$	$2\frac{1}{2}$ $+6\frac{9}{16}$ $9\frac{1}{16}$	$3\frac{1}{2}$ $+\frac{9}{10}$ $4\frac{2}{5}$	$1\frac{7}{12}$ $+1\frac{2}{3}$ $3\frac{1}{4}$	$7\frac{3}{16}$ $+6\frac{7}{8}$ $14\frac{1}{16}$	$14\frac{1}{2}$ $+10\frac{2}{5}$ $24\frac{9}{10}$	$4\frac{3}{4}$ $+6\frac{3}{8}$ $11\frac{1}{8}$
4.	$1\frac{1}{2}$ $+1\frac{1}{3}$ $2\frac{5}{6}$	$2\frac{3}{5}$ $+9\frac{1}{2}$ $12\frac{1}{10}$	$5\frac{1}{6}$ $+4\frac{1}{12}$ $9\frac{1}{4}$	$3\frac{1}{12}$ $+2\frac{3}{4}$ $5\frac{5}{6}$	$6\frac{1}{3}$ $+5\frac{1}{12}$ $11\frac{5}{12}$	$\frac{5}{12}$ $+1\frac{2}{3}$ $2\frac{1}{12}$	$11\frac{2}{3}$ $+15\frac{1}{2}$ $27\frac{1}{6}$	$1\frac{5}{6}$ $+8\frac{1}{2}$ $10\frac{1}{3}$
5.	$4\frac{1}{6}$ $1\frac{2}{3}$ $3\frac{2}{3}$ $+1\frac{2}{3}$ $11\frac{1}{6}$	$3\frac{1}{2}$ $4\frac{3}{4}$ $2\frac{1}{2}$ $+6\frac{1}{8}$ $16\frac{7}{8}$	$4\frac{1}{4}$ $2\frac{3}{8}$ $1\frac{5}{16}$ $+3\frac{1}{4}$ $11\frac{3}{16}$	$3\frac{3}{4}$ $4\frac{1}{16}$ $2\frac{5}{8}$ $+1\frac{3}{16}$ $11\frac{5}{8}$	6 $4\frac{4}{5}$ $2\frac{7}{10}$ $+1\frac{1}{2}$ 15	$5\frac{1}{2}$ $2\frac{2}{3}$ $6\frac{1}{12}$ $+3\frac{3}{4}$ 18	$22\frac{3}{8}$ $\frac{3}{4}$ $12\frac{1}{2}$ $+34\frac{5}{8}$ $70\frac{1}{4}$	$3\frac{1}{3}$ $2\frac{1}{2}$ $9\frac{5}{6}$ $+1\frac{1}{3}$ 17

⑬

6.	$4\frac{3}{4}$ $-2\frac{1}{8}$ $2\frac{5}{8}$	$8\frac{4}{5}$ $-4\frac{1}{5}$ $4\frac{3}{5}$	$5\frac{1}{2}$ -2 $3\frac{1}{2}$	$2\frac{7}{12}$ $-1\frac{1}{3}$ $1\frac{1}{4}$	$8\frac{9}{16}$ $-5\frac{1}{4}$ $3\frac{5}{16}$	$6\frac{7}{8}$ $-3\frac{1}{2}$ $3\frac{3}{8}$	$20\frac{1}{2}$ $-10\frac{1}{3}$ $10\frac{1}{6}$	$7\frac{3}{4}$ $-1\frac{1}{2}$ $6\frac{1}{4}$
7.	$6\frac{2}{3}$ $-\frac{1}{4}$ $6\frac{5}{12}$	$4\frac{7}{8}$ $-4\frac{1}{8}$ $\frac{3}{4}$	$2\frac{5}{6}$ $-\frac{1}{12}$ $2\frac{3}{4}$	$8\frac{9}{10}$ $-6\frac{1}{2}$ $2\frac{2}{5}$	$3\frac{11}{16}$ $-3\frac{3}{16}$ $\frac{1}{2}$	$9\frac{5}{6}$ $-7\frac{1}{3}$ $2\frac{1}{2}$	$13\frac{3}{4}$ $-7\frac{3}{8}$ $6\frac{3}{8}$	$6\frac{7}{8}$ $-2\frac{7}{8}$ 4
8.	$8\frac{5}{6}$ $-4\frac{1}{6}$ $4\frac{2}{3}$	$3\frac{4}{5}$ $-1\frac{1}{2}$ $2\frac{3}{10}$	$9\frac{7}{10}$ $-4\frac{1}{10}$ $5\frac{3}{5}$	$6\frac{11}{12}$ $-2\frac{1}{2}$ $4\frac{5}{12}$	$5\frac{9}{16}$ $-4\frac{1}{4}$ $1\frac{5}{16}$	$7\frac{5}{8}$ $-3\frac{1}{4}$ $4\frac{3}{8}$	$15\frac{1}{2}$ $-10\frac{1}{4}$ $5\frac{1}{4}$	$9\frac{5}{6}$ $-2\frac{1}{2}$ $7\frac{1}{3}$
9.	$7\frac{1}{2}$ $-4\frac{1}{2}$ 3	$3\frac{3}{4}$ $-2\frac{1}{4}$ $1\frac{1}{2}$	$8\frac{15}{16}$ $-6\frac{3}{4}$ $2\frac{3}{16}$	$7\frac{11}{12}$ $-6\frac{5}{12}$ $1\frac{1}{2}$	$9\frac{9}{10}$ $-2\frac{2}{5}$ $7\frac{1}{2}$	$4\frac{5}{6}$ $-2\frac{1}{3}$ $2\frac{1}{2}$	$20\frac{2}{3}$ $-14\frac{1}{3}$ $6\frac{1}{3}$	$8\frac{5}{6}$ $-7\frac{1}{3}$ $1\frac{1}{2}$
10.	$8\frac{3}{5}$ $-2\frac{1}{5}$ $6\frac{2}{5}$	$4\frac{1}{2}$ $-1\frac{1}{8}$ $3\frac{3}{8}$	$5\frac{7}{10}$ $-4\frac{1}{2}$ $1\frac{1}{5}$	$9\frac{13}{16}$ $-6\frac{7}{16}$ $3\frac{3}{8}$	$7\frac{7}{12}$ $-1\frac{1}{2}$ $6\frac{1}{12}$	$8\frac{1}{2}$ $-2\frac{1}{5}$ $6\frac{3}{10}$	$9\frac{3}{4}$ $-7\frac{2}{3}$ $2\frac{1}{12}$	$8\frac{3}{4}$ $-4\frac{3}{4}$ 4
11.	$9\frac{1}{2}$ $-1\frac{1}{8}$ $8\frac{3}{8}$	$6\frac{7}{8}$ $-3\frac{1}{4}$ $3\frac{5}{8}$	$8\frac{11}{12}$ $-4\frac{3}{4}$ $4\frac{1}{6}$	$7\frac{4}{5}$ $-5\frac{3}{10}$ $2\frac{1}{2}$	$3\frac{9}{16}$ $-1\frac{1}{16}$ $2\frac{1}{2}$	$6\frac{5}{8}$ $-3\frac{3}{8}$ $3\frac{1}{4}$	$4\frac{7}{8}$ $-4\frac{1}{8}$ $\frac{3}{4}$	$2\frac{5}{6}$ $-1\frac{1}{2}$ $1\frac{1}{3}$

More Practice

⑭

1.	4 $-\frac{2}{3}$ $3\frac{1}{3}$	8 $-\frac{3}{5}$ $7\frac{2}{5}$	6 $-\frac{1}{2}$ $5\frac{1}{2}$	2 $-\frac{9}{10}$ $1\frac{1}{10}$	3 $-\frac{5}{12}$ $2\frac{7}{12}$	7 $-\frac{11}{16}$ $6\frac{5}{16}$	10 $-\frac{5}{6}$ $9\frac{1}{6}$	12 $-\frac{3}{4}$ $11\frac{1}{4}$
2.	1 $-\frac{5}{6}$ $\frac{1}{6}$	7 $-1\frac{3}{4}$ $5\frac{1}{4}$	9 $-2\frac{2}{3}$ $6\frac{1}{3}$	5 $-4\frac{11}{16}$ $\frac{5}{16}$	4 $-2\frac{7}{10}$ $1\frac{3}{10}$	1 $-\frac{5}{12}$ $\frac{7}{12}$	15 $-7\frac{3}{8}$ $7\frac{5}{8}$	16 $-11\frac{1}{6}$ $4\frac{5}{6}$
3.	$6\frac{1}{4}$ $-2\frac{3}{4}$ $3\frac{1}{2}$	$9\frac{1}{6}$ $-4\frac{2}{3}$ $4\frac{1}{2}$	$5\frac{1}{8}$ $-4\frac{3}{4}$ $\frac{3}{8}$	$7\frac{3}{10}$ $-2\frac{4}{5}$ $4\frac{1}{2}$	$6\frac{5}{12}$ $-5\frac{5}{6}$ $\frac{7}{12}$	$4\frac{3}{16}$ $-1\frac{9}{16}$ $2\frac{5}{8}$	$12\frac{1}{2}$ $-10\frac{2}{3}$ $1\frac{5}{6}$	$14\frac{3}{8}$ $-10\frac{3}{4}$ $3\frac{5}{8}$
4.	$6\frac{5}{6}$ $-3\frac{5}{6}$ 3	$4\frac{1}{8}$ $-1\frac{5}{8}$ $2\frac{1}{2}$	$2\frac{1}{6}$ $-1\frac{2}{3}$ $\frac{1}{2}$	$5\frac{1}{10}$ $-3\frac{1}{2}$ $1\frac{3}{5}$	$8\frac{1}{10}$ $-6\frac{7}{10}$ $1\frac{2}{5}$	$4\frac{3}{16}$ $-2\frac{5}{8}$ $1\frac{9}{16}$	$11\frac{1}{4}$ $-9\frac{1}{2}$ $1\frac{3}{4}$	$17\frac{1}{6}$ $-12\frac{1}{2}$ $4\frac{2}{3}$
5.	$5\frac{1}{6}$ $-1\frac{5}{6}$ $3\frac{1}{3}$	$9\frac{3}{8}$ $-7\frac{3}{4}$ $1\frac{5}{8}$	$7\frac{1}{2}$ $-4\frac{1}{2}$ 3	$6\frac{5}{12}$ $-2\frac{11}{12}$ $3\frac{1}{2}$	$3\frac{3}{10}$ $-1\frac{4}{5}$ $1\frac{1}{2}$	$8\frac{1}{16}$ $-5\frac{7}{16}$ $2\frac{5}{8}$	$14\frac{1}{6}$ $-11\frac{2}{3}$ $2\frac{1}{2}$	$20\frac{1}{8}$ $-15\frac{7}{8}$ $4\frac{1}{4}$

⑮

Tell the missing numbers that go in the numerators:

6. $4\frac{1}{2} = 4\frac{?}{8}$ 4 $5\frac{1}{3} = 5\frac{?}{12}$ 4 $5\frac{2}{3} = 5\frac{?}{6} \overset{4}{=} 4\frac{?}{6}$ 10 $3\frac{1}{4} = 3\frac{?}{16} \overset{4}{=} 2\frac{?}{16}$ 20

7. $6\frac{1}{5} = 6\frac{?}{10}$ 2 $2\frac{1}{2} = 2\frac{?}{10}$ 5 $3\frac{3}{4} = 3\frac{?}{8} \overset{6}{=} 2\frac{?}{8}$ 14 $7\frac{3}{8} = 7\frac{?}{16} \overset{6}{=} 6\frac{?}{16}$ 22

8. $3\frac{5}{6} = 3\frac{?}{12}$ 10 $6\frac{5}{8} = 6\frac{?}{16}$ 10 $2\frac{1}{2} = 2\frac{?}{10} \overset{5}{=} 1\frac{?}{10}$ 15 $6\frac{7}{12} = 6\frac{?}{24} \overset{14}{=} 5\frac{?}{24}$ 38

9. $1\frac{3}{4} = 1\frac{?}{16}$ 12 $9\frac{1}{4} = 9\frac{?}{12}$ 3 $3\frac{4}{5} = 3\frac{?}{20} \overset{16}{=} 2\frac{?}{20}$ 36 $2\frac{3}{10} = 2\frac{?}{30} \overset{9}{=} 1\frac{?}{30}$ 39

10. $4\frac{2}{3} = 4\frac{?}{24}$ 16 $8\frac{1}{6} = 8\frac{?}{24}$ 4 $4\frac{1}{8} = 4\frac{?}{24} \overset{3}{=} 3\frac{?}{24}$ 27 $7\frac{7}{10} = 7\frac{?}{20} \overset{14}{=} 6\frac{?}{20}$ 34

11.	$5\frac{1}{2}$ $-3\frac{5}{6}$ $1\frac{2}{3}$	$4\frac{3}{4}$ $-3\frac{7}{8}$ $\frac{7}{8}$	$6\frac{1}{5}$ $-4\frac{1}{2}$ $1\frac{7}{10}$	$8\frac{1}{4}$ $-3\frac{7}{12}$ $4\frac{2}{3}$	$7\frac{3}{8}$ $-6\frac{9}{16}$ $\frac{13}{16}$	$9\frac{1}{2}$ $-6\frac{9}{10}$ $2\frac{3}{5}$	$18\frac{2}{5}$ $-11\frac{1}{2}$ $6\frac{9}{10}$	$11\frac{1}{2}$ $-10\frac{7}{8}$ $\frac{5}{8}$
12.	$3\frac{1}{2}$ $-1\frac{3}{4}$ $1\frac{3}{4}$	$8\frac{1}{3}$ $-7\frac{5}{6}$ $\frac{1}{2}$	$9\frac{1}{2}$ $-3\frac{5}{8}$ $5\frac{7}{8}$	$21\frac{1}{2}$ $-15\frac{4}{5}$ $5\frac{7}{10}$	$9\frac{1}{2}$ $-8\frac{15}{16}$ $\frac{9}{16}$	$7\frac{3}{16}$ $-3\frac{7}{8}$ $3\frac{5}{16}$	$3\frac{1}{4}$ $-2\frac{1}{3}$ $\frac{11}{12}$	$15\frac{1}{4}$ $-12\frac{5}{8}$ $2\frac{5}{8}$
13.	$6\frac{1}{3}$ $-2\frac{5}{6}$ $3\frac{1}{2}$	$9\frac{1}{2}$ $-3\frac{7}{8}$ $5\frac{5}{8}$	$8\frac{1}{4}$ $-1\frac{2}{3}$ $6\frac{7}{12}$	$5\frac{1}{4}$ $-3\frac{7}{12}$ $1\frac{2}{3}$	$7\frac{1}{5}$ $-1\frac{7}{10}$ $5\frac{1}{2}$	$9\frac{1}{2}$ $-5\frac{11}{16}$ $3\frac{13}{16}$	$10\frac{1}{2}$ $-9\frac{5}{6}$ $\frac{2}{3}$	$16\frac{1}{2}$ $-13\frac{4}{5}$ $2\frac{7}{10}$
14.	$9\frac{1}{4}$ $-1\frac{3}{8}$ $7\frac{7}{8}$	$4\frac{2}{3}$ $-3\frac{5}{6}$ $\frac{5}{6}$	$7\frac{1}{2}$ $-6\frac{5}{8}$ $\frac{7}{8}$	$8\frac{1}{2}$ $-5\frac{9}{10}$ $2\frac{3}{5}$	$6\frac{1}{4}$ $-5\frac{9}{16}$ $\frac{11}{12}$	$4\frac{1}{3}$ $-2\frac{11}{12}$ $1\frac{5}{12}$	$18\frac{1}{2}$ $-11\frac{5}{8}$ $6\frac{7}{8}$	$19\frac{3}{4}$ $-15\frac{7}{8}$ $3\frac{7}{8}$

⑯

1.	$3\frac{1}{6}$ $+2\frac{1}{4}$ $5\frac{5}{12}$	$4\frac{1}{8}$ $+3\frac{1}{6}$ $7\frac{7}{24}$	$8\frac{3}{4}$ $+2\frac{1}{10}$ $10\frac{17}{20}$	$3\frac{3}{8}$ $+7\frac{1}{12}$ $10\frac{11}{24}$	$5\frac{3}{10}$ $+1\frac{1}{4}$ $6\frac{11}{20}$	$8\frac{5}{6}$ $+2\frac{1}{4}$ $11\frac{1}{12}$	$12\frac{1}{8}$ $+15\frac{5}{6}$ $27\frac{23}{24}$	$7\frac{1}{4}$ $+7\frac{1}{6}$ $14\frac{5}{12}$
2.	$5\frac{1}{6}$ $+5\frac{3}{8}$ $10\frac{13}{24}$	$7\frac{5}{6}$ $+4\frac{5}{8}$ $12\frac{11}{24}$	$6\frac{3}{4}$ $+3\frac{1}{10}$ $9\frac{17}{20}$	$2\frac{3}{8}$ $+2\frac{1}{12}$ $4\frac{11}{24}$	$9\frac{1}{6}$ $+5\frac{7}{10}$ $14\frac{13}{15}$	$4\frac{1}{4}$ $+5\frac{1}{6}$ $9\frac{5}{12}$	$16\frac{3}{8}$ $+16\frac{1}{6}$ $32\frac{13}{24}$	$2\frac{1}{8}$ $+5\frac{5}{6}$ $7\frac{23}{24}$
3.	$9\frac{5}{6}$ $-1\frac{3}{4}$ $8\frac{1}{12}$	$2\frac{7}{8}$ $-1\frac{1}{6}$ $1\frac{17}{24}$	$8\frac{3}{4}$ $-6\frac{3}{10}$ $2\frac{9}{20}$	$4\frac{1}{10}$ $-2\frac{5}{8}$ $1\frac{19}{40}$	$5\frac{5}{12}$ $-3\frac{7}{8}$ $1\frac{13}{24}$	$8\frac{1}{6}$ $-4\frac{7}{8}$ $3\frac{7}{24}$	$17\frac{1}{4}$ $-15\frac{5}{6}$ $1\frac{5}{12}$	$9\frac{1}{4}$ $-4\frac{1}{6}$ $5\frac{1}{12}$
4.	$7\frac{1}{6}$ $-4\frac{3}{4}$ $2\frac{5}{12}$	$5\frac{3}{4}$ $-5\frac{1}{6}$ $\frac{7}{12}$	$9\frac{1}{8}$ $-2\frac{5}{12}$ $6\frac{17}{24}$	$1\frac{3}{4}$ $-\ \frac{3}{10}$ $1\frac{9}{20}$	$8\frac{3}{10}$ $-2\frac{5}{6}$ $5\frac{7}{15}$	$6\frac{5}{6}$ $-1\frac{7}{8}$ $4\frac{23}{24}$	$37\frac{5}{6}$ $-22\frac{1}{4}$ $15\frac{7}{12}$	$7\frac{3}{4}$ $-6\frac{1}{6}$ $1\frac{7}{12}$
5.	$8\frac{5}{6}$ $-7\frac{3}{4}$ $1\frac{1}{12}$	$9\frac{1}{8}$ $-1\frac{5}{6}$ $7\frac{7}{24}$	$7\frac{3}{4}$ $-3\frac{7}{10}$ $4\frac{1}{20}$	$4\frac{3}{8}$ $-2\frac{1}{12}$ $2\frac{7}{24}$	$6\frac{1}{6}$ $-3\frac{3}{10}$ $2\frac{13}{15}$	$4\frac{3}{4}$ $-1\frac{1}{6}$ $3\frac{7}{12}$	$19\frac{3}{8}$ $-13\frac{1}{6}$ $6\frac{5}{24}$	$8\frac{1}{8}$ $-2\frac{1}{6}$ $5\frac{23}{24}$

⑰

6.	$8\frac{1}{4}$ $1\frac{5}{6}$ $+4\frac{1}{2}$ $14\frac{7}{12}$	$2\frac{3}{8}$ $2\frac{1}{6}$ $+2\frac{1}{4}$ $6\frac{19}{24}$	$6\frac{1}{10}$ $2\frac{3}{4}$ $+5\frac{1}{2}$ $14\frac{7}{20}$	$1\frac{5}{12}$ $1\frac{3}{8}$ $+4\frac{1}{4}$ $7\frac{1}{24}$	$3\frac{1}{6}$ $4\frac{7}{10}$ $+2\frac{2}{3}$ $10\frac{8}{15}$	$4\frac{5}{6}$ $4\frac{1}{2}$ $+4\frac{3}{4}$ $14\frac{1}{12}$	$15\frac{1}{8}$ $7\frac{1}{2}$ $+6\frac{1}{6}$ $28\frac{19}{24}$	$2\frac{3}{4}$ $4\frac{1}{6}$ $+7\frac{1}{2}$ $14\frac{5}{12}$
7.	$3\frac{1}{2}$ $3\frac{1}{3}$ $+1\frac{1}{4}$ $8\frac{1}{12}$	$2\frac{5}{6}$ $6\frac{1}{3}$ $+4\frac{1}{4}$ $13\frac{5}{12}$	$1\frac{1}{12}$ $7\frac{1}{8}$ $+3\frac{1}{6}$ $11\frac{3}{8}$	$4\frac{1}{2}$ $8\frac{3}{10}$ $+4\frac{5}{6}$ $17\frac{19}{30}$	$2\frac{3}{4}$ $2\frac{7}{10}$ $+6\frac{1}{2}$ $11\frac{19}{20}$	$1\frac{3}{8}$ $1\frac{1}{6}$ $+3\frac{1}{3}$ $5\frac{7}{8}$	$10\frac{5}{6}$ $14\frac{1}{4}$ $+13\frac{1}{2}$ $38\frac{7}{12}$	$6\frac{1}{3}$ $2\frac{3}{8}$ $+7\frac{1}{6}$ $15\frac{7}{8}$
8.	$4\frac{5}{6}$ $2\frac{2}{3}$ $+4\frac{3}{4}$ $12\frac{1}{4}$	$6\frac{2}{3}$ $7\frac{1}{4}$ $+3\frac{1}{6}$ $17\frac{1}{12}$	$2\frac{3}{10}$ $2\frac{5}{8}$ $+2\frac{5}{6}$ $7\frac{91}{120}$	$5\frac{7}{12}$ $1\frac{1}{8}$ $+3\frac{3}{4}$ $10\frac{11}{24}$	$7\frac{1}{10}$ $4\frac{1}{6}$ $+5\frac{2}{3}$ $16\frac{14}{15}$	$9\frac{1}{4}$ $2\frac{5}{6}$ $+6\frac{1}{2}$ $18\frac{7}{12}$	$15\frac{7}{8}$ $18\frac{1}{6}$ $+11\frac{1}{3}$ $45\frac{3}{8}$	$9\frac{3}{4}$ $2\frac{5}{8}$ $+2\frac{5}{6}$ $15\frac{5}{24}$
9.	$1\frac{1}{2}$ $1\frac{5}{6}$ $1\frac{1}{4}$ $+3\frac{3}{8}$ $7\frac{23}{24}$	$3\frac{1}{6}$ $2\frac{2}{3}$ $2\frac{3}{4}$ $+\ \frac{7}{8}$ $9\frac{11}{24}$	$2\frac{5}{12}$ $5\frac{1}{2}$ $6\frac{1}{4}$ $+8\frac{5}{6}$ 23	$5\frac{1}{3}$ $1\frac{5}{6}$ $4\frac{3}{10}$ $+7\frac{1}{2}$ $18\frac{29}{30}$	$1\frac{1}{5}$ $2\frac{3}{4}$ $1\frac{3}{10}$ $+5\frac{1}{2}$ $10\frac{3}{4}$	$2\frac{1}{2}$ $3\frac{5}{6}$ $3\frac{1}{4}$ $+8\frac{2}{3}$ $18\frac{1}{4}$	$15\frac{3}{4}$ $10\frac{1}{3}$ $11\frac{1}{6}$ $+\ 9\frac{1}{8}$ $46\frac{3}{8}$	$8\frac{1}{2}$ $2\frac{1}{6}$ $1\frac{1}{4}$ $+4\frac{1}{3}$ $16\frac{1}{4}$
10.	$2\frac{1}{4}$ $5\frac{2}{3}$ $2\frac{5}{6}$ $+4\frac{5}{8}$ $15\frac{3}{8}$	$6\frac{1}{3}$ $1\frac{7}{8}$ $7\frac{5}{6}$ $+5\frac{1}{2}$ $21\frac{13}{24}$	$1\frac{3}{4}$ $8\frac{1}{3}$ $2\frac{1}{12}$ $+7\frac{3}{8}$ $19\frac{13}{24}$	$5\frac{1}{2}$ $6\frac{9}{10}$ $6\frac{1}{4}$ $+4\frac{4}{5}$ $23\frac{9}{20}$	$3\frac{1}{8}$ $8\frac{2}{3}$ $4\frac{5}{6}$ $+2\frac{3}{4}$ $19\frac{3}{8}$	$4\frac{1}{2}$ $9\frac{1}{3}$ $1\frac{3}{4}$ $+2\frac{1}{6}$ $17\frac{3}{4}$	$11\frac{1}{2}$ $17\frac{1}{4}$ $21\frac{1}{8}$ $+13\frac{2}{3}$ $63\frac{13}{24}$	$1\frac{1}{8}$ $6\frac{1}{2}$ $3\frac{1}{3}$ $+7\frac{1}{6}$ $18\frac{1}{8}$

More Practice

⑱

1. $\frac{1}{3} \times 9$ 3 $\frac{4}{5} \times 30$ 24 $\frac{7}{8} \times 48$ 42 $\frac{3}{4} \times 36$ 27 $\frac{2}{3} \times 6$ 4

2. $\frac{3}{4} \times 7$ $5\frac{1}{4}$ $\frac{5}{8} \times 11$ $6\frac{7}{8}$ $\frac{2}{3} \times 16$ $10\frac{2}{3}$ $\frac{9}{10} \times 13$ $11\frac{7}{10}$ $\frac{5}{6} \times 5$ $4\frac{1}{6}$

3. $\frac{5}{6} \times 9$ $7\frac{1}{2}$ $\frac{7}{16} \times 36$ $15\frac{3}{4}$ $\frac{5}{12} \times 15$ $6\frac{1}{4}$ $\frac{15}{16} \times 12$ $11\frac{1}{4}$ $\frac{5}{8} \times 6$ $3\frac{3}{4}$

4. $7 \times \frac{2}{5}$ $2\frac{4}{5}$ $11 \times \frac{3}{4}$ $8\frac{1}{4}$ $15 \times \frac{3}{8}$ $5\frac{5}{8}$ $17 \times \frac{5}{6}$ $14\frac{1}{6}$ $4 \times \frac{1}{3}$ $1\frac{1}{3}$

5. $6 \times \frac{1}{3}$ 2 $12 \times \frac{5}{6}$ 10 $60 \times \frac{7}{12}$ 35 $48 \times \frac{3}{16}$ 9 $4 \times \frac{7}{8}$ $3\frac{1}{2}$

6. $6 \times \frac{7}{16}$ $2\frac{5}{8}$ $16 \times \frac{5}{12}$ $6\frac{2}{3}$ $25 \times \frac{9}{10}$ $22\frac{1}{2}$ $14 \times \frac{7}{10}$ $9\frac{4}{5}$ $6 \times \frac{7}{8}$ $5\frac{1}{4}$

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7. $3 \times 8\frac{1}{2}$ $25\frac{1}{2}$ $11 \times 2\frac{1}{4}$ $24\frac{3}{4}$ $10 \times 1\frac{2}{3}$ $16\frac{2}{3}$ $13 \times 2\frac{5}{6}$ $36\frac{5}{6}$ $9 \times 4\frac{1}{2}$ $40\frac{1}{2}$

8. $9 \times 6\frac{2}{3}$ 60 $24 \times 2\frac{1}{8}$ 51 $20 \times 3\frac{1}{5}$ 64 $15 \times 3\frac{1}{3}$ 50 $6 \times 3\frac{2}{3}$ 22

9. $6 \times 1\frac{1}{4}$ $7\frac{1}{2}$ $15 \times 2\frac{1}{6}$ $32\frac{1}{2}$ $12 \times 1\frac{7}{8}$ $22\frac{1}{2}$ $18 \times 2\frac{1}{12}$ $37\frac{1}{2}$ $8 \times 5\frac{1}{6}$ $41\frac{1}{3}$

10. $1\frac{1}{4} \times 9$ $11\frac{1}{4}$ $3\frac{2}{3} \times 10$ $36\frac{2}{3}$ $6\frac{1}{4} \times 11$ $68\frac{3}{4}$ $1\frac{7}{8} \times 13$ $24\frac{3}{8}$ $3\frac{1}{3} \times 14$ $46\frac{2}{3}$

11. $5\frac{5}{6} \times 18$ 105 $3\frac{3}{5} \times 25$ 90 $2\frac{3}{4} \times 12$ 33 $1\frac{1}{5} \times 35$ 42 $1\frac{5}{6} \times 12$ 22

12. $3\frac{3}{4} \times 10$ $37\frac{1}{2}$ $5\frac{1}{4} \times 18$ $94\frac{1}{2}$ $5\frac{5}{6} \times 15$ $87\frac{1}{2}$ $6\frac{3}{4} \times 14$ $94\frac{1}{2}$ $6\frac{1}{8} \times 18$ $110\frac{1}{4}$

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13. $\frac{2}{3} \times \frac{2}{3}$ $\frac{4}{9}$ $\frac{3}{4} \times \frac{5}{8}$ $\frac{15}{32}$ $\frac{1}{5} \times \frac{3}{4}$ $\frac{3}{20}$ $\frac{3}{4} \times \frac{1}{2}$ $\frac{3}{8}$ $\frac{1}{2} \times \frac{3}{8}$ $\frac{3}{16}$

14. $\frac{3}{4} \times \frac{4}{5}$ $\frac{3}{5}$ $\frac{9}{10} \times \frac{2}{3}$ $\frac{3}{5}$ $\frac{5}{12} \times \frac{4}{5}$ $\frac{1}{3}$ $\frac{3}{10} \times \frac{5}{6}$ $\frac{1}{4}$ $\frac{5}{16} \times \frac{4}{5}$ $\frac{1}{4}$

15. $\frac{5}{6} \times \frac{4}{5}$ $\frac{2}{3}$ $\frac{9}{16} \times \frac{5}{6}$ $\frac{15}{32}$ $\frac{3}{10} \times \frac{4}{9}$ $\frac{2}{15}$ $\frac{9}{16} \times \frac{2}{3}$ $\frac{3}{8}$ $\frac{5}{12} \times \frac{3}{4}$ $\frac{5}{16}$

16. $\frac{3}{4} \times \frac{3}{4}$ $\frac{9}{16}$ $\frac{1}{10} \times \frac{5}{8}$ $\frac{1}{16}$ $\frac{15}{16} \times \frac{2}{3}$ $\frac{5}{8}$ $\frac{11}{12} \times \frac{3}{4}$ $\frac{11}{16}$ $\frac{5}{16} \times \frac{2}{5}$ $\frac{1}{8}$

17. $\frac{1}{8} \times \frac{4}{5}$ $\frac{1}{10}$ $\frac{9}{10} \times \frac{5}{6}$ $\frac{3}{4}$ $\frac{3}{10} \times \frac{2}{3}$ $\frac{1}{5}$ $\frac{9}{10} \times \frac{5}{12}$ $\frac{3}{8}$ $\frac{3}{10} \times \frac{5}{6}$ $\frac{1}{4}$

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18. $\frac{1}{3} \times 2\frac{1}{2}$ $\frac{5}{6}$ $\frac{2}{3} \times 4\frac{1}{2}$ 3 $\frac{3}{4} \times 3\frac{1}{2}$ $2\frac{5}{8}$ $\frac{1}{2} \times 1\frac{2}{5}$ $\frac{7}{10}$ $\frac{1}{2} \times 1\frac{2}{3}$ $\frac{5}{6}$

19. $\frac{5}{16} \times 6\frac{2}{5}$ 2 $\frac{3}{8} \times 9\frac{1}{3}$ $3\frac{1}{2}$ $\frac{9}{10} \times 8\frac{1}{3}$ $7\frac{1}{2}$ $\frac{3}{4} \times 1\frac{1}{3}$ 1 $\frac{1}{5} \times 3\frac{1}{8}$ $\frac{5}{8}$

20. $1\frac{1}{4} \times \frac{3}{4}$ $\frac{15}{16}$ $2\frac{1}{3} \times \frac{1}{4}$ $\frac{7}{12}$ $1\frac{5}{6} \times \frac{5}{8}$ $1\frac{7}{48}$ $3\frac{2}{3} \times \frac{1}{2}$ $1\frac{5}{6}$ $7\frac{1}{2} \times \frac{4}{5}$ 6

21. $4\frac{1}{6} \times \frac{9}{10}$ $3\frac{3}{4}$ $8\frac{3}{4} \times \frac{4}{5}$ 7 $1\frac{3}{5} \times \frac{5}{12}$ $\frac{2}{3}$ $3\frac{1}{3} \times \frac{3}{10}$ 1 $8\frac{1}{4} \times \frac{5}{6}$ $6\frac{7}{8}$

22. $7\frac{1}{2} \times 2\frac{2}{5}$ 18 $1\frac{1}{4} \times 7\frac{1}{5}$ 9 $2\frac{3}{5} \times 1\frac{1}{6}$ $3\frac{1}{30}$ $2\frac{1}{4} \times 1\frac{2}{5}$ $3\frac{3}{20}$ $3\frac{1}{8} \times 3\frac{1}{3}$ $10\frac{5}{12}$

23. $3\frac{1}{5} \times 1\frac{1}{6}$ $3\frac{11}{15}$ $9\frac{3}{4} \times 3\frac{1}{3}$ $32\frac{1}{2}$ $4\frac{1}{6} \times 4\frac{1}{5}$ $17\frac{1}{2}$ $5\frac{5}{8} \times 3\frac{1}{5}$ 18 $3\frac{3}{4} \times 2\frac{2}{5}$ 9

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1. $3 \div \frac{1}{4}$ 12 $8 \div \frac{1}{5}$ 40 $12 \div \frac{1}{3}$ 36 $10 \div \frac{1}{8}$ 80 $7 \div \frac{2}{3}$ $10\frac{1}{2}$

2. $8 \div \frac{2}{3}$ 12 $12 \div \frac{3}{4}$ 16 $10 \div \frac{4}{5}$ $12\frac{1}{2}$ $15 \div \frac{9}{10}$ $16\frac{2}{3}$ $6 \div \frac{3}{5}$ 10

3. $7 \div \frac{3}{4}$ $9\frac{1}{3}$ $11 \div \frac{5}{8}$ $17\frac{3}{5}$ $12 \div \frac{7}{10}$ $17\frac{1}{7}$ $13 \div \frac{3}{5}$ $21\frac{2}{3}$ $8 \div \frac{1}{3}$ 24

4. $\frac{7}{8} \div \frac{3}{5}$ $1\frac{11}{24}$ $\frac{15}{16} \div \frac{4}{5}$ $1\frac{11}{64}$ $\frac{2}{3} \div \frac{1}{16}$ $10\frac{2}{3}$ $\frac{5}{8} \div \frac{3}{5}$ $1\frac{1}{24}$ $\frac{2}{3} \div \frac{4}{5}$ $\frac{5}{6}$

5. $\frac{7}{8} \div \frac{3}{4}$ $1\frac{1}{6}$ $\frac{3}{5} \div \frac{4}{5}$ $\frac{3}{4}$ $\frac{5}{16} \div \frac{5}{8}$ $\frac{1}{2}$ $\frac{3}{4} \div \frac{1}{12}$ 9 $\frac{4}{5} \div \frac{1}{10}$ 8

6. $\frac{3}{4} \div \frac{3}{10}$ $2\frac{1}{2}$ $\frac{5}{6} \div \frac{3}{8}$ $2\frac{2}{9}$ $\frac{3}{4} \div \frac{5}{6}$ $\frac{9}{10}$ $\frac{3}{10} \div \frac{7}{8}$ $\frac{12}{35}$ $\frac{1}{4} \div \frac{9}{16}$ $\frac{4}{9}$

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7. $1\frac{3}{4} \div \frac{1}{2}$ $3\frac{1}{2}$ $2\frac{1}{2} \div \frac{1}{8}$ 20 $2\frac{2}{3} \div \frac{1}{6}$ 16 $6\frac{1}{5} \div \frac{1}{10}$ 62 $1\frac{1}{3} \div \frac{2}{3}$ 2

8. $3\frac{1}{3} \div \frac{2}{3}$ 5 $2\frac{1}{12} \div \frac{5}{6}$ $2\frac{1}{2}$ $3\frac{3}{4} \div \frac{3}{8}$ 10 $2\frac{1}{10} \div \frac{3}{5}$ $3\frac{1}{2}$ $3\frac{1}{2} \div \frac{7}{8}$ 4

9. $1\frac{1}{8} \div \frac{4}{5}$ $1\frac{13}{32}$ $2\frac{5}{8} \div \frac{2}{3}$ $3\frac{15}{16}$ $1\frac{2}{5} \div \frac{3}{4}$ $1\frac{13}{15}$ $7\frac{1}{5} \div \frac{1}{6}$ $43\frac{1}{5}$ $8\frac{1}{4} \div \frac{11}{12}$ 9

10. $7\frac{1}{2} \div \frac{5}{6}$ 9 $3\frac{1}{3} \div \frac{5}{6}$ 4 $1\frac{1}{6} \div \frac{2}{3}$ $1\frac{3}{4}$ $2\frac{1}{2} \div \frac{2}{5}$ $6\frac{1}{4}$ $3\frac{3}{8} \div \frac{9}{10}$ $3\frac{1}{4}$

11. $2\frac{1}{2} \div \frac{3}{4}$ $3\frac{1}{3}$ $3\frac{3}{4} \div \frac{2}{3}$ $5\frac{5}{8}$ $5\frac{1}{4} \div \frac{7}{8}$ 6 $2\frac{3}{4} \div \frac{5}{8}$ $4\frac{2}{5}$ $1\frac{7}{8} \div \frac{9}{16}$ $3\frac{1}{3}$

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12. $\frac{1}{3} \div 4$ $\frac{1}{12}$ $\frac{5}{8} \div 2$ $\frac{5}{16}$ $\frac{3}{4} \div 2$ $\frac{3}{8}$ $\frac{5}{6} \div 3$ $\frac{5}{18}$ $\frac{1}{3} \div 2$ $\frac{1}{6}$

13. $\frac{3}{4} \div 6$ $\frac{1}{8}$ $\frac{9}{16} \div 3$ $\frac{3}{16}$ $\frac{7}{16} \div 7$ $\frac{1}{16}$ $\frac{5}{8} \div 10$ $\frac{1}{16}$ $\frac{5}{6} \div 2$ $\frac{5}{12}$

14. $\frac{9}{10} \div 6$ $\frac{3}{20}$ $\frac{4}{5} \div 10$ $\frac{3}{25}$ $\frac{15}{16} \div 9$ $\frac{5}{48}$ $\frac{9}{16} \div 12$ $\frac{3}{64}$ $\frac{2}{5} \div 4$ $\frac{1}{10}$

15. $4\frac{1}{5} \div 2$ $2\frac{1}{10}$ $4\frac{2}{3} \div 5$ $\frac{14}{15}$ $1\frac{1}{8} \div 4$ $\frac{9}{32}$ $2\frac{1}{5} \div 2$ $1\frac{1}{10}$ $1\frac{1}{2} \div 8$ $\frac{3}{16}$

16. $5\frac{1}{3} \div 8$ $\frac{2}{3}$ $4\frac{4}{5} \div 4$ $1\frac{1}{5}$ $2\frac{2}{5} \div 6$ $\frac{2}{5}$ $4\frac{1}{5} \div 7$ $\frac{3}{5}$ $5\frac{2}{5} \div 9$ $\frac{3}{5}$

17. $3\frac{3}{5} \div 4$ $\frac{9}{10}$ $6\frac{2}{3} \div 8$ $\frac{5}{6}$ $6\frac{1}{4} \div 15$ $\frac{5}{12}$ $2\frac{2}{5} \div 8$ $\frac{3}{10}$ $2\frac{1}{4} \div 3$ $\frac{3}{4}$

㉕

18. $6 \div 2\frac{1}{2}$ $2\frac{2}{5}$ $12 \div 1\frac{1}{8}$ $10\frac{2}{3}$ $8 \div 1\frac{1}{4}$ $6\frac{2}{5}$ $15 \div 2\frac{2}{3}$ $5\frac{5}{8}$ $10 \div 8\frac{1}{3}$ $1\frac{1}{5}$

19. $5 \div 1\frac{7}{8}$ $2\frac{2}{3}$ $18 \div 2\frac{1}{4}$ 8 $10 \div 2\frac{2}{3}$ $3\frac{3}{4}$ $20 \div 3\frac{1}{3}$ 6 $15 \div 4\frac{1}{6}$ $3\frac{3}{5}$

20. $\frac{1}{4} \div 1\frac{1}{3}$ $\frac{3}{16}$ $\frac{2}{3} \div 1\frac{1}{4}$ $\frac{8}{15}$ $\frac{1}{2} \div 1\frac{1}{5}$ $\frac{5}{12}$ $\frac{3}{4} \div 1\frac{2}{3}$ $\frac{9}{20}$ $\frac{15}{16} \div 2\frac{1}{2}$ $\frac{3}{8}$

21. $\frac{5}{8} \div 3\frac{3}{4}$ $\frac{1}{6}$ $\frac{9}{10} \div 3\frac{3}{4}$ $\frac{6}{25}$ $\frac{5}{6} \div 3\frac{1}{3}$ $\frac{1}{4}$ $\frac{9}{16} \div 3\frac{3}{4}$ $\frac{3}{20}$ $\frac{3}{10} \div 4\frac{1}{5}$ $\frac{1}{14}$

22. $7\frac{1}{2} \div 1\frac{1}{3}$ $5\frac{5}{8}$ $4\frac{1}{6} \div 1\frac{2}{3}$ $1\frac{1}{2}$ $4\frac{1}{2} \div 1\frac{1}{3}$ $3\frac{3}{8}$ $6\frac{2}{3} \div 3\frac{1}{3}$ 2 $8\frac{3}{4} \div 6\frac{2}{3}$ $1\frac{5}{16}$

23. $5\frac{1}{4} \div 3\frac{1}{2}$ $1\frac{1}{2}$ $7\frac{1}{2} \div 2\frac{2}{5}$ $3\frac{1}{8}$ $10\frac{1}{2} \div 4\frac{2}{3}$ $2\frac{1}{4}$ $3\frac{3}{4} \div 1\frac{1}{5}$ $3\frac{1}{8}$ $1\frac{1}{3} \div 5\frac{1}{3}$ $\frac{1}{4}$

More Practice

㉖

1.	3.65 .84 .99 +1.43 6.91	6.1 49.6 7.8 +81.9 145.4	7.36 6.48 5.48 +1.69 21.01	.453 .361 .222 +.795 1.831	1.592 8.558 2.697 +1.239 14.086	8.038 4.745 5.043 +1.196 19.022
2.	4.7 63.8 1.0 27.7 73.7 +91.8 262.7	1.97 .48 6.18 .61 1.42 +5.97 16.63	3.30 5.59 .93 7.03 .24 +3.46 20.55	.793 .850 .171 .405 .068 +.223 2.510	8.612 .446 9.374 .513 8.631 +5.270 32.846	5.370 9.862 4.684 9.456 8.546 +2.785 40.703

3. 3.7 + 2.9 + 8.1 + 5.2 + 7.6 + 15.8 + 11.9 55.2

4. 3.16 + 4.51 + 6.25 + 4.37 + 3.75 + 8.87 + 2.25 33.16

5. .29 + .85 + .63 + .18 + .38 + .56 + .15 + .39 + .35 3.78

6. 6.66 + 3.37 + 1.13 + 7.75 + 4.51 + 2.12 + 7.61 + 9.28 42.43

7. 3.186 + 4.612 + 1.325 + .863 + .425 + 4.792 + 6.375 21.578

8. .50 + 1.12 + 2.25 + .33 + 4.54 + 6.67 + 12.22 27.63

9. 2.38 + 19.5 + 82.7 + 60.93 + 12.6 + 18.9 + 23.5 220.51

10. 16.5 + 25.625 + 68.75 + 4 + 32.25 + 14 + 11.375 172.500

11. 4.0625 + 1.625 + 11 + 4.75 + 1.3125 + 2.875 + 4.6 30.2250

㉗

12.	62.7 −19.5 43.2	94.2 −45.3 48.9	8.65 −.98 7.67	1.05 −.47 .58	4.143 −.396 3.747	7.330 −1.292 6.038
13.	93.7 −26.9 66.8	33.2 −8.5 24.7	9.47 −1.85 7.62	3.00 −2.75 .25	4.195 −1.938 2.257	7.354 −2.195 5.159
14.	96.4 −14.9 81.5	70.1 −30.7 39.4	6.01 −.92 5.09	8.30 −7.95 .35	9.954 −2.315 7.639	5.151 −3.523 1.628

15. 34.1 − 16.7 17.4 3.75 − 1.5 2.25 $12 − $4.37 $7.63 $25 − $11.45 $13.55

16. 93.8 − 6.42 87.38 62.9 − 37.95 24.95 $18 − $6.54 $11.46 $46 − $29.87 $16.13

17. 1.06 − .385 .675 4.81 − 3.1 1.71 $20 − $9.89 $10.11 $70 − $62.28 $7.72

18. .937 − .618 .319 7.25 − 6.44 .81 $25 − $8.26 $16.74 $30 − $25.75 $4.25

㉘

1. 6 × 3.18 19.08   7 × 54.2 379.4   36 × .81 29.16   27 × 8.5 229.5   49 × .608 29.792
2. 9 × 1.03 9.27   9 × .059 .531   42 × 9.6 403.2   19 × 2.6 49.4   84 × 8.31 698.04
3. 7 × .904 6.328   5 × .786 3.930   38 × .45 17.10   45 × 3.9 175.5   96 × .042 4.032
4. 3 × 3.07 9.21   3 × 9.61 28.83   56 × 5.7 319.2   60 × .97 58.20   53 × 3.24 171.72
5. 9 × .042 .378   7 × 9.03 63.21   16 × .21 3.36   31 × .68 21.08   27 × .087 2.349

㉙

6. .6 × 5.7 3.42   .12 × 9.8 1.176   1.7 × 1.9 3.23   .25 × .52 .1300   3.7 × 8.42 31.154
7. .8 × 4.6 3.68   5.4 × .25 1.350   .94 × 3.7 3.478   7.4 × .38 2.812   8.8 × 49.6 436.48
8. .3 × 9.2 2.76   .08 × 9.5 .760   2.5 × .48 1.200   .69 × .51 .3519   .38 × .702 .26676
9. .2 × .98 .196   .09 × 7.7 .693   .82 × 1.6 1.312   .09 × 9.7 .873   3.9 × 6.17 24.063
10. .7 × .37 .259   .47 × 6.2 2.914   3.7 × 4.2 15.54   4.3 × .86 3.698   .98 × .496 .48608
11. .9 × .54 .486   3.6 × 5.4 19.44   2.4 × .31 .744   .87 × .19 .1653   4.6 × 6.09 28.014
12. .5 × 8.3 4.15   2.1 × 1.7 3.57   .36 × 8.2 2.952   1.8 × 6.9 12.42   .79 × 8.06 6.3674
13. .8 × 7.4 5.92   .91 × 6.2 5.642   3.3 × 4.7 15.51   .35 × 1.3 .455   2.1 × 1.75 3.675

㉚

14. .46 × .04 .0184   .07 × .24 .0168   .04 × .43 .0172   .16 × .25 .0400   .02 × .043 .00086
15. .08 × .06 .0048   .06 × .07 .0042   .06 × .09 .0054   .03 × .12 .0036   .12 × .125 01500
16. .97 × .06 .0582   .12 × .31 .0372   .02 × .46 .0092   1.3 × .027 .0351   1.4 × .005 .0070
17. .5 × .013 .0065   .11 × .12 .0132   .09 × .08 .0072   .46 × .04 .0184   6.5 × .096 .6240
18. .8 × .007 .0056   .04 × 1.2 .048   .07 × 1.7 .119   .08 × .75 .0600   .08 × .502 .04016

㉛

19. $\overset{2.36}{6\overline{)14.16}}$ $\overset{8.64}{7\overline{)60.48}}$ $\overset{5.96}{9\overline{)53.64}}$ $\overset{4.68}{28\overline{)131.04}}$ $\overset{.358}{59\overline{)21.122}}$
20. $\overset{14.5}{9\overline{)130.5}}$ $\overset{37.6}{4\overline{)150.4}}$ $\overset{30.7}{2\overline{)61.4}}$ $\overset{.112}{34\overline{)3.808}}$ $\overset{9.46}{33\overline{)312.18}}$
21. $\overset{.213}{8\overline{)1.704}}$ $\overset{.317}{5\overline{)1.585}}$ $\overset{2.76}{3\overline{)8.28}}$ $\overset{5.43}{73\overline{)396.39}}$ $\overset{2.71}{42\overline{)113.82}}$
22. $\overset{3.12}{6\overline{)18.72}}$ $\overset{77.2}{3\overline{)231.6}}$ $\overset{31.2}{9\overline{)280.8}}$ $\overset{76.2}{26\overline{)1981.2}}$ $\overset{2.76}{61\overline{)168.36}}$
23. $\overset{.375}{5\overline{)1.875}}$ $\overset{1.68}{7\overline{)11.76}}$ $\overset{38.7}{6\overline{)232.2}}$ $\overset{2.31}{74\overline{)170.94}}$ $\overset{40.2}{42\overline{)1688.4}}$
24. $\overset{5.94}{4\overline{)23.76}}$ $\overset{.173}{8\overline{)1.384}}$ $\overset{5.81}{5\overline{)29.05}}$ $\overset{6.75}{25\overline{)168.75}}$ $\overset{3.67}{48\overline{)176.16}}$

More Practice

㉜

Divide until there is no remainder. Annex zeros if necessary:

1. $\overset{42.125}{8\overline{)337.000}}$ $\overset{6.75}{36\overline{)243.00}}$ $\overset{18.75}{28\overline{)525.00}}$ $\overset{21.25}{12\overline{)255.00}}$ $\overset{8.6875}{32\overline{)278.0000}}$

2. $\overset{138.5}{6\overline{)831.0}}$ $\overset{13.64}{25\overline{)341.00}}$ $\overset{12.5}{34\overline{)425.0}}$ $\overset{22.8}{35\overline{)798.0}}$ $\overset{7.25}{84\overline{)609.00}}$

3. $\overset{178.75}{4\overline{)715.00}}$ $\overset{14.5}{12\overline{)174.0}}$ $\overset{4.5625}{96\overline{)438.0000}}$ $\overset{6.75}{44\overline{)297.00}}$ $\overset{17.5}{38\overline{)665.0}}$

4. $\overset{13.68}{5\overline{)68.40}}$ $\overset{.9625}{64\overline{)61.6000}}$ $\overset{.24375}{48\overline{)11.70000}}$ $\overset{3.125}{28\overline{)87.500}}$ $\overset{.58}{75\overline{)43.50}}$

5. $\overset{4.3375}{8\overline{)34.7000}}$ $\overset{.324}{75\overline{)24.300}}$ $\overset{.8625}{56\overline{)48.3000}}$ $\overset{.85}{52\overline{)44.20}}$ $\overset{.48125}{64\overline{)30.80000}}$

6. $\overset{7.54}{5\overline{)37.70}}$ $\overset{1.45}{36\overline{)52.20}}$ $\overset{1.225}{44\overline{)53.900}}$ $\overset{.0975}{72\overline{)7.0200}}$ $\overset{1.924}{25\overline{)48.100}}$

7. $\overset{5.7625}{8\overline{)46.1000}}$ $\overset{.912}{25\overline{)22.800}}$ $\overset{.675}{48\overline{)32.400}}$ $\overset{.425}{32\overline{)13.600}}$ $\overset{.475}{24\overline{)11.400}}$

㉝

Divide. In ex. 8–11, find the quotients correct to the nearest tenth; in ex. 12–14, find the quotients correct to the nearest hundredth:

8. $\overset{44.4}{9\overline{)400}}$ $\overset{40.3}{3\overline{)121}}$ $\overset{12.9}{28\overline{)362}}$ $\overset{14.3}{44\overline{)629}}$ $\overset{2.9}{63\overline{)182}}$

9. $\overset{3.4}{7\overline{)23.6}}$ $\overset{243.3}{4\overline{)973}}$ $\overset{27.9}{31\overline{)865}}$ $\overset{4.3}{74\overline{)318}}$ $\overset{5.2}{48\overline{)250}}$

10. $\overset{4.7}{6\overline{)28}}$ $\overset{7.6}{8\overline{)60.7}}$ $\overset{2.1}{65\overline{)138}}$ $\overset{5.7}{22\overline{)125}}$ $\overset{3.9}{87\overline{)343}}$

11. $\overset{25.9}{9\overline{)233}}$ $\overset{132.1}{7\overline{)925}}$ $\overset{1.5}{13\overline{)20.1}}$ $\overset{.2}{11\overline{)1.85}}$ $\overset{1.1}{17\overline{)19.2}}$

12. $\overset{209.75}{4\overline{)839}}$ $\overset{228.33}{3\overline{)685}}$ $\overset{16.47}{15\overline{)247}}$ $\overset{5.88}{17\overline{)100}}$ $\overset{16.19}{16\overline{)259}}$

13. $\overset{21.83}{6\overline{)131}}$ $\overset{1.38}{6\overline{)8.25}}$ $\overset{4.41}{64\overline{)282}}$ $\overset{5.36}{55\overline{)295}}$ $\overset{22.74}{19\overline{)432}}$

14. $\overset{12.96}{7\overline{)90.7}}$ $\overset{.37}{8\overline{)2.92}}$ $\overset{.63}{39\overline{)24.7}}$ $\overset{11.86}{59\overline{)700}}$ $\overset{16.25}{28\overline{)455}}$

㉞

Change these fractions to decimals:

15. $\frac{3}{4}$ .75 $\frac{4}{5}$ .8 $\frac{13}{20}$ .65 $\frac{7}{8}$ .875 $\frac{3}{16}$ .1875 $\frac{7}{10}$ .7 $\frac{21}{25}$ .84 $\frac{37}{50}$ .74 $\frac{9}{16}$ .5625 $\frac{7}{20}$ .35

Change to decimals correct to the nearest hundredth:

16. $\frac{3}{7}$ .43 $\frac{7}{9}$ .78 $\frac{19}{31}$ .61 $\frac{6}{7}$ .86 $\frac{16}{23}$ .70 $\frac{4}{19}$ .21 $\frac{31}{83}$ .37 $\frac{27}{59}$ .46 $\frac{13}{17}$ .76 $\frac{11}{12}$ .92

17. $\frac{2}{3}$ .67 $\frac{1}{9}$ .11 $\frac{13}{19}$ .68 $\frac{8}{9}$ .89 $\frac{11}{37}$ .30 $\frac{37}{53}$ .70 $\frac{17}{18}$ .94 $\frac{23}{61}$ .38 $\frac{60}{89}$ .67 $\frac{13}{15}$ .87

More Practice

㉟

Divide until there is no remainder. Annex zeros if necessary:

1. $\overset{283}{.76\overline{)215.08}}$ $\overset{137}{3.2\overline{)438.4}}$ $\overset{368}{2.4\overline{)883.2}}$ $\overset{31}{.157\overline{)4.867}}$

2. $\overset{394}{9.8\overline{)3861.2}}$ $\overset{17}{.63\overline{)10.71}}$ $\overset{413}{2.2\overline{)908.6}}$ $\overset{32}{.256\overline{)8.192}}$

3. $\overset{4.53}{3.5\overline{)15.855}}$ $\overset{14.2}{.79\overline{)11.218}}$ $\overset{6.54}{4.6\overline{)30.084}}$ $\overset{87.4}{.51\overline{)44.574}}$

4. $\overset{4.31}{2.8\overline{)12.068}}$ $\overset{2.64}{.63\overline{)1.6632}}$ $\overset{12.6}{8.4\overline{)105.84}}$ $\overset{6.43}{.97\overline{)6.2371}}$

5. $\overset{3.91}{.18\overline{).7038}}$ $\overset{67.3}{2.4\overline{)161.52}}$ $\overset{5.48}{.47\overline{)2.5756}}$ $\overset{27.5}{6.9\overline{)189.75}}$

6. $\overset{105}{.95\overline{)99.75}}$ $\overset{9.4\underline{2}}{3.8\overline{)35.796^{0}}}$ $\overset{40.8}{6.4\overline{)261.12}}$ $\overset{13.6}{7.9\overline{)107.44}}$

7. $\overset{4.37\underline{5}}{1.8\overline{)7.875^{0}}}$ $\overset{32.9}{.56\overline{)18.424}}$ $\overset{74.9}{3.5\overline{)262.15}}$ $\overset{3.15}{9.5\overline{)29.925}}$

㊱

Divide. In ex. 8–10, find quotients correct to the nearest tenth; in ex. 11–13, find quotients correct to the nearest hundredth:

8. $\overset{28.3}{.6\overline{)17}}$ $\overset{209.3}{4.6\overline{)963}}$ $\overset{2.1}{.65\overline{)1.38}}$ $\overset{1086.9}{.84\overline{)913}}$ $\overset{310.7}{2.81\overline{)873}}$

9. $\overset{43.3}{.9\overline{)39}}$ $\overset{715.5}{.58\overline{)415}}$ $\overset{2.5}{4.6\overline{)11.7}}$ $\overset{76.9}{9.3\overline{)715}}$ $\overset{2.3}{91.6\overline{)208}}$

10. $\overset{138.6}{.7\overline{)97}}$ $\overset{10.3}{7.9\overline{)81.6}}$ $\overset{27.0}{3.7\overline{)100}}$ $\overset{660.7}{.61\overline{)403}}$ $\overset{42.4}{8.74\overline{)371}}$

11. $\overset{96.67}{.3\overline{)29}}$ $\overset{1119.44}{.36\overline{)403}}$ $\overset{7.82}{.28\overline{)2.19}}$ $\overset{8.23}{4.8\overline{)39.5}}$ $\overset{37.10}{.372\overline{)13.8}}$

12. $\overset{57.78}{.9\overline{)52}}$ $\overset{1.09}{8.9\overline{)9.7}}$ $\overset{.65}{1.9\overline{)1.24}}$ $\overset{487.10}{.31\overline{)151}}$ $\overset{9.44}{28.7\overline{)271}}$

13. $\overset{2.14}{.7\overline{)1.5}}$ $\overset{81.22}{7.4\overline{)601}}$ $\overset{9.55}{5.1\overline{)48.7}}$ $\overset{8.46}{8.2\overline{)69.4}}$ $\overset{23.18}{36.5\overline{)846}}$

㊲

14. $\overset{.02}{7\overline{).14}}$ $\overset{.013}{43\overline{).559}}$ $\overset{.0022}{91\overline{).2002}}$ $\overset{.031}{8.5\overline{).2635}}$ $\overset{160}{.52\overline{)83.2}}$

15. $\overset{.04}{.8\overline{).032}}$ $\overset{74.2}{.08\overline{)5.936}}$ $\overset{.021}{3.7\overline{).0777}}$ $\overset{.0032}{96\overline{).3072}}$ $\overset{.005}{3.8\overline{).019}}$

16. $\overset{.075}{6\overline{).45}}$ $\overset{.012}{7.2\overline{).0864}}$ $\overset{.0096}{21\overline{).2016}}$ $\overset{.076}{.04\overline{).00304}}$ $\overset{.07}{5.6\overline{).392}}$

17. $\overset{.07}{.5\overline{).035}}$ $\overset{3780}{.04\overline{)151.2}}$ $\overset{.0027}{35\overline{).0945}}$ $\overset{4670}{.09\overline{)420.3}}$ $\overset{850}{.05\overline{)42.5}}$

18. $\overset{54}{.6\overline{)32.4}}$ $\overset{1735}{.08\overline{)138.8}}$ $\overset{670}{.28\overline{)187.6}}$ $\overset{2730}{.08\overline{)218.4}}$ $\overset{930}{.07\overline{)65.1}}$

More Practice

38

1. 15% of 80 12   4% of 450 18   2% of 800 16   16% of \$300 \$48

2. 22% of 50 11   9% of 200 18   5% of 260 13   19% of \$150 \$28.50

3. 60% of 35 21   6% of 250 15   7% of 400 28   32% of \$500 \$160

4. 24% of 75 18   3% of 400 12   9% of 300 27   80% of \$620 \$496

5. 18% of 75 13.5   8% of 500 40   4% of 850 34   11% of \$350 \$38.50

39

6. 50% of 62 31   25% of 52 13   75% of \$240 \$180   25% of \$9.60 \$2.40

7. 25% of 84 21   75% of 72 54   50% of \$155 77.50   75% of \$4.80 \$3.60

8. 75% of 68 51   50% of 86 43   25% of \$150 \$37.50   50% of \$3.40 \$1.70

9. 25% of 48 12   75% of 88 66   50% of \$216 \$108   25% of \$1.64 \$.41

10. 75% of 64 48   25% of 36 9   75% of \$280 \$210   50% of \$7.98 \$3.99

40

11. 90% of 80 72 $66\frac{2}{3}\%$ of \$477 \$318 $83\frac{1}{3}\%$ of \$25.80 \$21.50

12. 60% of 45 27 $16\frac{2}{3}\%$ of \$966 \$161 $12\frac{1}{2}\%$ of \$64.40 \$8.05

13. 80% of 95 76 $33\frac{1}{3}\%$ of \$690 \$230 $62\frac{1}{2}\%$ of \$75.28 \$47.05

14. 20% of 85 17 $37\frac{1}{2}\%$ of \$480 \$180 $66\frac{2}{3}\%$ of \$86.31 \$57.54

15. 40% of 45 18 $87\frac{1}{2}\%$ of \$528 \$462 $16\frac{2}{3}\%$ of \$43.32 \$7.22

16. 30% of 70 21 $12\frac{1}{2}\%$ of \$848 \$106 $33\frac{1}{3}\%$ of \$49.11 \$16.37

41

Find the discount (1) and the net price (2) if you buy articles having these marked prices and discounts:

(1), (2) **17.** \$12.50, 10% \$1.25, \$11.25   \$12.75, $33\frac{1}{3}\%$ \$4.25, \$8.50   \$75.00, \$425.00 \$500.00, 15% ✓

18. \$15.60, 25% \$3.90, \$11.70   \$14.40, $12\frac{1}{2}\%$ \$1.80, \$12.60   \$70.60, \$211.80 \$282.40, 25%

19. \$12.80, 25% \$3.20, \$9.60   \$24.80, $37\frac{1}{2}\%$ \$9.30, \$15.50   \$31.15, \$124.60 \$155.75, 20%

20. \$16.00, 50% \$8.00, \$8.00   \$25.35, $33\frac{1}{3}\%$ \$8.45, \$16.90   \$193.25, \$193.25 \$386.50, 50%

21. \$24.00, 30% \$7.20, \$16.80   \$46.00, $12\frac{1}{2}\%$ \$5.75, \$40.25   \$21.56, \$194.04 \$215.60, 10%

22. \$18.75, 20% \$3.75, \$15.00   \$19.65, $33\frac{1}{3}\%$ \$6.55, \$13.10   \$73.05, \$170.45 \$243.50, 30%

23. \$42.40, 25% \$10.60, \$31.80   \$12.80, $37\frac{1}{2}\%$ \$4.80, \$8.00   \$43.90, \$131.70 \$175.60, 25%

㊷

Find the answers correct to the nearest cent:

1. 3% of \$13.60 \$.41 9% of \$11.20 \$1.01 24% of \$36.45 \$8.75

2. 2% of \$18.15 \$.36 5% of \$27.22 \$1.36 50% of \$11.75 \$5.88

3. 1% of \$76.30 \$.76 8% of \$36.35 \$2.91 15% of \$17.32 \$2.60

4. 4% of \$14.35 \$.57 3% of \$30.40 \$.91 75% of \$32.70 \$24.53

5. 6% of \$12.25 \$.74 7% of \$68.50 \$4.80 10% of \$24.82 \$2.48

㊸

Estimate the answers:

6. 84% of \$120 \$100 63% of \$480 \$300 39% of \$250 \$100

7. 33% of \$198 \$66 37% of \$240 \$90 62% of \$880 \$550

8. 24% of \$520 \$130 11% of \$625 \$62.50 38% of \$168 \$63

9. 66% of \$390 \$260 38% of \$488 \$183 34% of \$438 \$146

10. 21% of \$170 \$34 12% of \$288 \$36 81% of \$255 \$204

㊹

Find the answers correct to the nearest cent:

11. 225% of \$966 \$2173.50 180% of \$375 \$675 110% of \$50.55 \$55.61

12. 210% of \$326 \$684.60 200% of \$148 \$296 112% of \$36.49 \$40.87

13. 425% of \$162 \$688.50 175% of \$665 \$1163.75 165% of \$80.75 \$133.24

14. 112% of \$175 \$196.00 120% of \$444 \$532.80 220% of \$36.65 \$80.63

15. 150% of \$655 \$982.50 125% of \$331 \$413.75 250% of \$11.75 \$29.38

㊺

Find what per cent the first number is of the second:

16. 28, 80 35% 14, 16 $87\frac{1}{2}\%$ 18, 50 36% 14, 56 25% 15, 125 12%

17. 33, 75 44% 25, 75 $33\frac{1}{3}\%$ 13, 20 65% 21, 60 35% 13, 104 $12\frac{1}{2}\%$

18. 16, 48 $33\frac{1}{3}\%$ 36, 80 45% 18, 72 25% 36, 48 25% 65, 250 26%

19. 27, 36 75% 18, 75 24% 19, 38 50% 15, 40 $37\frac{1}{2}\%$ 32, 160 20%

20. 57, 60 95% 27, 60 45% 26, 40 65% 80, 96 $83\frac{1}{3}\%$ 75, 625 12%

21. 36, 75 48% 18, 24 75% 28, 42 $66\frac{2}{3}\%$ 63, 84 75% 24, 120 20%

22. 20, 30 $66\frac{2}{3}\%$ 14, 50 28% 48, 80 60% 60, 72 $83\frac{1}{3}\%$ 66, 275 24%

More Practice

㊻

Change these decimals to the nearest whole per cent:

1. .192 19% .018 2% .254 25% .417 42% .293 29% .299 30% .245 25%

2. .086 9% .454 45% .007 1% .061 6% .516 52% .274 27% .398 40%

3. .325 33% .075 8% .149 15% .356 36% .289 29% .480 48% .997 100%

4. .072 7% .053 5% .167 17% .248 25% .391 39% .008 1% .350 35%

㊼

Find what per cent the first number is of the second:

5. 20, 16 125% 92, 92 100% 175, 100 175% 411, 137 300% 240, 144 $166\frac{2}{3}\%$

6. 50, 25 200% 46, 23 200% 168, 120 140% 352, 320 110% 420, 210 200%

7. 88, 40 220% 90, 36 250% 210, 175 120% 350, 175 200% 460, 115 400%

8. 36, 24 150% 80, 60 $133\frac{1}{3}\%$ 415, 166 250% 600, 120 500% 370, 185 200%

㊽

(inc.) (dec.)

Find the per cent of increase or decrease that the second number shows over the first number:

9. \$25, \$35 40% inc. \$250, \$350 40% inc. \$125, \$150 20% inc. \$5.00, \$4.75 5% dec.

10. \$12, \$16 $33\frac{1}{3}\%$ inc. \$185, \$370 100% inc. \$250, \$200 20% dec. \$8.00, \$6.88 14% dec.

11. \$20, \$17 15% dec. \$200, \$150 25% dec. \$180, \$150 $16\frac{2}{3}\%$ dec. \$4.50, \$6.00 $33\frac{1}{3}\%$

12. \$40, \$35 $12\frac{1}{2}\%$ dec. \$240, \$204 15% dec. \$120, \$240 100% inc. \$3.60, \$8.10 125% inc.

13. \$15, \$18 20% inc. \$144, \$162 $12\frac{1}{2}\%$ inc. \$150, \$100 $33\frac{1}{3}\%$ dec. \$7.00, \$5.25 25% dec.

㊾

The first number is the profit; the second is the selling price. The profit is what per cent of the selling price?

14. \$8, \$64 $12\frac{1}{2}\%$ \$12, \$96 $12\frac{1}{2}\%$ \$1.20, \$12.00 10% \$3.00, \$20.00 15%

15. \$9, \$90 10% \$10, \$80 $12\frac{1}{2}\%$ \$1.25, \$10.00 $12\frac{1}{2}\%$ \$1.50, \$25.00 6%

16. \$4, \$32 $12\frac{1}{2}\%$ \$11, \$55 20% \$3.50, \$17.50 20% \$3.36, \$42.00 8%

17. \$9, \$45 20% \$10, \$40 25% \$4.00, \$25.00 16% \$7.80, \$65.00 12%

18. \$4, \$16 25% \$12, \$60 20% \$6.00, \$50.00 12% \$6.30, \$45.00 14%

19. \$6, \$36 $16\frac{2}{3}\%$ \$13, \$65 20% \$4.50, \$25.00 18% \$9.00, \$50.00 18%

20. \$9, \$72 $12\frac{1}{2}\%$ \$11, \$88 $12\frac{1}{2}\%$ \$9.00, \$75.00 12% \$4.00, \$20.00 20%

21. \$7, \$70 10% \$16, \$96 $16\frac{2}{3}\%$ \$2.80, \$14.00 20% \$8.80, \$55.00 16%

㊿

Using the formula, find the interest correct to the nearest cent:

1. $600, 3%, 8 mo. $12 — $300, 2%, 11 mo. $5.50 — $475, 5%, 6 mo. $11.88
2. $360, 5%, 7 mo. $10.50 — $275, 6%, 10 mo. $13.75 — $495, 3%, 7 mo. $8.66
3. $450, 4%, 4 mo. $6.00 — $336, 3%, 11 mo. $9.24 — $660, 2%, 4 mo. $4.40
4. $800, 6%, 9 mo. $36 — $525, 4%, 10 mo. $17.50 — $485, 2%, 5 mo. $4.04

(51)

Find the interest on the following:

5. $500 at 4% for 90 da. $5.00 — $350 at 6% for 24 da. $1.40
6. $900 at 3% for 66 da. $4.95 — $275 at 4% for 54 da. $1.65
7. $840 at 5% for 45 da. $5.25 — $490 at 3% for 60 da. $2.45
8. $660 at 2% for 30 da. $1.10 — $680 at 5% for 90 da. $8.50
9. $780 at 6% for 54 da. $7.02 — $250 at 4% for 36 da. $1.00

(52)

10.	3 ft. 2 in. 4 ft. 9 in. +1 ft. 3 in. 9 ft. 2 in.	2 da. 7 hr. 1 da. 9 hr. +3 da. 9 hr. 7 da. 1 hr.	6 ft. 8 in. 2 ft. 9 in. +1 ft. 5 in. 10 ft. 10 in.	3 hr. 10 min. 1 hr. 40 min. +2 hr. 35 min. 7 hr. 25 min.
11.	6 ft. 1 in. 4 ft. 3 in. +2 ft. 6 in. 12 ft. 10 in.	2 lb. 9 oz. 1 lb. 8 oz. +3 lb. 4 oz. 7 lb. 5 oz.	4 qt. 1 pt. 6 qt. +2 qt. 1 pt. 13 qt.	6 yd. 15 in. 9 yd. 22 in. +5 yd. 16 in. 21 yd. 17 in.
12.	3 ft. 1 in. −2 ft. 7 in. 6 in.	9 ft. −4 ft. 6 in. 4 ft. 6 in.	6 bu. −2 bu. 2 pk. 3 bu. 2 pk.	15 yd. −11 yd. 14 in. 3 yd. 22 in.
13.	8 ft. 5 in. −6 ft. 9 in. 1 ft. 8 in.	9 bu. −5 bu. 1 pk. 3 bu. 3 pk.	8 qt. −4 qt. 1 pt. 3 qt. 1 pt.	10 gal. −2 gal. 3 qt. 7 gal. 1 qt.
14.	6 ft. 2 in. ×5 30 ft. 10 in.	6 lb. 2 oz. ×4 24 lb. 8 oz.	3 ft. 4 in. ×3 10 ft.	2 hr. 10 min. ×6 13 hr.
15.	2 ft. 4 in. ×5 11 ft. 8 in.	3 bu. 2 pk. ×6 21 bu.	5 lb. 8 oz. ×7 38 lb. 8 oz.	7 hr. 15 min. ×5 36 hr. 15 min.
16.	7 ft. 2 in. ×3 21 ft. 6 in.	4 da. 8 hr. ×7 30 da. 8 hr.	9 bu. 2 pk. ×6 57 bu.	3 hr. 12 min. ×5 16 hr.

Pages 242-243 provide ready-reference pages of tables of measure.

Tables for Reference

Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
43,560 square feet = 1 acre (A.)
160 square rods = 1 acre
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Measures of Weight

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

Measures of Time

60 seconds = 1 minute
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)

12 months = 1 year (yr.)
365 days = 1 year
366 days = 1 leap year
100 years = 1 century

Liquid Measure

2 cups = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Tables for Reference

Counting Articles

12 units = 1 dozen (doz.)
12 dozen = 1 gross
20 units = 1 score

Counting Paper

24 or 25 sheets = 1 quire
20 quires = 1 ream
480 or 500 sheets = 1 ream

Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
90 degrees = 1 right angle
180 degrees = 1 straight angle

Weight per Bushel (in many states)

Apples..............	48 lb.	Corn (on cob).......	70 lb.
Barley..............	48 lb.	Oats...............	32 lb.
Beans..............	60 lb.	Peas...............	60 lb.
Buckwheat..........	48 lb.	Potatoes...........	60 lb.
Clover seed.........	60 lb.	Rye................	56 lb.
Corn (shelled)........	56 lb.	Wheat..............	60 lb.

Useful Equivalents

1 bushel = about 2150 cu. in.
1 bushel = about $1\frac{1}{4}$ cu. ft.
1 cubic foot = about 0.8 bu.
1 gallon = 231 cu. in.
1 cubic foot = about $7\frac{1}{2}$ gal.
1 barrel = $31\frac{1}{2}$ gal.
1 barrel flour = 196 lb.
1 ton hard coal = about 35 cu. ft.
1 ton soft coal = about 42 cu. ft.
1 long ton = 2240 lb.
1 cord of wood = 128 cu. ft.
1 cu. ft. water = about $62\frac{1}{2}$ lb.
1 cu. ft. ice = about $57\frac{1}{2}$ lb.
1 sea mile = 1.15 land miles

Suggestions to Teachers

Improvement Tests. The timed tests described on pages 46 to 50 are called Improvement Tests. There are 72 of these tests in this book. The purpose of these tests is to keep alive and to perfect the computational skill with whole numbers that the pupils have learned previously, and to accomplish this task in a comparatively few minutes each week, so that ample time is left in each class period for the study of new topics. The procedure in giving these tests is described below and on pages 46 to 50. For a complete list of the pages upon which Improvement Tests appear, see the Index of this book.

How to Give Improvement Tests. The Improvement Tests are arranged in sets, each set containing three tests of the same kind and of equal difficulty. See the set of three addition tests on page 47. The procedure in giving a test, such as Addition Test 1a on page 47, is as follows:

(1) Start all the pupils at the same time by saying "Go." Stop all pupils promptly at the end of 4 min., which is the time assigned for this Addition Test. Use a watch with a second hand to keep the time. For addition and subtraction tests the answers should be written along the edge of a sheet of folded paper, as described on page 48.

(2) Read the answers. Then have each pupil count the number of exercises he has right and find his score on a scale of 10, using the Scoring Table on page 345. A partially completed exercise does not count.

(3) The pupil should keep a graphic record of his scores on these tests as described on page 50.

Never give more than one test of a set per day. For example, if Addition Test 1a, on page 47, is given today, Addition Tests 1b and 1c should be given on subsequent days. In general, give 1 set (3 tests) at most per week. There are 24 sets (72 tests) in all; to distribute these tests uniformly over the school year, it is suggested that one set (3 tests) be given, on the average, about every 6 school days. After a little practice a test can be taken, scored, and recorded within 10 minutes.

The time allowed for each test is a reasonable one. If the pupil has had sufficient practice in the fundamental operations, he should have no trouble in completing the test in the assigned time. It is important that each test of a set be given exactly the same amount of time; otherwise there will be no dependable means of knowing whether the pupil is improving or not.

The graphic record of scores, which each pupil keeps, enables him quickly to see whether his skill is improving. This record also makes it possible for the pupil to compare his skill in addition with that in subtraction or multiplication, thus showing him the operation upon which he needs the most practice.

The writing of answers on folded paper applies only to tests on addition and subtraction. For tests on multiplication and division, the exercises should be copied on a sheet of paper *before* the test is given. In copying the exercises the pupil should spread them over the sheet to allow ample space for the work.

Number Facts. In the early part of Grade 7 the pupils should be given the tests on pages 4, 12, 16, and 21 to see how well they know the fundamental number facts. A mastery of these facts is necessary for success in arithmetic.

The tests just mentioned are to be given orally to each individual pupil. As the pupil reads each fact on any given test, he gives *only the answer*. The teacher should note all the facts for which the pupil gives unsatisfactory responses. At the end of the test the teacher should give the pupil a list of these particular facts, with instructions to study them carefully. An unsatisfactory response is one that is incorrect or that is given with much hesitation, even though it is correct. The time limit for each test is 2 to 3 minutes, depending upon the ability of the pupil and the part of the school year in which the test is given.

More Practice. Pages 325–341 of this book give supplementary practice material that can be used in several ways as the teacher sees fit. References to these sets of exercises have been made on certain pages of the text where it is possible that more practice of a given type may be needed. The teacher can decide when to use this material. The supplementary exercises can also be used for the purpose of review on any given topic where review is desired.

SCORING TABLE FOR IMPROVEMENT TESTS

NUMBER OF EXAMPLES IN TEST	NUMBER OF EXAMPLES RIGHT															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	0	3	5	8	10											
5	0	2	4	6	8	10										
6	0	2	3	5	7	8	10									
7	0	1	3	4	6	7	9	10								
8	0	1	3	4	5	6	8	9	10							
9	0	1	2	3	4	6	7	8	9	10						
10	0	1	2	3	4	5	6	7	8	9	10					
12	0	1	2	3	3	4	5	6	7	8	8	9	10			
15	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10

For directions for using this table, see page 49.

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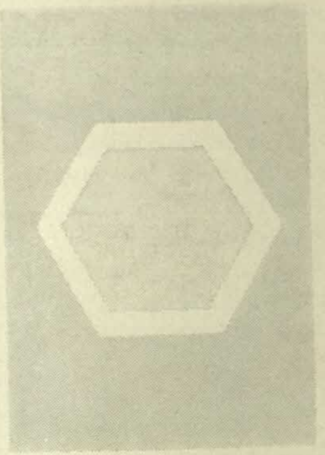

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American Arithmetic
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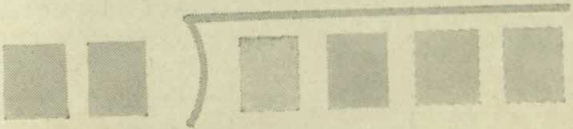

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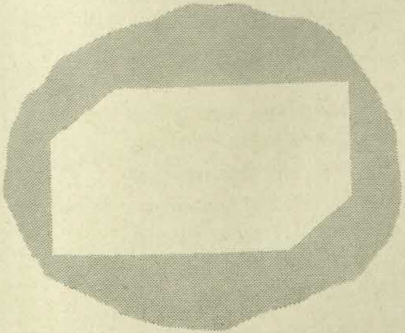

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EP 10 9 8 7 6 5 4 3 2 1

Use of This Guide. This Guide, together with the Upton-Fuller-Whitson *American Arithmetic, Second Edition, Grade 7, Discovering Mathematics*, offers to the teacher a course in modern methods of teaching mathematics in Grade 7. The Guide presents the principles of teaching, while *American Arithmetic, Grade 7* illustrates these principles through the actual materials of instruction. This combination provides a more comprehensive discussion of methods in mathematics than is found in the usual methods textbook.

The Guide gives suggestions and helps on the teaching of each page of *American Arithmetic, Grade 7* and often refers to specific exercises in the text. To use the Guide successfully, you should study carefully each text exercise or explanation to which reference is made in the Guide.

Since a thorough acquaintance with a textbook helps the teacher in using that textbook effectively, certain outstanding features of this series of books are given below.

Philosophy Underlying These Books. The objectives of *American Arithmetics* are simply stated:

(1) To teach pupils to understand the structure of our number system and the four fundamental processes with whole numbers, fractions, and decimals

(2) To teach pupils properly to interpret and to solve the problems of everyday life that involve quantitative data

(3) To develop in pupils sufficient skill in computation to enable them to solve the problems of everyday life with reasonable speed and accuracy

(4) To develop in pupils an appreciation of the nature of mathematics

The attainment of these objectives comes *through insight and intelligent understanding* rather than through blind following of rules and mechanical procedures. At appropriate grade levels, concepts and methods of algebra and geometry are introduced to assist in attaining these objectives.

Meaning and Understanding. Meaning and understanding in mathematics go much farther than using objects in the primary grades to make number meaningful, or explaining the process of carrying in addition. Meaning and understanding, properly treated, must pervade all the work in mathematics throughout the elementary and junior high schools. The constant aim of this series of texts is to teach mathematics so that the pupils understand and enjoy it. Throughout these books special attention is given to the following:

1. Basic Concepts and Terms of Mathematics. The meanings of fundamental concepts relating to whole numbers, fractions, and decimals, as well as the definitions of the technical terms of mathematics, are explained with unusual clarity.

2. Comprehension of the Fundamental Processes. This feature includes clear explanations of such topics as the addition of unlike fractions, the division of a fraction by a fraction, the division of a decimal by a decimal, and so on.

3. Meaningful Processes through Motivating Problems. In these textbooks each new topic or process is made meaningful through a motivating problem related to the interests of the pupils. This method shows why the new process is needed.

4. Fundamental Principles. Fundamental principles of mathematics are simply explained and applied throughout this series of books. These principles replace arbitrary rules.

5. Meaningful Problems. All problems and projects relate to the interests and experiences of the pupils and are written in language that they can understand.

6. Our Number System. The principles underlying our number system are emphasized in each book of the series.

Number Principles and Relationships. One of the outstanding features of this series of textbooks is the use of number principles and relationships in the teaching of the basic number facts. These principles and relationships greatly simplify learning for the pupil and help him to attain a degree of mastery and accomplishment that is not possible by other methods. In this work, if the pupil forgets the answer to a number fact, he can get it quickly *through his own thinking* by using a simple number principle or relationship. This method makes it unnecessary for him to get the answer by counting objects, or by looking it up in a table, or by asking the teacher for it. For a discussion of some of these number principles and relationships see *American Arithmetic, Grade 7*, pages 5, 6, 13, 16, 17, and 22. For further details, see the Guides for Grades 3 through 6.

In these texts, number relationships and fundamental principles are used effectively also in teaching the four processes with whole numbers, fractions, and decimals; in the work on percentage; in problem solving; and in the formulas for mensuration. This use of number principles is an outstanding feature of these books.

Long Division. Long division is by far the most difficult topic with whole numbers. Since long division is frequently used in the work of Grade 7, this topic is fully retaught in *American Arithmetic, Grade 7*.

In many textbooks long division is taught by a method which requires frequent correction of each quotient figure before the correct figure is found. An example of this inefficient way of doing long division is described below in Method A:

Method A. In the work at the right, each quotient figure is found by dividing each partial dividend by the *first figure of the divisor*. For example, to find the first quotient figure, you divide 9 by 2, which gives 4 as a trial quotient. Since 4 is too large, you then try 3, which is correct. To get the next quotient figure, you divide 13 by 2, getting 6, which is too large. You try 5, which is also too large. Then you try 4, and that is correct. For the next quotient figure you divide 21 by 2, which gives 10; but no quotient figure can be larger than 9, so you

$$\begin{array}{r} 347 \\ 28\overline{)9734} \\ \underline{84} \\ 133 \\ \underline{112} \\ 214 \\ \underline{196} \\ 18 \end{array}$$

try 9. You find that 9 is too large, so you try 8; 8 is also too large, so you try 7, which is correct. It should be noted that the correct quotient figure was obtained each time *only after two or three trials*.

By contrast, in *American Arithmetic, Grades 5 and 6*, and also in *American Arithmetic, Grade 7*, the pupils are taught to do the example as shown in Method B:

Method B. Instead of dividing each partial dividend by 2, divide in each case by 3, which is 1 *more than the first figure of the divisor*. In other words, you think of 28 as being close to 30; hence, you get each trial quotient figure by dividing by 3 instead of 2. To get the first quotient figure, divide 9 by 3; this gives 3, which is correct. To get the second quotient figure, divide 13 by 3; this gives 4, which is correct. To get the next quotient figure, divide 21 by 3; this gives 7, which is correct. By following this method, you see that the correct quotient figure is found each time *on the first trial*.

In the example shown, the divisor is 28. In the *American Arithmetics*, if the divisor were 26, 27, or 29 instead of 28, the example would be done also by Method B, because each of these divisors is closer to 30 than to 20. In fact, for any 2-figure divisor having a second figure of 6, 7, 8, or 9, each quotient figure is estimated by dividing by 1 more than the first figure of the divisor. See page 29 of *American Arithmetic, Grade 7*. On the other hand, if the second figure of a 2-figure divisor is 1, 2, 3, 4, or 5, each quotient figure is estimated by dividing by the first figure of the divisor. See page 27 of *American Arithmetic, Grade 7*. It is thus seen that with the method of long division taught in the *American Arithmetics*, each quotient figure is found by dividing by the *first figure* of the divisor when the two-figure divisor ends in 1, 2, 3, 4, or 5, and by dividing by 1 *more than the first figure* when the two-figure divisor ends in 6, 7, 8, or 9. This method of long division is very efficient since it gives the correct quotient figure *on first trial* in 80% of all cases; in the remaining 20% of cases, with minor exceptions, the correct quotient figure is found after a single correction. In other words, the necessity of correcting each quotient figure several times does not occur when long division is taught according to the method given in the *American Arithmetics*. This method of long division is the result of years of research on the part of Professor Upton. The research was reported in detail in the *Tenth Yearbook of the National Council of Teachers of Mathematics* in an article entitled "Making Long Division Automatic."

Projects. *American Arithmetic, Grade 7* provides many challenging projects in which mathematics is related to everyday activities and is also correlated in interesting ways with transportation, banking, postal information, and public services such as the telephone, the telegraph, electricity, gas, and so on.

Vocabulary and Language. Problems and explanations are written in clear language, and the vocabulary has been selected with unusual care. The technical words of mathematics are clearly defined and are reviewed in special exercises; see pages 13, 18, 23, 51, 69, 99, and so on.

Maintenance Program. Full provision has been made to maintain skills in computation and problem solving as follows:

1. Reteaching of Important Topics. *American Arithmetic, Grade 7* devotes the first three chapters to a review and reteaching of the four processes with whole numbers, fractions, and decimals.

2. Diagnostic Tests. Diagnostic tests with keyed references to remedial exercises are given at the end of each chapter. See pages 42, 80, 120, 150, 177, 178, 210, 238, 264, 286, 322, and 323.

3. Problem Tests. A Problem Test designed to test the pupil's ability in problem solving is given in each chapter. See pages 41, 79, 119, 149, 176, 209, 237, 263, 285, and 321.

4. Chapter Reviews. Reviews covering the new work of each chapter are provided. See pages 40, 78, 118, 148, 208, 236, and 284.

5. Oral and Written Reviews. Reviews covering important topics previously taught are frequently given. For examples, see pages 28, 30, 36, 38, 60, 68, 90, 98, 103, 117, 175, 183, and so on.

6. Improvement Tests. A series of 72 Improvement Tests, covering the four operations with whole numbers, is distributed over the year's work. These tests provide a most efficient and interesting means of maintaining skills with whole numbers while the pupil is studying new topics. See pages 47, 57, 69, 87, 99, 109, 127, 143, 155, 167, 185, 193, 203, and so on.

7. Mixed Practice. Computational practice on the four fundamental processes is amply provided for. See pages 87, 108, 175, 221, 280, 318, and 320.

8. Maintenance of Mathematics Vocabulary. Reviews which give an opportunity to check understanding of words essential to mathematics are provided. See pages 69, 99, 203, and 283.

The Work of Grade 7. The aim of the year's work is to deepen the pupil's understanding of our number system and to acquaint the pupil with the importance of mathematics and some of its larger uses in everyday life. The subject matter includes the number system, applications of mathematics, informal geometry, graphs, and formulas. You will find that *American Arithmetic, Grade 7* and *American Arithmetic, Grade 8* include many of the topics and methods of presentation that have been recommended by national groups for the improvement of the mathematics curriculum in these grades. The work of Grade 7 is outlined below:

1. The Number System. The study of the structure of the number system is continued and expanded in Grade 7. The commutative principle for addition and for multiplication and other previously taught principles are reviewed. The associative principle for addition and for multiplication and the distributive principle are introduced by name and given further development. All generalizations are developed inductively and then stated algebraically. The relationship of the inverse operations subtraction and division to addition and multiplication is expressed algebraically and emphasized. Place value is carefully reviewed and exponents are used to express the value of a number in terms of its digits. Number systems having bases other than ten are presented to increase the pupil's appreciation of the structure of our number system.

2. Applications of Mathematics. This book is outstanding in respect to its treatment of the applications of mathematics to everyday life. These applications, which may be called consumer mathematics, include travel by train and air; standard time zones; communication by telephone, telegraph, and letter; shipping by parcel post and express; budgets; installment buying; everyday banking; borrowing money; and the use of electricity and gas.

Percentage and its applications are presented with great clarity. Since percentage differs from decimals largely in the language in which its problems are expressed, many exercises are given for the purpose of acquainting the pupil with the technical expressions that are peculiar to this topic. The computational difficulties of percentage, which are closely related to those of decimals, are carefully graded. Complicated per cents are postponed until Grade 8.

3. Informal Geometry and the Formula. Form study, geometric constructions, designs, and scale drawing are presented through exercises involving observation and drawing. The terms *line*, *ray*, and *segment* are explained and used throughout the text. The formulas for finding the areas of a rectangle, a parallelogram, and a triangle, the circumference of a circle, and the volume of a rectangular prism are developed experimentally, thus giving the pupil confidence in them.

In the work with formulas, unknown factors of products are found by using the relationship of multiplication to division. The approximate nature of measurement is discussed and illustrated, as well as precision of measurement and the meaning of the greatest possible error in a measurement.

4. Graphs. The making and interpretation of bar, line, and circle graphs, such as those that appear in newspapers and magazines, are included. This work is carefully graded, starting with simple cases and gradually proceeding to those that are more difficult.

Suggestions to Teachers. Before starting the year's work, read "Suggestions to Teachers" on pages 344–345 of *American Arithmetic, Grade 7*. These suggestions contain valuable teaching hints and give various helps for teaching specific pages of the text.

Key. All answers are given on the appropriate pages in the Guide. These answers will be a convenience for those teachers not using the annotated edition, who may prefer to transfer them to the textbook pages for ready reference. As a further convenience, problems of several steps and other difficult examples are worked out throughout the book.

Chapter 1

Aims of Chapter 1. The major aims of Chapter 1 are to:

1. Review round numbers and teach a new way to write them.
2. Review the fundamental addition, subtraction, multiplication, and division facts.
3. Reteach the commutative principle for addition.
4. Teach the associative principle for addition.
5. Review the addition and subtraction of whole numbers.
6. Encourage different methods of rapid recall in the addition and subtraction of whole numbers.
7. Review making and counting change.
8. Reteach the commutative principle for multiplication.
9. Teach the associative principle for multiplication.
10. Teach the distributive principle.
11. Review the multiplication of whole numbers.
12. Review division with a 1-figure divisor.
13. Teach how to cast out nines.
14. Teach how to use casting out nines for checking multiplication and division.
15. Reteach long division.
16. Review and reteach averages.

Reteaching of Fundamental Operations in Grade 7. The experience of a large number of teachers has shown that in Grade 7 there are always some pupils who display weaknesses in the fundamental operations with whole numbers, fractions, and decimals. Thus, there is a need in Grade 7 for a review of these topics and for reteaching the more difficult aspects of some of these topics. *American Arithmetic, Grade 7* undertakes in the first three chapters to provide such instruction. In this work basic concepts, relationships, and generalizations are emphasized. The *commutative*, *associative*, and *distributive principles* are used to deepen the pupil's understanding of the structure of our number system. For the same purpose, number systems with bases other than ten are presented in Chapter 3, and the concept of place value is thoroughly reviewed. Applications to everyday life are made to show the usefulness of each topic.

Chapter 1 reviews and reteaches the four fundamental operations with whole numbers. The amount of time needed for this chapter will depend upon the ability and previous achievement of the pupils and will vary from class to class. For average pupils, this treatment represents a brief refresher course in whole numbers. More capable pupils can proceed rapidly, omitting any topics they have mastered. For pupils of lesser ability, this reteaching is supplemented by a liberal number of practice exercises, which will be found on pages 325–328 of the text. It is highly important that any computational weaknesses on the part of the pupils be remedied in Grades 7 and 8, since these years represent the last opportunity many pupils have to study arithmetic.

Page 1

Aim: To present an interesting project using very large numbers

Suggestions: In preparing for the problems on page 1, review the meaning of numbers of 4 places, 5 places, and so on. Put 3804 on the chalkboard and ask the pupils to read it in three different ways: 3 thousand, 8 hundred, 4; 38 hundred 4; and three, eight, o, four. Also give some practice in writing numbers as you read such numbers as "nine million, two hundred forty-three thousand, nine." Ask pupils to bring in clippings showing numbers in the millions and billions. Ask them also to give the names of two periods representing numbers larger than billions. A dictionary will supply this information.

Key: **1.** 65 billion; 10,000,000,000; 10,000,000; 10,000. **2.** 35 thousand, 582 thousand; 100,000; 7000; 80,000. **3.** $826,300,000. **4.** (1) About 2 times; (2) $4\frac{1}{3}$ times; (3) 65,000,000,000 − 15,000,000,000 = 50,000,000,000 (pieces of mail); 50,000,000,000 ÷ 50 = 1,000,000,000 (pieces of mail). **5.** $11\frac{1}{2}$ times.

Page 2

Aim: To reteach rounding off numbers

Suggestions: First give some class review and practice in rounding off numbers to the nearest thousand; for example, 2943; 8427; 9928; 1742; 6394. Next, provide practice in rounding off these numbers to the nearest hundred: 683; 847; 269; 594; 319. Then give practice in rounding off to the nearest ten, using these numbers: 36; 58; 22; 43; 89. The number 2943 might be rounded off first to the nearest thousand (3000), then to the nearest hundred (2900), and finally to the nearest ten (2940). Give more practice in rounding off 4-place numbers in these different ways, using these numbers: 4827; 5092; 8726; 4992; 1999. After this preparation, take up the work on page 2. In everyday life large numbers like those on page 2 are frequently rounded off if the exact number is unnecessary. Have pupils bring in some large round numbers from the newspaper.

Key: **2.** 18,000; 23,000; 211,000; 80,000; 6000. **3.** 24,000,000,000. **4.** 22,000, 16,000, 47,000, 11,000, 9000; 20,000, 20,000, 50,000, 10,000, 10,000. **5.** 20,000,000; 6,000,000; 8,000,000; 3,000,000. **6.** 19,800,000, 6,300,000, 7,500,000, 3,100,000; 19,830,000, 6,290,000, 7,500,000, 3,080,000. **7.** 2,616,000; 1,709,000; 3,206,000; 1,620,000.

Page 3

Aim: To teach a short way to write very large round numbers

Suggestions: This short way of writing large round numbers is explained in ex. 1. By this method 3,400,000,000 is written more briefly as 3.4 billion; likewise, 5,600,000 is written as 5.6 million. It is evident that the shorter forms, 3.4 billion and 5.6 million, are easier to read than the longer forms. To read 3,400,000,000, the reader has to count the number of periods in the number to make sure that it represents *billions* rather than *millions;* but he is saved this extra effort if the number is written as 3.4 billion. This way of writing numbers is usually confined to round numbers that can be expressed by writing a 1-place decimal before the word *million* or *billion*, as in 6.2 million and 4.7 billion. Occasionally one finds such numbers written by using a 2-place or a 3-place decimal, as in 3.25 million and 7.328 billion; but such usage is exceptional. This method of writing large numbers is frequently used in newspapers, magazines, and books. Ask the pupils to watch for these numbers and bring them to class.

Workbook Reference: Upton-Uhlinger: Arithmetic Workshop, Book 7, pages 1 and 2 (published by American Book Company)

Key: **2.** $12,300,000; 215,700,000 lb. **3.** 6,700,000; $114,300,000; $1,800,000,000; 26,400,000,000; $2,900,000,000. **4.** $3.9 million; 6.8 billion; $27.4 million; 123.6 billion; 8.7 million; 7.9 million; 5.3 billion; 2.1 billion. **6.** 13,148,273,160 (13 billion, 148 million, 273 thousand, 160); 5,826,914 (5 million, 826 thousand, 914); 16,182,198,000 (16 billion, 182 million, 198 thousand); 238,412,143 (238 million, 412 thousand, 143). **7.** 13.1 billion; 5.8 million; 16.2 billion; 238.4 million.

Page 4

Aim: To test each pupil's mastery of the 100 addition facts

Suggestions: Read the paragraph about number facts on page 345 of the text. If you find it impossible to test each pupil, enlist the help of several better pupils and have each of them test several other pupils. At this time of the year allow 3 minutes for the test, but later in the year allow only $2\frac{1}{2}$ or 2 minutes. Each pupil should have a list of the facts he has not mastered, with instructions to work for their mastery. In a week or two give those pupils who failed to make 100% on the test another chance to give the answers in 3 minutes.

Key: **1.** 12; 11; 4; 13; 4; 9; 8; 17; 12; 8. **2.** 3; 10; 5; 10; 8; 7; 5; 12; 2; 4. **3.** 8; 15; 1; 7; 6; 8; 9; 5; 13; 9. **4.** 4; 5; 14; 7; 14; 9; 10; 11; 3; 12. **5.** 13; 6; 7; 9; 13; 5; 8; 14; 9; 17. **6.** 2; 6; 18; 7; 12; 7; 15; 10; 16; 10. **7.** 8; 16; 11; 3; 12; 8; 5; 4; 10; 6. **8.** 13; 9; 15; 13; 9; 16; 11; 2; 3; 14. **9.** 15; 7; 8; 11; 1; 9; 10; 6; 7; 11. **10.** 14; 11; 9; 12; 6; 6; 0; 11; 10; 10.

Page 5

Aim: To reemphasize the commutative principle for addition

Suggestions: Encourage the pupils to suggest different ways to recall addition facts. At this time of the year many will need to discuss different ways of seeing combinations. Some of the exercises suggest uses of the associative principle as well as of the commutative principle.

Key: **1.** 12; 12; 15; 15; The order in which two addends are combined does not change the sum. **2.** $8 + 6 = 14$; $4 + 7 = 11$. **3.** $5 + 6 = 5 + (5 + 1) = (5 + 5)$

+ 1 = 10 + 1 = 11; 6 + 7 = 6 + (6 + 1) = (6 + 6) + 1 = 12 + 1 = 13; 8 + 9 = 8 + (8 + 1) = (8 + 8) + 1 = 16 + 1 = 17. **4.** 9 + 6 = 9 + (1 + 5) = (9 + 1) + 5 = 10 + 5 = 15; 7 + 9 = (6 + 1) + 9 = 6 + (1 + 9) = 6 + 10 = 16; 9 + 5 = 9 + (1 + 4) = (9 + 1) + 4 = 10 + 4 = 14. **5.** 7; 4; 5; 0; n; The sum of 0 and any other number is the other number. **7.** 22; 32; 42; 52; 62.

Page 6

Aim: To teach the associative principle for addition

Suggestions: Encourage the comparison of the ease in adding numbers by means of different groupings. Grouping for mental addition frequently differs from the grouping used in writing numbers. Explore different uses of both the associative and the commutative principles. Such exploration will give additional practice and is much more interesting than ordinary drill. Proper use of these two principles should develop speed and accuracy with greater ease. Work toward this end.

Key: **1.** (2 + 5) + 4 = 7 + 4 = 11, 2 + (5 + 4) = 2 + 9 = 11; (8 + 1) + 5 = 9 + 5 = 14, 8 + (1 + 5) = 8 + 6 = 14; (5 + 8) + 7 = 13 + 7 = 20, 5 + (8 + 7) = 5 + 15 = 20; (4 + 9) + 9 = 13 + 9 = 22, 4 + (9 + 9) = 4 + 18 = 22. **2.** When adding three numbers, you can group the first two numbers together and add their sum to the third, or you can group the last two numbers together and add the first to their sum. (8 + 9) + 7 = 17 + 7 = 24, 8 + (9 + 7) = 8 + 16 = 24; (14 + 5) + 3 = 19 + 3 = 22, 14 + (5 + 3) = 14 + 8 = 22; (73 + 84) + 7 = 157 + 7 = 164, 73 + (84 + 7) = 73 + 91 = 164; (29 + 71) + 18 = 100 + 18 = 118, 29 + (71 + 18) = 29 + 89 = 118. **4.** Associative; commutative; commutative; associative. **6.** 78; 91; 62; 54; 73; 96; 65; 70. **7.** 31; 23; 26; 60; 49; 81; 56; 29. **8.** (a) 80, 93, 64, 56, 75, 98, 67, 72; 33, 25, 28, 62, 51, 83, 58, 31; (b) 77, 90, 61, 53, 72, 95, 64, 69; 30, 22, 25, 59, 48, 80, 55, 28; (c) 79, 92, 63, 55, 74, 97, 66, 71; 32, 24, 27, 61, 50, 82, 57, 30.

Page 7

Aim: To reteach column addition, the terms used in addition, and the checking of column addition

Suggestions: Place on the chalkboard an example in column addition having 6 or 7 addends that are three-place numbers. Give the class about 1 minute to find the sum and check the work. Emphasize the importance of checking all computation. You may safely assume that not all the pupils found the correct sum; therefore, stress the need of mastering the 100 addition facts on page 4.

Ex. 3 on page 7 shows the pupils a way to break a long example into two parts. Have half the class try ex. 6 by adding the long column just as it is; have the other half try the same exercise by breaking it into two parts as was done in ex. 3. Start all pupils at a given time and see which group has the better record both in speed and in accuracy.

Additional practice is given on page 325 in the group of exercises marked with a white 1 in a red circle. You will notice that at the bottom of page 7 there is the first reference to the extra practice exercises on pages 325–341. Such references will be found on a number of pages in the text. The white number in the red circle refers you to the set of practice exercises which is related to the work of the page. These practice exercises should be assigned in accordance with each pupil's individual need. A pupil who is especially weak in an operation should be given extra practice distributed over several weeks so that he can be perfecting his skill in this operation while he is studying other topics in his arithmetic class. Distributed practice is often more effective than a short period of concentrated practice. This extra practice material can be used at any time to review a topic or to give a few such exercises for a test.

Key: **4.** \$3.82. **5.** 58. **6.** 614.

Page 8

Aim: To give practice in column addition

Suggestions: For most pupils in the class, half the exercises on page 8 will probably be enough practice. Divide the class into two groups and assign one part of ex. 1–11 to the first group and the other part to the second group. The extra practice mentioned at the bottom of the page can be used for remedial practice where needed, and for reviews and tests.

Workbook Reference: Arithmetic Workshop, Book 7, page 3

Key: **1.** 3124; 1794; 2921; 3180; 2146; 3325; 2369. **2.** 4392; 4119; 3197; 4092; 4535; 5106; 4246. **3.** 41,786; 36,829; 32,118; 31,440; 34,205; 31,365. **4.** 766. **5.** 701. **6.** \$166.48. **7.** \$187.34. **8.** \$208.35. **9.** \$190.97. **10.** \$185.85. **11.** \$107.79.

Page 9

Aim: To give some one-step problems in which column addition is required

Suggestions: Besides the problems on page 9, ask each pupil in the class to make up two good problems using column addition in which there are at least nine addends, each of which is under \$1.00. Pupils should work their problems before class and have the answers ready. In class call on different pupils to read a problem slowly while the rest of the class prepares to work it.

Key: **1.** \$2.78. **2.** \$2.91. **3.** 56¢.

Pages 10–11

Aim: To review the making and counting of change

Suggestions: Show the pupils that making change is done by the process of addition and not of subtraction. It is a process of making successive additions. You start with the amount of the purchase and add coins successively to this amount until you reach the amount that was given to the clerk. The coins that are used in this successive addition represent the correct change. Make up problems and give each pupil a chance to count change. For example, say to Jim, "You had $3.00 and spent $2.19 for a book. Count the change as a clerk might do." Also give some more practice on exercises like 11 and 12, where a buyer gives pennies with a larger coin to avoid receiving more pennies in change. Here are more such exercises: "Peggy gave 1 quarter and 1 penny for a purchase costing 16¢. Find her change." "Jack gave 3 quarters and 3 pennies for a purchase costing 68¢. What change did he get?" "Jean gave 1 half dollar and 2 pennies for a purchase costing 37¢. Find her change."

Workbook Reference: Arithmetic Workshop, Book 7, page 5

Key: **2.** $1.37; $1.40; $1.50; $2.00; $3.00; $4.00; $5.00. **3.** No; 2 pennies, 2 dimes, 2 quarters. **4.** 1 penny, 2 dimes; 79, 80, 90, $1.00. **5.** 4 pennies, 1 nickel, 1 dime, 1 half dollar. **6.** Yes; (8 coins) 4 pennies, 1 nickel, 1 dime, 2 quarters; (9 coins) 4 pennies, 3 nickels, 2 quarters; (10 coins) 4 pennies, 2 nickels, 3 dimes, 1 quarter; (13 coins) 4 pennies, 5 nickels, 4 dimes; (17 coins) 4 pennies, 13 nickels; (69 coins) 69 pennies. **7.** $2.16, $2.20, $2.25, $2.50, $3.00, $4.00, $5.00; $3.75, $4.00, $5.00; $2.55, $2.65, $2.75, $3.00, $4.00, $5.00; $1.24, $1.25, $1.50, $2.00, $3.00, $4.00, $5.00; $4.85, $4.90, $5.00; $1.08, $1.10, $1.15, $1.25, $1.50, $2.00, $3.00, $4.00, $5.00. **8.** $3.05, $3.15, $3.25, $3.50, $4.00, $5.00; $.29, $.30, $.40, $.50, $1.00, $2.00, $3.00, $4.00, $5.00; $3.39, $3.40, $3.50, $4.00, $5.00; $2.12, $2.15, $2.25, $2.50, $3.00, $4.00, $5.00; $.10, $.15, $.25, $.50, $1.00, $2.00, $3.00, $4.00, $5.00; $4.25, $4.50, $5.00. **9.** $1.49, $1.50, $2.00, $3.00, $4.00, $5.00; $4.15, $4.25, $4.50, $5.00; $.46, $.50, $1.00, $2.00, $3.00, $4.00, $5.00; $3.37, $3.40, $3.50, $4.00, $5.00; $2.61, $2.65, $2.75, $3.00, $4.00, $5.00; $1.83, $1.85, $1.90, $2.00, $3.00, $4.00, $5.00. **10.** 40 nickels, 30 dimes, 20 quarters. **11.** Yes; she didn't want pennies in change; 1 nickel. **12.** 1 nickel and 1 quarter, or 3 dimes.

Page 12

Aim: To test each pupil's mastery of the 100 subtraction facts

Suggestion: See the suggestions for page 4 which refer to the addition facts. Those suggestions apply also to the subtraction facts.

Key: **1.** 0; 1; 9; 4; 8; 7; 6; 2; 5; 5. **2.** 7; 4; 2; 3; 7; 9; 0; 8; 3; 8. **3.** 5; 7; 4; 7; 3; 6; 4; 9; 2; 4. **4.** 2; 8; 5; 9; 1; 6; 3; 7; 3; 0. **5.** 3; 2; 5; 0; 6; 9; 2; 1; 1; 1. **6.** 8; 6; 9; 1; 6; 2; 8; 6; 9; 0. **7.** 2; 0; 4; 8; 0; 1; 7; 5; 3; 0. **8.** 9; 7; 1; 7; 9; 5; 4; 1; 1; 3. **9.** 3; 2; 9; 5; 6; 0; 8; 2; 6; 3. **10.** 8; 6; 5; 0; 4; 8; 7; 4; 4; 5.

Page 13

Aims: To show the relation of subtraction facts to addition facts, to review the terms used in subtraction, and to reteach checking subtraction

Suggestions: As an oral exercise ask different pupils to tell how they should think if they forget 7 from 13, 9 from 16, and so on. Point out that the method of thinking in ex. 1 will let the pupil help himself to get the answer to a subtraction fact if he has forgotten it. Emphasize the generalizations in ex. 3 and 4.

At this grade level not all the pupils in a given class may be using the same method of subtraction. A pupil should continue to use the method he was originally taught if he finds it satisfactory.

In *American Arithmetics* the *decomposition* or *take-away borrow* method of subtraction is taught. However, because of the interest of teachers in the *additive* method of subtraction, this method is explained in "Suggestions to Teachers" at the end of the texts for Grades 3, 4, 5, and 6. The basic concept of additive subtraction is to find the number which should be added to the subtrahend to give the minuend. Changing or borrowing is not required in this method, and for that reason the additive method is often easier for pupils to learn than the take-away method. The model example on page 13 is explained here by the additive method of subtraction.

8000 **Minuend**
3946 **Subtrahend**
4054 **Remainder**

Think, "6 + 4 = 10." Write 4 in ones place and add 1 ten to 4 tens, making 5 tens.

Think, "5 + 5 = 10." Write 5 in tens place and add 1 hundred to 9 hundreds, making 10 hundreds.

Think, "10 + 0 = 10." Write 0 in hundreds place and add 1 thousand to 3 thousands, making 4 thousands.

Think, "4 + 4 = 8." Write 4 in thousands place. The remainder is 4054. Check by adding 4054 to 3946 to see if the sum is 8000.

Key: **1.** 7 and 4 make 11; 6 and 8 make 14; 8 and 9 make 17. **2.** 8, 8, 9, 8; 8, 8, 9, 8. **3.** 7; 5; 9. 0 subtracted from any number gives the number. **4.** 0; 0. Any number subtracted from itself gives 0. **7.** 19; 40; 92; 14. **8.** 27; 56; 83; 152. **9.** 36; 57; 56; 463.

Page 14

Aim: To give practice on subtracting larger numbers and on word problems requiring subtraction

Suggestions: In most cases two rows of abstract exercises together with the word problems (ex. 5–9) are enough to assign. Let half the class do ex. 1 and 2 with ex. 5–9, and the other half do ex. 3 and 4 with ex. 5–9. There is more practice on subtraction on page 326, which you may use in any way you see fit.

Workbook Reference: Arithmetic Workshop, Book 7, page 4

Key: **1.** 1801; 2593; $686.68; $157.91; $7.82. **2.** 112; 363; $444.86; $589.37; $4430.07. **3.** 8627; 3956; $307.77; $246.34; $4121.39. **4.** 697; 7; $845.74; $384.31; $78.60. **5.** $10.35. **6.** $19.57. **7.** $58.01. **8.** $6.93. **9.** 27,500 trees.

Page 15

Aim: To give problems using the various forms of the language of subtraction

Suggestion: Since Grade 3, pupils have been given problems stated in the various forms of language that is peculiar to subtraction. The problems on page 15 represent each of the more important types of problems in which the answer is obtained by subtraction.

Key: **1.** 1725 workers. **2.** \$53.64. **3.** 7445. **4.** \$3625. **5.** 4298 sq. mi. **6.** 1905. **7.** \$134.26. **8.** \$.54.

Pages 16–17

Aims: To review the multiplication facts; to reteach the commutative principle for multiplication (CPM); to teach the associative principle for multiplication (APM); to teach the distributive principle (DP)

Suggestions: Give oral practice in finding answers to multiplication facts. Use the commutative and/or the associative principle to fix facts that cause difficulty. Make a 10 by 10 multiplication chart. To the left of the 10 rows place the digits 0 through 9. Above the columns place the digits 0 through 9. Where a column and a row intersect, write the product of the number of the column and the number of the row. Notice the relationships of the multiplication facts as shown on the chart. Illustrate the uses of the principles by the products on the chart.

Stress these principles and search for examples where the use of these principles enables one to do mental multiplication easily. Multiplication can be distributed over the sum of more than two addends, e.g., 8×115 can be thought of as $8 \times (100 + 10 + 5)$, which equals $(8 \times 100) + (8 \times 10) + (8 \times 5)$, giving $800 + 80 + 40$, or 920. Encourage the pupils to use different combinations for the example above, such as: $8 \times 115 = 8 \times (110 + 5) = (8 \times 110) + (8 \times 5) = 880 + 40 = 920$.

Key: **1.** (1) 27, 30, 0, 40, 3, 20, 0, 72, 32, 15; (2) 2, 21, 0, 16, 16, 6, 6, 27, 0, 3; (3) 12, 56, 0, 10, 9, 7, 20, 0, 42, 8; (4) 4, 4, 48, 0, 45, 8, 24, 30, 0, 32; (5) 42, 0, 12, 14, 36, 4, 7, 48, 0, 72; (6) 0, 5, 81, 12, 35, 0, 54, 25, 63, 21; (7) 0, 63, 18, 2, 35, 15, 6, 0, 9, 8; (8) 40, 0, 56, 36, 14, 64, 18, 1, 0, 49; (9) 54, 6, 12, 24, 0, 18, 6, 0, 10, 28; (10) 45, 28, 18, 36, 5, 8, 0, 24, 24, 9. **2.** 42; 42; yes; yes. The order of multiplying two factors does not change their product. **3.** $6 \times 9 = 54$; $8 \times 5 = 40$; $9 \times 7 = 63$; $4 \times 7 = 28$. **4.** 0; 0; 0; 0. Any number times 0 equals 0; 0 times any number equals 0. **5.** 3; 8. Any number times 1 equals that number; 1 times any number equals that number. **6.** $(2 \times 3) \times 5 = 6 \times 5 = 30$, $2 \times (3 \times 5) = 2 \times 15 = 30$; $(1 \times 3) \times 2 = 3 \times 2 = 6$, $1 \times (3 \times 2) = 1 \times 6 = 6$; $(3 \times 6) \times 4 = 18 \times 4 = 72$, $3 \times (6 \times 4) = 3 \times 24 = 72$; $(5 \times 4) \times 2 = 20 \times 2 = 40$, $5 \times (4 \times 2) = 5 \times 8 = 40$. **7.** When multiplying three numbers, you can group the first two factors together and multiply their product and the third factor, or you can group the last two factors together and multiply the first factor and this product; $(3 \times 6) \times 7 = 18 \times 7 = 126$, $3 \times (6 \times 7) = 3 \times 42 = 126$; $(5 \times 1) \times 8 = 5 \times 8 = 40$, $5 \times (1 \times 8) = 5 \times 8 = 40$; $(2 \times 3) \times 12 = 6 \times 12 = 72$, $2 \times (3 \times 12) = 2 \times 36 = 72$; $(15 \times 2) \times 7 = 30 \times 7 = 210$, $15 \times (2 \times 7) = 15 \times 14 = 210$. **8.** APM;

CPM; APM; multiplication performed. **9.** $2 \times (3+4) = 2 \times 7 = 14, 2 \times (3+4) = (2 \times 3) + (2 \times 4) = 6 + 8 = 14; 4 \times (2+7) = 4 \times 9 = 36, 4 \times (2+7) = (4 \times 2) + (4 \times 7) = 8 + 28 = 36; 8 \times (6+3) = 8 \times 9 = 72, 8 \times (6+3) = (8 \times 6) + (8 \times 3) = 48 + 24 = 72$. **10.** $7 \times (4+8) = 7 \times 12 = 84, 7 \times (4+8) = (7 \times 4) + (7 \times 8) = 28 + 56 = 84; 9 \times (2+5) = 9 \times 7 = 63, 9 \times (2+5) = (9 \times 2) + (9 \times 5) = 18 + 45 = 63; 6 \times (8+9) = 6 \times 17 = 102, 6 \times (8+9) = (6 \times 8) + (6 \times 9) = 48 + 54 = 102; 14 \times (2+5) = 14 + 7 = 98, 14 \times (2+5) = (14 \times 2) + (14 \times 5) = 28 + 70 = 98$. **12.** $5 \times 17 = 5 \times (10+7) = (5 \times 10) + (5 \times 7) = 50 + 35 = 85$.

Page 18

Aim: To review multiplication of large numbers, the checking of such exercises, and the names of the terms used in multiplication

Suggestions: On the chalkboard place four multiplication examples with 3-place multiplicands and multipliers. Have one case of 0 at the end of the multiplier and one case of 0 in the middle of the multiplier. Ask four pupils to work these examples. Then discuss the examples with the class. Make sure that all pupils know the parts of each example by asking them to read the multiplier, the multiplicand, the product, and the partial products. When multiplying two numbers, the pupils should write each partial product in such a position that its right-hand figure will come directly below the figure by which they are multiplying. For example, in ex. 1 on this page, the partial product 1488 is so written that the right-hand 8 comes directly below the 4 of the multiplier. Call attention to the examples in ex. 5, which show the approved practice for computing when the multiplier contains a 0. In the third of these examples, notice that the multiplier, 740, is so written under 218 that the 0 stands out at the right. In the second example in ex. 5, the second partial product is 0 × 596. How much is this and where is it written? You see that if you are careful to write the third partial product in the correct place, it is not necessary to write 0 as the partial product for tens place of the multiplier. Emphasize the use of the term *factor* in those situations where it is appropriate.

Key: **3.** 670; 280; 920; 410; 1170; 2340; 56,790; 285,040. **4.** 4700; 8300; 27,400; 347,800; 1,489,700. **6.** 13,260; 271,128; 358,340; 433,359. **7.** 15,675; 305,492; 815,409; 286,160. **8.** 59,220; 328,790; 106,953; 528,781. **9.** 56,289; 148,716; 545,360; 211,554.

Page 19

Aim: To review problems without numbers

Suggestions: If pupils have difficulty in solving these problems, suggest that they supply numbers for each problem, solve it, and then tell in general terms what they did. Since some problems have more than one step, there should be an answer for each step. Problems of this nature provide you with an excellent opportunity to learn how well your pupils are doing mathematical thinking.

Workbook Reference: Arithmetic Workshop, Book 7, page 51

Key: **1.** Add the cost of the articles; then subtract the sum from the amount you had at first. **2.** Multiply. **3.** Multiply. **4.** Divide. **5.** Multiply. **6.** Divide. **7.** Add. **8.** Add all the numbers; then divide by 6. **9.** Subtract.

Page 20

Aim: To review problem solving especially where multiplication is used

Suggestions: Give the class two problems to work, as follows:

(1) Jim bought 4 neckties. They cost \$.75, \$.95, \$.69, and \$1.19. How much did Jim spend for these 4 ties?

(2) Mary bought 4 handkerchiefs. They cost \$.39 apiece. How much did Mary spend for the 4 handkerchiefs?

Discuss with the class the reason for using addition in problem 1 and multiplication in problem 2, and have someone try to make a generalization about when multiplication is to be used.

Key: **1.** 580 × \$.35 = \$203.00. **2.** 150 × 13 mi. × 1950 mi. **3.** 18 × 24 = 432 (peach trees). **4.** $3\frac{1}{2}$ × 220 mi. = 770 mi. **5.** 3 × \$.55 = \$1.65, $\frac{1}{2}$ × \$.68 = \$.34; \$1.65 + \$.34 = \$1.99. **6.** 1465 × \$.08 = \$117.20. **7.** $\frac{1}{4}$ × \$2460 = \$615; \$2460 − \$615 = \$1845. **8.** $\frac{1}{5}$ × 64,500,000 bu. = 12,900,000 bu. **9.** 250 × 60 lb. = 15,000 lb.; 15,000 ÷ 100 = 150 (bags).

Page 21

Aim: To test each pupil's mastery of the 90 even division facts

Suggestion: See the suggestions for the test on page 4.

Key: **1.** 4; 0; 2; 6; 2; 8. **2.** 2; 1; 0; 7; 5; 7. **3.** 0; 9; 8; 4; 3; 5. **4.** 2; 1; 9; 6; 6; 6. **5.** 5; 0; 5; 8; 5; 3. **6.** 3; 8; 4; 7; 3; 5. **7.** 1; 0; 9; 6; 5; 4. **8.** 0; 1; 2; 6; 9; 4. **9.** 7; 0; 7; 9; 7; 2. **10.** 0; 1; 6; 9; 4; 4. **11.** 3; 3; 7; 4; 8; 8. **12.** 1; 1; 8; 3; 3; 8. **13.** 2; 0; 6; 8; 2; 7. **14.** 1; 6; 5; 7; 3; 9. **15.** 4; 1; 9; 2; 5; 9.

Page 22

Aims: To show how multiplication facts and division facts are related and to give practice on some uneven division facts

Suggestions: Give some oral class practice by asking a pupil how he should think if he forgets $7\overline{)56}$. The pupil should answer, "How many 7's make 56? I know that eight 7's make 56, so 56 ÷ 7 = 8." Use a number of the other even division facts for such practice. Ex. 3 is very important. Often when using a formula, the pupil will need to find an *unknown factor* of a product.

Practice on uneven division facts is important as a help in division with carrying. Go over the exercises on page 22 orally, asking different pupils for the answers. If more practice is needed, here are other uneven facts which can be given orally so that the pupils will not see the example in written form:

$6\overline{)9}$	$3\overline{)17}$	$9\overline{)71}$	$7\overline{)55}$	$5\overline{)38}$	$8\overline{)65}$	$6\overline{)57}$
$3\overline{)7}$	$9\overline{)21}$	$7\overline{)31}$	$5\overline{)13}$	$8\overline{)71}$	$6\overline{)22}$	$9\overline{)89}$
$5\overline{)4}$	$8\overline{)23}$	$4\overline{)39}$	$7\overline{)44}$	$9\overline{)59}$	$5\overline{)19}$	$7\overline{)60}$
$9\overline{)5}$	$6\overline{)29}$	$5\overline{)46}$	$9\overline{)40}$	$4\overline{)19}$	$6\overline{)41}$	$8\overline{)61}$
$7\overline{)8}$	$9\overline{)38}$	$6\overline{)34}$	$8\overline{)30}$	$5\overline{)29}$	$7\overline{)33}$	$9\overline{)78}$
$4\overline{)7}$	$7\overline{)68}$	$9\overline{)16}$	$4\overline{)25}$	$8\overline{)50}$	$9\overline{)51}$	$6\overline{)50}$

Key: TOP: **1.** Six 8's make 48; nine 6's make 54; eight 7's make 56. **2.** 7, 7, 9, 4; 7, 7, 9, 4. **3.** 9; 9; 3; 4. **4.** Yes; 0; 0. **5.** $5 \times 1 = 5$; 3; 7. Any number divided by 1 equals that number. **6.** 1; 1; 1. BOTTOM: **1.** 1 and 2 over; 5 and 1 over; 3 and 1 over; 9 and 1 over; 2 and 2 over; 7 and 2 over; 9 and 4 over. **2.** 2 and 1 over; 8 and 3 over; 8 and 1 over; 4 and 2 over; 6 and 1 over; 3 and 1 over; 5 and 1 over. **3.** 0 and 3 over; 2 and 1 over; 4 and 4 over; 6 and 1 over; 6 and 2 over; 9 and 2 over; 5 and 2 over.

Page 23

Aim: To review division with 1-figure divisors, the checking of these examples, and the terms used in division

Suggestions: Give some class practice in picking out the dividend, the divisor, the quotient, and the remainder in examples you have placed on the chalkboard. Then erase the quotients and the remainders and see whether the pupils can give the quotients by doing the work mentally. Many pupils in Grade 7 should be able to do mental short division, especially with the divisors 2, 3, 4, and 5. However, you need not require that the work be done mentally. For ex. 3–9 tell the pupils they can do the work in either the long form or the short form.

Key: **3.** 220 mi. **4.** \$3.15. **5.** 1529 R5; 738; 4317; 7039 R2; 6105. **6.** 2086; 792 R5; 1024; 5927; 9486. **7.** 627; 842 R3; 1205; 9317; 4009. **8.** 834; 2958 R1; 1097 R5; 6003 R5; 8734. **9.** 3007 R2; 1567; 1168; 6930 R3; 4591.

Page 24

Aim: To discover how to find the residue when casting out nines

Suggestions: Lead the pupils to explore rapid ways of finding the residue when casting out nines. Be sure to bring out that the relationship presented on this page holds only for nines. The pupils may discover how it aids in determining divisibility by three or in finding the remainder when dividing by 3. It will take continued practice in casting out nines in all operations with whole numbers to gain real facility in casting out nines by inspection.

Key: **1.** 2; 0; 4; 1; 8. **2.** 4; 4; 2; 0; 1; 4; 2; no. **3.** 1; 1; 1; 2; 2; 3; 3; 4; 5; 6. The remainder is equal to the sum of the digits. **4.** 3; 3; 8; 8; 0; 0; 7; 7; 7; 7. **5.** 5; 7; 8; 6; 8; 8; 6; 7; 7. **6.** 2; 4; 8; 4; 6; 5; 2. **7.** 2; 3; 2; 5; 3; 3; 1.

Pages 25–26

Aim: To teach casting out 9's as a new method of checking examples in multiplication

Suggestions: There are three methods of checking exercises in multiplication. The first and probably the most used method is to go over the work a second time to see that it is correct. The second method is to interchange the multiplicand and the multiplier and multiply again, as explained in ex. 1 on page 18. This check is an excellent one although it requires more time than the first method. The third method is casting out 9's, which is also called "the check of 9's." It is fully described on pages 25 and 26 of the text. This is the quickest method of checking multiplication in cases where the multiplier contains two or more figures. In exercises having a 1-place multiplier, the most satisfactory check is the first one described above; namely, to go over the work again. The check of 9's is an extremely useful and rapid check and is recommended for use, particularly in the longer exercises in multiplication. On rare occasions the check of 9's fails to detect an error in a multiplication example, as explained in ex. 4 on page 26. In this case a partial product is misplaced, and the final product is, therefore, incorrect. The check of 9's, however, did not detect this mistake. In fact, whenever a partial product is misplaced, the check of 9's will not disclose the error. This weakness on the part of the 9's check does not disqualify it from use, but it does show the importance of seeing that each partial product is in the right place before applying the check of 9's.

You must remember that no method of checking is perfect. If you check multiplication by going over the work again, or by reversing the factors, you have no guarantee that the result is correct. You merely have a reason for believing that the answer is *probably* correct. In everyday life, one of the favorite methods of checking computations of all kinds is *to go over the work again;* but in such cases, if a mistake was made in the original computation, it may easily be repeated in the checking.

Casting out 9's is the most rapid check, not only for multiplication, but also for division. On this account it is strongly recommended in the seventh and eighth grades as a check for these operations. Casting out 9's is not recommended as a regular check for addition and subtraction, since it requires more time to check either of these operations by 9's than by the usual methods of checking addition and subtraction.

Workbook Reference: Arithmetic Workshop, Book 7, page 6

Key: Page 25 **8.** 583,895; 597,432; 298,102; 3,380,740. **9.** 145,222; 231,495; 242,403; 1,776,138. **10.** 455,794; 456,456; 88,963; 5,774,442.

Page 26 **2.**

338 5
247 4
2366 20 (2)
1352
676
83486 (2)

4.

4253 5
1078 7
34024 35 (8)
29771
4253
4584734 (8)

5. 47,970; 280,141; 3,270,348; 8,570,439. **6.** 63,648; 151,682; 5,576,627; 28,738,362. **7.** 28,952; 260,700; 800,935; 35,892,306. **8.** 9234; 479,576; 534,285; 14,102,000. **9.** 22,356; 338,724; 856,128; 43,947,516.

Page 27

Aim: To reteach division with 2-figure divisors ending in 1, 2, 3, 4, or 5

The Teaching of Long Division. Long division is the most difficult operation with whole numbers, not only for children but also for adults. The most difficult part of long division is that connected with the finding of the correct quotient figures. In this book, long division is taught by a method that gives the correct quotient figure *on first trial* in 80% of all cases. In the remaining 20% of cases, the correct quotient figure is found, with very minor exceptions, after a single correction of the trial quotient figure. The procedure so often used in schoolrooms, by which the pupil tries first one quotient figure and then another until he finds the correct one, is thus avoided.

In order to obtain such high efficiency in finding quotient figures, this book uses two rules for estimating quotient figures. One of these rules applies to 2-figure divisors ending in 1 to 5 and is stated on page 27 of the text; the other rule applies to 2-figure divisors ending in 6 to 9 and is stated on page 29. In this Guide these rules, for convenience, will be called Rule I (divisors ending in 1 to 5) and Rule II (divisors ending in 6 to 9).

The treatment of long division as given in this series of arithmetics is one of its outstanding features. For a more detailed discussion of this highly efficient method of long division, see the Introduction of this Guide.

Suggestions: The work in long division as presented on pages 27–33 of this text is a reteaching of the work in long division that was given in the sixth grade. When the work on page 27 is first presented to the class, put the model example in ex. 1 on the chalkboard and go through each step of finding each quotient figure. In this work remind the pupils that there are five steps in finding each quotient figure, which are as follows:

(1) *Divide* the partial dividend by the divisor.
(2) *Multiply* the divisor by the quotient figure.
(3) *Subtract.*
(4) *Compare* the remainder with the divisor. The remainder should always be less than the divisor.
(5) *Bring down* the next figure of the dividend.

Of these five steps, emphasize the importance of Step 4, in which the remainder is compared with the divisor. If the quotient figure is correct, the remainder *will always be less* than the divisor. On the other hand, if the remainder equals the divisor, or is greater than the divisor, then the quotient figure is too small and should be made 1 larger. Success in long division depends upon the careful inspection of each remainder to see that it is smaller than the divisor. If not, the quotient figure requires correction.

After the model example given at the top of page 27 has been carefully explained, the method of checking the work should also be explained. Every example in long division should be checked as a routine procedure.

The exercises on page 27 all have divisors ending in 1, 2, 3, 4, or 5. Hence Rule I on that page should be used to estimate quotient figures.

Key: **2.** 55 R5; 59 R5; 367; 213. **3.** 326; 84; 785 R11; 375 R13. **4.** 89; 67 R11; 296 R16; 605 R22. **5.** 89; 308 R5; 840 R7; 998. **6.** 92; 54; 638; 406 R13.

Page 28

Aim: To give a quick oral review, especially on measures taught in the preceding grades

Suggestions: First give some practice in counting by 6's, 7's, 8's, and 9's by beginning with the number itself. Then give the oral counting in ex. 1–3 in which the beginning and ending numbers are supplied. More of this kind of counting can be assigned by saying, "Beginning with 6, count by 9's until I tell you to stop." You can easily make up exercises of this sort as you need them. In connection with the work on measures, ask such questions as, "How many feet are there in 1 mile?" "How many quarts are there in 1 peck?"

Key: **1.** 4, 11, 18, 25, 32, 39, 46, 53, 60; 109, 116, 123, 130, 137, 144, 151, 158, 165, 172. **2.** 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, 99; 110, 118, 126, 134, 142, 150, 158, 166, 174, 182. **3.** 7, 16, 25, 34, 43, 52, 61, 70, 79, 88; 152, 161, 170, 179, 188, 197, 206, 215, 224. **4.** (Adding 6) 49, 33, 62, 94, 101, 80, 45, 68; (adding 7) 50, 34, 63, 95, 102, 81, 46, 69; (adding 8) 51, 35, 64, 96, 103, 82, 47, 70; (adding 9) 52, 36, 65, 97, 104, 83, 48, 71. **5.** (Adding 6) 100, 74, 41, 85, 28, 52, 63, 89; (adding 7) 101, 75, 42, 86, 29, 53, 64, 90; (adding 8) 102, 76, 43, 87, 30, 54, 65, 91; (adding 9) 103, 77, 44, 88, 31, 55, 66, 92. **6.** \$2.00 + \$.60 + \$.50 + \$.13 = \$3.23. **7.** 53. **8.** 13¢; 26¢. **9.** 2 × 19¢ = 38¢; \$1.00 − \$.38 = \$.62. **10.** 6 4-cent apples, 1 cent over. **11.** 58¢. **12.** 12; $\frac{1}{2}$; 20. **13.** 5280; 16; $\frac{3}{4}$. **14.** 3; 8; $\frac{1}{4}$. **15.** 16; 16; $1\frac{1}{4}$. **16.** 6; $\frac{2}{3}$; 18. **17.** 48; $1\frac{1}{2}$; 120. **18.** 14; 4; 6000. **19.** 18; 180; 120. **20.** 30; 6; 2640.

Page 29

Aim: To reteach division with 2-figure divisors ending in 6, 7, 8, or 9

Suggestions: Attention is called to the rule on page 29 which states that when the two-figure divisor ends in 6, 7, 8, or 9, you should divide by 1 *more than the* *first figure* of the divisor to estimate each figure of the quotient. For example, if the divisor is 56, 57, 58, or 59, you divide by 6 to estimate each quotient figure. By following Rule II on page 29 for its group of divisors, together with Rule I on page 27 for its particular group of divisors, you will obtain the correct quotient figure *on first trial* in 80% of all cases. In the remaining 20% of cases, in general, only one correction of the quotient figure will be necessary. Only in very rare situations will it be necessary to make two corrections of the quotient figure.

The model example given at the top of page 29 should be carefully explained and checked. Notice that the quotient is a 4-figure number. The practice exercises have 3-figure and 4-figure quotients.

Key: **2.** 541 R3; 3527; 4725. **3.** 3188 R11; 4823; 7519. **4.** 883 R5; 9475; 3728. **5.** 768 R5; 2914; 7562. **6.** 1728; 2534; 7582. **7.** 614 R35; 6728; 3718.

Page 30

Aim: To give more division practice for those who need it

Suggestions: Assign only part of this page, since there are more exercises than are needed for all pupils. For example, give the boys all even-numbered exercises and the girls all odd-numbered exercises. Be sure to require checking.

Key: **1.** \$50.00 − \$37.25 = \$12.75; \$12.75 ÷ \$.75 = 17 (wk.). **2.** \$13.16 ÷ 28 = \$.47. **3.** 237 mi. ÷ 6 = $39\frac{1}{2}$ mi. **4.** From 8:30 A.M. to 1:30 P.M. is 5 hr.; 190 mi. ÷ 5 = 38 mi. **5.** 93; 656; 504. **6.** 335; 667; 403 R12. **7.** 144; 537; 694 R25. **8.** 87; 547; 669 R8. **9.** 65; 483; 222. **10.** 257 R11; 562 R12; 664. **11.** 145; 662 R5; 386. **12.** 205; 470; 547. **13.** 127; 728; 895. **14.** 256 R31; 663; 708. **15.** 594; 930 R15; 753.

Page 31

Aims: To review tests of divisibility and to use them in simplifying fractions

Suggestions: The four tests of divisibility given on this page are very useful. Remember that a test of divisibility, such as the test for divisibility by 9, is merely a short way of finding whether a number is exactly divisible by 9, without actually dividing the given number by 9 to find out. One of the main uses of tests of divisibility is in changing fractions to lowest terms, as described in ex. 5.

Key: **5.** Yes. **6.** $\frac{3}{5}$; $\frac{2}{3}$; $\frac{16}{29}$; $\frac{3}{4}$; $\frac{5}{11}$; $\frac{5}{7}$; $\frac{3}{8}$; $\frac{8}{9}$. **7.** $\frac{14}{19}$; $\frac{2}{3}$; $\frac{7}{8}$; $\frac{3}{8}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{9}{11}$; $\frac{4}{7}$. **8.** $\frac{3}{5}$; $\frac{2}{3}$; $\frac{5}{6}$; $\frac{2}{5}$; $\frac{13}{16}$; $\frac{16}{21}$; $\frac{6}{13}$; $\frac{43}{77}$.

Pages 32–33

Aim: To reteach division with 3-figure divisors

Suggestions: The method of doing long division with 3-figure divisors as presented on these pages has the same high efficiency as the method used in this book for 2-figure divisors. That is, this method gives the correct quotient figure on first trial in 80% of all cases. In the remaining 20% of cases the correct quotient figure is found, with minor exceptions, after a single correction of the trial quotient figure. Page 32 gives the rule for estimating quotient figures when the second figure of the 3-figure divisor is 1, 2, 3, 4, or 5. For convenience this rule is called Rule I and corresponds to Rule I as used for 2-figure divisors, which is given on page 27 of the text. Page 33 gives Rule II, which is the rule for estimating quotient figures when the second figure of the 3-figure divisor is 6, 7, 8, or 9. This rule corresponds to Rule II as used for 2-figure divisors on page 29. The exercises on page 32 apply the work on tests of divisibility, since many fractions in the quotients can be changed to lowest terms.

Workbook Reference: Arithmetic Workshop, Book 7, pages 7 and 8

Key: Page 32 **2.** $9\frac{3}{8}$; 124; 70; $196\frac{11}{53}$. **3.** $6\frac{7}{8}$; $87\frac{1}{2}$; $54\frac{12}{103}$; 628. **4.** $9\frac{1}{3}$; 96; 79; 259. **5.** 24; $27\frac{11}{93}$; 75; $221\frac{4}{91}$. **6.** 25; 58; $243\frac{5}{9}$; 588. **7.** $31\frac{3}{7}$; 39; 255; 351.

Page 33 **2.** 526; 982; 87; $20.73. **3.** 860 R12; 609; 34 R560; $2.07 R$1.38. **4.** 2457; 1847; 408; $9.00 R$1.04. **5.** 451; 1243 R24; 136; $6.07. **6.** 2038; 725 R28; 58; $8.90 R63¢.

Pages 34–35

Aim: To review and reteach the two types of problems with averages

Suggestions: In ex. 1 on page 34 a two-step problem in averages is explained, while in ex. 2 a one-step problem in averages is illustrated. It is important to distinguish between these two types of average problems. In the two-step problem you first add and then divide. In the one-step problem it is not necessary to add the numbers since the sum is given; hence, you divide only. Attention should be called to ex. 1, 3, and 4, in which pupils must properly interpret the answers by giving results that are sensible.

Workbook Reference: Arithmetic Workshop, Book 7, page 9

Key: **2.** $683 \div 135 = 5\frac{8}{135}$, or 5 (tickets). **3.** 2 hr. + $1\frac{1}{2}$ hr. + $2\frac{1}{2}$ hr. + 1 hr. = 7 hr.; 7 hr. ÷ 4 = $1\frac{3}{4}$ hr. **5.** $1\frac{3}{4}$ hr. + 2 hr. + $2\frac{1}{4}$ hr. + 1 hr. + 1 hr. = 8 hr.; 8 hr. ÷ 5 = $1\frac{3}{5}$ hr. **6.** $2.50 + $3.75 + $1.25 + $3.50 = $11.00; $11.00 ÷ 4 = $2.75. **7.** 1170 ÷ 3 = 390 (tickets).

Page 36

Aims: To review division of all types and to extend to divisors of four figures

Suggestions: Although 2-figure and 3-figure divisors are the ones most used, there are real cases where larger divisors are sometimes needed. Caution pupils to work carefully in order not to make mistakes when using these large divisors. Checking should be required.

Key: **1.** $6255.50 ÷ 3568 = $1.75 R$11.50, or $1.75. **2.** 4775 mi. ÷ 25 = 191 mi. **3.** $65.74 ÷ $.19 = 346 (notebooks). **4.** 15,348 ÷ 2568 = 5 R2508, or 6 (times). **5.** 304; 528; 38 R4; 183. **6.** 142; 219 R60; 48; 483. **7.** 700 R31; 496; 428; 600 R15. **8.** 827; 783; 51 R7; 619 R13.

Page 37

Aim: To teach casting out 9's as a new method of checking division

Suggestions: The check of 9's is here applied to long division. The check numbers, of course, are obtained in each case by the method described on page 24 of the text. Notice that the check is done by multiplication and not by division.

In the exercises on page 37 there are no divisors that give a check figure of 0. However, such a check figure may appear in some examples. When the check figure of the divisor is zero and there is *no remainder*, as illustrated at the right, you can see that *any* number will check as the quotient for such a division example. This happens because 0 times any check figure is 0. You might, therefore, conclude that casting out 9's is of no value in such examples. However, if a pupil makes an error as he works such an example, he is almost certain to bring in a remainder and thus obtain different final check numbers, as shown at the right. Therefore an error in this kind of example will usually be detected by the check of casting out 9's.

The check of 9's is strongly recommended for division in Grades 7 and 8.

Correct

```
          (7)
(0)       826          9's check
    27)22302 (0)           7
       216                 0
        70                ---
        54                (0)
       162
       162
       ---
```

Incorrect

```
          (8)
(0)       863          9's check
    27)22302 (0)           8
       206                 0
       170                ---
       162                 0
         82                1
         82               ---
         --               (1)
          1
         (1)
```

Key: **2.**

```
         (0)
   (2)   927          9's check
   38)35226 (0)           0
      342                 2
       102               ---
        76                0
       266
       266
       ---
```

3. 672 R7; 380; 928. **4.** 778; 762 R137; 206 R141. **5.** 601 R8; 821; 225. **6.** 840 R13; 604; 708 R53.

Page 38

Aim: To give both oral and written practice on the work covered in Chapter 1

Suggestions: In class go over the oral work very rapidly and give any assistance that may be needed. Assign the written practice as you think necessary. Not all pupils need work all the exercises.

Key: **1.** (Dividing by 5) 5, 9 R3, 6 R3, 5 R3, 8, 3 R3, 12, 1 R3; (dividing by 6) 4 R1, 8, 5 R3, 4 R4, 6 R4, 3, 10, 1 R2. **2.** (Dividing by 5) 2 R2, 11, 3 R4, 18, 3, 7 R1, 11 R3, 0 R3; (dividing by 6) 2, 9 R1, 3 R1, 15, 2 R3, 6, 9 R4, 0 R3. **3.** 21; 212; 320; 103; 203; 70. **4.** 101; 61; 62; 204; 71; 20. **5.** 52; 78; 100; 61; 83; 76. **6.** 24; 108; 17; 36. **7.** 22; 112; 40; 55. **8.** 21; 126; 62; 72. **9.** 9; 5; 83; 48. **10.** 6; 1; 21; 61. **11.** 147,980; 229,565; 68,600; 486,570; 674,485. **12.** 88,723; 205,690; 124,335; 1,037,967; 2,433,282. **13.** 109,220; 809,260; 571,900; 7,397,720; 3,421,080. **14.** 80,192; 34,368; 32,041; 1,009,739; 552,394. **15.** 67; 389; 1424. **16.** 47; 50; 1003. **17.** 84; 35; 1793. **18.** 18; 46; 309.

Page 39

Aim: To review important problem patterns in division

Suggestion: In class go over the problems with the pupils and ask what process will be used to solve each problem.

Key: **1.** \$348 ÷ 12 = \$29. **2.** 153 ÷ 17 = 9 (jars). **3.** 666 ÷ 18 = 37 (gal.). **4.** 2405 ÷ 481 = 5 (hr.). **5.** 30 min. ÷ 18 = $1\frac{2}{3}$ min. **6.** 2250 lb. ÷ 15 = 150 lb. **7.** 6000 ÷ $1\frac{1}{4}$ = 4800 (mi.). **8.** \$495 ÷ \$55 = 9 (mo.). **9.** 8640 ÷ 12,960 = $\frac{2}{3}$.

Page 40

Aim: To summarize the work reviewed or retaught in Chapter 1

Suggestions: Be sure that any errors made on page 40 are discussed with the individual pupils who made them. If necessary, explain the topic again and see that a few practice exercises are done correctly. The test on page 42 contains the same type of examples.

Key: **1.** 2,000,000; 8,000,000; 10,000,000. **2.** 60,000,000; 30,000,000; 90,000,000. **3.** 2,000,000,000; 8,000,000,000; 6,000,000,000. **4.** 3000, 6599, 9225, 5742, 8195; 3775, 7374, 10,000, 6517, 8970. **5.** 5997, 6716, 5000, 7719, 5771; 1182, 1901, 185, 2904, 956. **6.** 72,611; 1,406,592; 56,000,000. **7.** 1715 R3; 805 R914; 737 R2593. **8.** \$4.23, \$4.25, \$4.50, \$5.00; \$3.90, \$4.00, \$5.00; \$2.76, \$2.80, \$2.90, \$3.00, \$4.00, \$5.00; \$4.42, \$4.45, \$4.50, \$5.00; \$3.15, \$3.25, \$3.50, \$4.00, \$5.00; \$1.38, \$1.40, \$1.50, \$2.00, \$3.00, \$4.00, \$5.00. **9.** 6173 + 4092 + 7384 + 5227 = 22,876; 22,876 ÷ 4 = 5719. **10.** \$2.00 + \$1.50 + \$1.10 + \$.50 + \$.15 = \$5.25; \$5.25 ÷ \$.35 = 15 (tickets). **11.** $\frac{7}{8}$; $\frac{1}{5}$; 8. **12.** 20; 44; 8. **13.** $3\frac{2}{3}$; $\frac{4}{5}$; 27.

Page 41

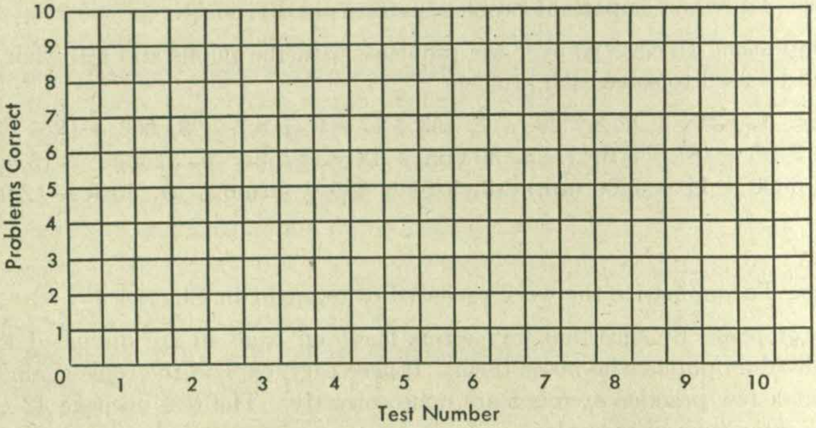
Aim: To present Problem Test 1

Suggestions: Allow pupils ample time to read and to complete these problems. After you have scored the problems, provide an opportunity for the pupils to make a record of problem-test scores. As successive tests are taken, remind the pupils to enter their ratings. As the year goes on, each pupil will thus have a way of comparing his test scores.

Such a record can be kept by means of a bar graph on which the color of the bar indicates the rating. For instance, a rating of "You need help" could be colored red, a "Fair" rating could be brown, a "Good" rating could be green, and an "Excellent" rating could be blue. The chart should be planned for ten problem tests and might appear as shown below:

GRADE SEVEN PROBLEM–TEST RECORD

If your rating is "You need help," color the column *red.*
If your rating is "Fair," color the column *brown.*
If your rating is "Good," color the column *green.*
If your rating is "Excellent," color the column *blue.*



On how many tests were you a blue-ribbon winner?

Workbook Reference: Arithmetic Workshop, Book 7, page 11

Key: **1.** 30 + 25 + 26 + 29 + 24 = 134; 134 ÷ 5 = 26 R4, or 27 (pupils). **2.** 15 × 24 pieces = 360 pieces; 360 pieces + 10 pieces = 70 pieces. **3.** From 2:07 P.M. to 4:22 P.M. is 135 min. or $2\frac{1}{4}$ hr.; $2\frac{1}{4}$ × 480 mi. = 1080 mi. **4.** $9\frac{1}{2}$ × \$.60 = \$5.70; \$6.95 − \$5.70 = \$1.25. **6.** \$1.50 + \$.35 + \$.60 + \$.15 = \$2.60; \$.10 + \$.45 + \$.06 + \$1.00 + \$.08 = \$1.69; \$2.60 − \$1.69 = \$.91. **7.** \$12.60 ÷ 36 = \$.35. **8.** 8 × \$.26 = \$2.08; \$5.00 − \$2.08 = \$2.92. **9.** 37,564 − 6525 = 31,039 (people). **10.** 144 × \$.05 = \$7.20; \$7.20 − \$4.50 = \$2.70.

Page 42

Aim: To provide a diagnostic test with page references for remedial work

Suggestions: Page 42 is a diagnostic test covering the four processes with whole numbers covered in Chapter 1. Observe how this diagnostic test is constructed. Column addition is tested by a set of examples of equal difficulty. If a pupil gets the right answer on all but one of the examples in ex. 1, the wrong answer was probably caused by an accidental error. But if he misses two or three examples in this set, something fundamental is probably wrong. In such a case, endeavor to discover the reason for the errors. In some instances an incomplete mastery of the addition facts may be responsible; if this is true, give the pupil a list of the facts that are causing him trouble and point out the number relationships involved. In other cases, the pupil may not understand a step involved in the process, such as carrying, in which case reteaching is necessary. After correcting the fundamental difficulties, assign additional practice on the page listed at the end of the row.

Key: **1.** 3646; 4016; 3441; 3927; 3775. **2.** 1790; 3477; $248.09; $2849.69. **3.** 5284; 1650; $244.28; $6135.65. **4.** 153,908; 325,872; 2,821,830; 1,004,003; 1,708,098. **5.** 6,907,420; 15,751,872; 35,136,360; 7,689,600; 7,502,121. **6.** 579; 779 R15; 700 R31. **7.** 435; 834; 259. **8.** 386¼; 643. **9.** 503 R270; 558. **10.** 3587; 846.

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Chapter 2

Aims of Chapter 2. The major aims of Chapter 2 are to:

1. Review the meaning of, and the arithmetical terms related to, fractions.
2. Reteach simplification of fractions and changing of a mixed number to an improper fraction.
3. Reteach addition of fractions with like denominators.
4. Reteach addition of fractions with unlike denominators.
5. Reteach subtraction of mixed numbers where the fraction in the minuend is larger than the fraction in the subtrahend.
6. Reteach subtraction of mixed numbers where the fraction in the minuend is smaller than the fraction in the subtrahend.
7. Reteach and extend work on least common denominator in addition and subtraction of fractions.
8. Reteach all cases of the multiplication of fractions and mixed numbers.
9. Reteach all cases of the division of fractions and mixed numbers.
10. Show how to take and score improvement tests in whole numbers.

Pages 43–45

Aim: To present a practical project on train travel, including the reading of timetables

Suggestions: Have pupils bring to class some bus, train, and airplane timetables, and let them practice finding the time and distance for certain trips.

Key: Page 43 **1.** \$6.13. **2.** \$97.00; \$41.45. **3.** \$10.80.

Page 44 **1.** Thursday, 9:55 A.M. **2.** 10:25 P.M.; 10:45 P.M.; 20 min. **3.** 7:00 P.M., 5 min.; 2:10 A.M., 5 min.; 3:50 A.M., 5 min. **4.** 11:50 P.M.; 9:55 A.M. **5.** 18 hr. 40 min.

Page 45 **1.** 1023 mi.; 20 min.; yes; 54 mph. **2.** 20 min.; 5 min.; 5 min.; 5 min.; 17 hr. 45 min.; 18 hr.; 57 mph. **3.** 7 hr. 10 min.; 64 mph. **4.** 574 mi.; 52 mph. **5.** Wednesday, 7:00 P.M.; 25 hr.; 55 mph.

Pages 46–50

Aim: To show how to take and score improvement tests

Purpose of Improvement Tests. The teacher of mathematics in the seventh grade has three large tasks which have to be completed during the year:

(1) To review the four operations with whole numbers sufficiently to maintain and to develop further the computational skills that the pupils have already learned

(2) To review the four operations with common fractions and decimals

(3) To teach the new work of Grade 7, which includes new topics in percentage with applications, informal geometry, new formulas, and certain social uses of arithmetic

Unless you plan your work carefully, you may find that tasks (1) and (2) take so much time that they greatly interfere with task (3). In order to reduce task (1) to a minimum, the authors have devised a series of improvement tests which give the pupils the necessary practice on whole numbers in less than 30 min. a week. These tests are described on pages 46–50 and 344 of the text. These improvement tests have been used successfully by many hundreds of teachers who are unanimous in their verdict that they are the most effective means yet found of improving the pupil's skill in computation. One important fact about these tests is that the pupils themselves grow more enthusiastic about them the longer they use them, whereas with many other types of tests the interest of the pupils soon dies out. These improvement tests are first introduced in *American Arithmetic, Grade 6* and are continued throughout this text.

Description of Improvement Tests. There are 72 improvement tests in this book, and these tests are distributed uniformly over the year's work. The tests are arranged in sets of 3 tests each; hence, there are 24 sets of such tests. They cover the addition, subtraction, multiplication, and division of whole numbers. For a complete list of these tests, see the Index under "Improvement Tests." A preliminary orientation concerning these tests is given on page 46 of the text, which the pupils should read carefully at this time. A set of improvement tests, consisting of 3 addition tests, is shown on page 47. The nature of these tests is evident by an examination of the 3 tests. Each test has 8 exercises, all the exercises being of equal difficulty. If a pupil gets a score of 9 on Test 1a and a score of 10 on Test 1b, it is evident that he has done better on Test 1b than on Test 1a, since the tests are of equal difficulty.

Though there are 3 tests on page 47, these 3 tests should not all be given on the same day. Only one test of a set is to be taken on a given day. The reason for this caution is to avoid fatigue and to minimize the time devoted to practice on whole numbers. In general, one test should be given every second or third day; hence, the 3 tests on page 37 will be spread over a period of 5 to 7 days.

The number of exercises in each test is indicated by a small number in a circle, which is printed near the last exercise in the test. For example, at the end of Test 1a on page 47 you find ⑧, which means that there are 8 exercises in this test. On the other hand, in Subtraction Test 1a, given on page 57, there are 15 exercises in each test.

How to Give Improvement Tests. The method of giving improvement tests is described in detail on page 344 of the text. These directions should be studied carefully. In timing the tests, use a watch with a second hand so that the pupils can be stopped promptly at the end of the time assigned to the test. In giving the addition tests on page 47, allow exactly 4 minutes for each test so that the results on these tests may be comparable. If you allow 4 minutes for Test 1a but permit the time for Test 1b to be 4½ minutes, the results on Test 1b cannot be reliably compared with those on Test 1a. The purpose of these tests is to measure improvement in computing with whole numbers, and this measurement can be made only if the tests are given under the same conditions each time.

In taking an addition test such as one of those on page 47, the pupils should not copy the examples before working them. Instead, they are to write the answers along the edge of a piece of folded paper, as described on page 48 of the text. Folded paper is used only for the tests on addition and subtraction. For tests on multiplication and division, such as those given on pages 69 and 87, the exercises should be copied on a sheet of paper *before* the test is given. In copying the exercises, the pupils should spread them out over the sheet to allow ample space for the work. The time assigned for a test on multiplication or division is the time actually allowed for taking the test *after* the copying has been completed.

Scoring Tests. All improvement tests are scored on a scale of 10, and the actual scoring is done by the aid of a scoring table, which is shown on page 49. For convenience, this scoring table is also printed on page 345 of the text. The pupils must be carefully instructed in using this table to find their scores. Detailed instructions for finding scores are given on page 49.

Keeping Records. Each pupil should keep a graphic record of his scores on each improvement test, as explained on page 50 of the text. This record should include all tests taken during the school year. A Record Book can easily be made from a few sheets of squared paper. If the school does not have squared paper, a pad of such paper should be purchased and a few sheets distributed to each pupil. It will be seen in the graph on page 50 that 3 scores are kept for each set of tests. Since there are 24 sets of tests in all in this book, each pupil need have only enough squared paper to provide for the scores on 24 sets.

The plan of having each pupil keep a graphic record of his scores has a very favorable psychological effect upon him. If he finds that his score on the second test of a set is not above that on the first test, he immediately puts forth all his energy to improve his record on the third test. The pupil is always trying to better his previous performance, and this element maintains a high degree of interest in this work throughout the school year. The pupil also gains considerable satisfaction in studying his scores from time to time during the year. In the middle of the year his scores should be better, in general, than those at the first of the year; and at the end of the year he should be doing still better.

A pupil will find it helpful to compare his scores on subtraction tests with those on addition tests. If he finds that he does not do so well on the subtraction tests as on the addition tests, he will see that he needs more practice on subtraction. Similar comparisons may be made with respect to the tests on multiplication and division.

Key: Page 47 **1.** 462; 503; 510; 312; 438; 340; 488; 457. **2.** 373; 537; 405; 316; 429; 478; 550; 457. **3.** 421; 389; 486; 467; 530; 441; 421; 327.

Page 50 **2.** 10; Nov. 5. **3.** 7 on Nov. 12; 9 on Nov. 16; 8 on Nov. 18.

Page 51

Aim: To review the three meanings of a fraction and the special names connected with fractions, such as *numerator, denominator, proper fraction, improper fraction, mixed number*

Suggestions: On the chalkboard review some diagrams that show the meaning of $\frac{3}{4}$, $\frac{4}{4}$, $\frac{5}{4}$. This review can be given by using circles divided into fourths. The first will have 3 parts colored to show $\frac{3}{4}$, the second will be entirely colored to show $\frac{4}{4}$, and the third diagram will show 1 whole circle and $\frac{1}{4}$ of another circle. Page 51 is to be done in class and should be supplemented by more practice of the kind given in ex. 7 and 9.

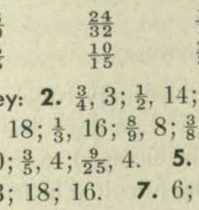
Workbook Reference: Arithmetic Workshop, Book 7, page 12

Key: **9.** Proper fractions: $\frac{3}{10}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{1}{3}$; improper fractions: $\frac{6}{5}$, $\frac{12}{12}$, $\frac{11}{3}$, $\frac{10}{7}$, $\frac{8}{8}$.

Page 52

Aim: To reteach the fundamental principle of fractions.

Suggestions: Draw a circle on the board and divide it into 4 equal parts. Shade 3 of the parts. Now draw two dashed lines to divide the circle into 8 equal parts. Ask the pupils to study this diagram and tell what equal fractions it shows. They will bring out the fact that $\frac{3}{4} = \frac{6}{8}$ and that $\frac{1}{4} = \frac{2}{8}$. In other words, $\frac{3}{4}$ and $\frac{6}{8}$ have the same value. Draw other circles with 6ths and 12ths, 4ths and 16ths, and so on. Then take up ex. 1, which gives a way to find equivalent fractions without using diagrams. Here are more fractions to change to lowest terms if extra practice is needed:



$\frac{16}{20}$ $\frac{24}{32}$ $\frac{10}{12}$ $\frac{32}{48}$ $\frac{10}{18}$ $\frac{15}{25}$ $\frac{12}{36}$ $\frac{18}{45}$ $\frac{45}{100}$

$\frac{15}{25}$ $\frac{10}{15}$ $\frac{20}{30}$ $\frac{54}{72}$ $\frac{21}{27}$ $\frac{25}{40}$ $\frac{24}{96}$ $\frac{35}{50}$ $\frac{72}{100}$

Key: **2.** $\frac{3}{4}$, 3; $\frac{1}{2}$, 14; $\frac{4}{5}$, 5; $\frac{1}{3}$, 9; $\frac{3}{8}$, 4; $\frac{5}{6}$, 4; $\frac{1}{3}$, 10; $\frac{1}{4}$, 15; $\frac{3}{4}$, 25. **3.** $\frac{3}{4}$, 5; $\frac{2}{3}$, 6; $\frac{9}{10}$, 2; $\frac{1}{3}$, 18; $\frac{1}{3}$, 16; $\frac{8}{9}$, 8; $\frac{3}{8}$, 10; $\frac{1}{14}$, 5; $\frac{1}{2}$, 50. **4.** $\frac{3}{5}$, 3; $\frac{2}{3}$, 12; $\frac{5}{6}$, 9; $\frac{1}{3}$, 15; $\frac{1}{6}$, 16; $\frac{1}{3}$, 20; $\frac{4}{5}$, 10; $\frac{3}{5}$, 4; $\frac{9}{25}$, 4. **5.** $\frac{2}{7}$, 6; $\frac{1}{4}$, 6; $\frac{2}{5}$, 7; $\frac{5}{8}$, 4; $\frac{2}{3}$, 7; $\frac{1}{4}$, 12; $\frac{1}{5}$, 18; $\frac{7}{8}$, 5; $\frac{7}{20}$, 5. **6.** 8; 15; 33; 18; 16. **7.** 6; 28; 15; 44; 95. **8.** 8; 21; 49; 12; 90. **9.** $\frac{660}{5280} = \frac{1}{8}$ (mi.). **10.** $\frac{20}{60} = \frac{1}{3}$ (hr.).

Page 53

Aim: To reteach changing an improper fraction to a mixed number and a mixed number to an improper fraction

Suggestions: On the chalkboard use diagrams of pies to show that 2 whole pies cut into sixths and $\frac{5}{6}$ of another pie is the same as $\frac{17}{6}$. In other words, $2\frac{5}{6}$ and $\frac{17}{6}$ are two ways of writing the same thing. Sometimes it is more convenient to use one form than the other. In the addition of fractions, an answer might be $\frac{8}{3}$, which can be changed to $2\frac{2}{3}$. In the multiplication of mixed numbers by fractions or mixed numbers, it is often necessary to change a mixed number to an improper fraction. Ex. 4 should be emphasized since it shows how a whole number can be written in the form of a fraction with a denominator of 1. This form will be useful later in the multiplication and division of fractions. Page 53 can all be done orally.

Workbook Reference: Arithmetic Workshop, Book 7, page 13

Key: **2.** $3\frac{1}{3}$; $3\frac{1}{8}$; $1\frac{1}{2}$; 2; $1\frac{1}{2}$; 6; $1\frac{1}{4}$; $5\frac{1}{2}$; $2\frac{1}{3}$. **3.** $1\frac{1}{2}$; $2\frac{1}{2}$; $1\frac{1}{8}$; $3\frac{1}{4}$; $1\frac{7}{8}$; 1; 4; $1\frac{9}{10}$; $1\frac{3}{4}$. **5.** $\frac{7}{2} \div \frac{1}{2} = 7$ (persons). **6.** $\frac{13}{2}$; $\frac{11}{4}$; $\frac{41}{8}$; $\frac{14}{3}$; $\frac{31}{5}$; $\frac{31}{8}$; $\frac{51}{8}$; $\frac{13}{10}$. **7.** $\frac{21}{8}$; $\frac{25}{6}$; $\frac{26}{3}$; $\frac{29}{4}$; $\frac{22}{7}$; $\frac{11}{2}$; $\frac{47}{5}$; $\frac{31}{12}$. **8.** $\frac{10}{3}$; $\frac{29}{5}$; $\frac{23}{6}$; $\frac{19}{2}$; $\frac{35}{4}$; $\frac{19}{8}$; $\frac{17}{2}$; $\frac{59}{16}$. **9.** $\frac{37}{7}$; $\frac{19}{3}$; $\frac{15}{8}$; $\frac{17}{6}$; $\frac{25}{3}$; $\frac{18}{5}$; $\frac{19}{4}$; $\frac{73}{12}$.

Page 54

Aim: To reteach addition of fractions with like denominators

Suggestions: Start with chalkboard work in which you have pupils add 4 apples and 7 apples, then 3 eights and 4 eights, and finally $\frac{2}{5}$ and $\frac{1}{5}$. This method brings out the fact that you can add the numerators of fractions that have the same name or the same denominator. Page 54 gives models showing the sums of fractions making less than 1, making 1, and making more than 1. Some simplifying of fractions is included. Go over these models with the class before assigning ex. 3–5. If more practice on this topic is needed, it can be found on page 328.

Key: **3.** 17; $8\frac{1}{2}$; 10; $10\frac{2}{3}$; $11\frac{1}{2}$; $5\frac{4}{5}$; $12\frac{1}{8}$. **4.** $6\frac{2}{3}$; $3\frac{2}{3}$; 9; $13\frac{1}{4}$; $7\frac{1}{3}$; 11; $10\frac{3}{5}$. **5.** $10\frac{1}{4}$; $12\frac{5}{8}$; $12\frac{1}{2}$; $17\frac{1}{4}$; 11; $7\frac{3}{16}$; $14\frac{7}{12}$.

Page 55

Aim: To reteach addition of fractions with unlike denominators

Suggestions: Show that 2 quarters and 3 dimes can be added only by changing both to cents. Since fractions can be added only when they have the same denominator, those with unlike denominators must first be changed to fractions with like denominators. The change is made by using the principle of fractions stated on page 52 of the text. If more practice on the addition of fractions is necessary, it will be found on page 329.

Workbook Reference: Arithmetic Workshop, Book 7, page 14

Key: **3.** $5\frac{1}{8}$; $9\frac{5}{6}$; $5\frac{9}{10}$; $11\frac{1}{2}$; $10\frac{1}{2}$; $8\frac{3}{4}$; $6\frac{7}{16}$. **4.** $10\frac{3}{8}$; $9\frac{5}{6}$; $6\frac{1}{3}$; $6\frac{1}{2}$; $15\frac{1}{5}$; $4\frac{1}{4}$; $14\frac{1}{3}$. **5.** $15\frac{7}{8}$; 10; 14; $12\frac{1}{2}$; $5\frac{1}{12}$; $16\frac{1}{5}$; $8\frac{1}{4}$.

Page 56

Aim: To reteach the subtraction of mixed numbers where the fraction in the minuend is larger than the fraction in the subtrahend

Suggestion: The suggestions given for page 55 apply as well to the subtraction of fractions on this page.

Key: **2.** $3\frac{7}{8}$ yd. $-$ $1\frac{4}{8}$ yd. $=$ $2\frac{3}{8}$ yd. **3.** $4\frac{1}{8}$; $3\frac{2}{3}$; $1\frac{1}{2}$; $3\frac{1}{2}$; $1\frac{3}{4}$; $1\frac{1}{2}$; $3\frac{13}{16}$. **4.** $\frac{1}{6}$; $2\frac{1}{6}$; $3\frac{1}{2}$; $2\frac{1}{2}$; $1\frac{5}{12}$; $5\frac{1}{3}$; $2\frac{7}{16}$. **5.** $2\frac{3}{8}$; $3\frac{1}{2}$; $7\frac{5}{8}$; $5\frac{1}{4}$; $2\frac{1}{8}$; $2\frac{1}{2}$; $7\frac{5}{6}$. **6.** $1\frac{11}{20}$; $3\frac{1}{2}$; $2\frac{1}{2}$; $4\frac{2}{5}$; 6; $1\frac{1}{4}$; $1\frac{1}{10}$. **7.** $4\frac{3}{10}$; $6\frac{5}{12}$; $4\frac{1}{3}$; $6\frac{2}{5}$; $3\frac{4}{15}$; $7\frac{1}{6}$; $5\frac{1}{12}$. **8.** $1\frac{3}{8}$; 4; $1\frac{1}{4}$; 2; $1\frac{2}{3}$; 0; $4\frac{1}{3}$. **9.** $5\frac{1}{6}$; 0; $2\frac{1}{6}$; $3\frac{5}{12}$; $8\frac{1}{8}$; $1\frac{1}{8}$; $2\frac{3}{8}$.

Page 57

Aim: To give the first set of improvement tests in the subtraction of whole numbers

Suggestion: This set of improvement tests in subtraction should be administered in the same way as the set of improvement tests in addition given on page 47 of the text.

Key: **1.** 484; 1493; 1308; 4793; 4999. **2.** 5641; 1579; 877; 2566; 3237. **3.** 2579; 1919; 2210; 6855; 1342. **4.** 2898; 1957; 686; 1665; 4887. **5.** 885; 2463; 579; 256; 556. **6.** 1964; 1525; 3999; 3347; 2233. **7.** 6276; 5620; 4770; 4349; 548. **8.** 179; 8439; 5626; 5885; 5479. **9.** 3747; 2892; 1748; 1631; 262.

Page 58

Aim: To reteach subtraction of mixed numbers where the fraction in the subtrahend is larger than the fraction in the minuend

Suggestions: You can visualize this step for the pupils by drawing 3 pies and $\frac{1}{4}$ of another pie. You wish to serve $\frac{1}{4}$ pie to each of three people or $\frac{3}{4}$ pie in all. To do this, cut one whole pie into fourths. Then you have 2 pies and $\frac{5}{4}$ pies, or $2\frac{5}{4}$ pies. This shows that $3\frac{1}{4}$ pies equals $2\frac{5}{4}$ pies. The mixed numbers are two ways of writing the same thing. Give some practice on such changes as $4\frac{1}{4} = 3\frac{?}{4}$, $2\frac{5}{8} = 1\frac{?}{8}$, $5\frac{3}{16} = 4\frac{?}{16}$. The work on this page is sometimes called subtraction with borrowing because the fraction in the minuend is smaller than the fraction in the subtrahend. It is not necessary to use the term *borrowing* unless you prefer to do so. In the model example in ex. 1, think of $8\frac{1}{4}$ as changed to $7\frac{5}{4}$ and $2\frac{1}{2}$ as changed to $2\frac{2}{4}$. More practice on this type of subtraction can be found on page 330.

Workbook Reference: Arithmetic Workshop, Book 7, page 15

Key: **3.** $3\frac{3}{4}$; $6\frac{1}{2}$; $\frac{3}{10}$; $5\frac{1}{4}$; $\frac{5}{8}$; $1\frac{2}{5}$; $\frac{1}{4}$. **4.** $\frac{3}{8}$; $\frac{1}{2}$; $3\frac{13}{16}$; $3\frac{5}{16}$; $\frac{1}{2}$; $2\frac{3}{4}$; $5\frac{9}{16}$. **5.** $3\frac{7}{8}$; $1\frac{2}{3}$; $2\frac{5}{6}$; $2\frac{1}{2}$; $5\frac{4}{5}$; $3\frac{13}{16}$; $1\frac{7}{10}$. **6.** $1\frac{7}{8}$; $2\frac{1}{3}$; $\frac{5}{8}$; $2\frac{2}{5}$; $3\frac{3}{4}$; $\frac{1}{2}$; $1\frac{11}{12}$.

Page 59

Aim: To reteach the most difficult case of subtraction of mixed numbers in which the fraction in the subtrahend is larger than the fraction in the minuend

Suggestions: The exercises on this page are more difficult than those on page 58 because two changes must be made in the minuend, one to change the fraction to the common denominator and the other to make the fraction larger by changing 1 to an improper fraction and combining it with the fraction already there. Exercises like 1–4 on page 59 help the pupil to make changes in the form of the minuend so that the fraction in the minuend will be larger than the fraction in the subtrahend and will also have the common denominator. In the model in ex. 5 pupils must think of $3\frac{1}{2}$ as $3\frac{4}{8}$ because the denominator of the other fraction is 8. Then they must think of $3\frac{4}{8}$ as $2\frac{12}{8}$.

Workbook Reference: Arithmetic Workshop, Book 7, page 16

Key: **1.** 2, 6; 9, 21; 4, 16. **2.** 4, 10; 10, 22; 12, 28. **3.** 2, 10; 5, 15; 9, 21. **4.** 3, 9; 10, 26; 8, 18. **6.** $\frac{5}{8}$; $2\frac{2}{3}$; $2\frac{13}{16}$; $2\frac{11}{12}$; $4\frac{5}{16}$; $\frac{3}{4}$; $4\frac{15}{16}$. **7.** $1\frac{7}{12}$; $4\frac{1}{2}$; $4\frac{3}{4}$; $\frac{7}{16}$; $2\frac{9}{10}$; $1\frac{9}{16}$; $1\frac{7}{10}$. **8.** $2\frac{5}{6}$; $2\frac{7}{8}$; $4\frac{7}{10}$; $1\frac{3}{4}$; $2\frac{9}{16}$; $7\frac{3}{5}$; $\frac{7}{12}$. **9.** $1\frac{3}{8}$; $2\frac{5}{6}$; $2\frac{11}{12}$; $3\frac{3}{10}$; $2\frac{5}{6}$; $\frac{13}{16}$; $6\frac{11}{16}$. **10.** $3\frac{5}{8}$; $\frac{5}{8}$; $5\frac{5}{6}$; $3\frac{9}{16}$; $5\frac{4}{5}$; $3\frac{9}{16}$; $2\frac{5}{12}$.

Page 60

Aim: To give a quick oral review of preceding work

Suggestion: No pencils and paper should be used in this work. A very few minutes should be enough to cover the entire page. If any one example gives trouble, make another nearly like it to assure an understanding of the point.

Key: **1.** $\frac{1}{2}$; $\frac{3}{4}$. **2.** $1\frac{1}{2}$. **3.** $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$ or $\frac{3}{4}$; $2 - \frac{3}{4} = 1\frac{1}{4}$. **4.** 7; $8\frac{2}{3}$; $2\frac{3}{10}$. **5.** $\frac{2}{3}$; $\frac{1}{2}$; $\frac{2}{5}$. **6.** \$.75 + \$.15 = \$.90, or 9 dimes. **7.** 5 × 25¢ = 125¢, or \$1.25; \$1.25 + \$.45 = \$1.70. **8.** 54; 48; 41; 70; 42; 79. **9.** 80 min. **10.** 1742; 1950; 625. **11.** 2 hr.; 4 hr.; 3 hr.; 8 hr. **12.** 25 4-cent stamps; 50 4-cent stamps and 50 3-cent stamps. **13.** 59. **14.** $2\frac{2}{5}$; $3\frac{1}{3}$; $3\frac{1}{4}$; $4\frac{1}{2}$; $2\frac{2}{9}$; $1\frac{5}{6}$; $2\frac{1}{2}$; $3\frac{1}{7}$; $2\frac{1}{2}$. **15.** $\frac{3}{4}$, $\frac{15}{20}$, and $\frac{9}{12}$. **16.** $\frac{2}{3}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{7}{8}$; $\frac{4}{5}$. **17.** From 3:25 P.M. to 5:10 P.M. is 105 min., or 1 hr. 45 min. **18.** $5\frac{1}{2}$ ft. − $1\frac{1}{2}$ ft. = 4 ft. **19.** \$.79, \$.80, \$.90, \$1.00.

Pages 61–62

Aim: To reteach and extend an efficient method of finding the least common denominator

Suggestions: On page 61 the model example in ex. 1, which is $\frac{1}{4} + \frac{1}{6}$, is worked in two ways. In the first way, which is shown at A, the common denominator of $\frac{1}{4}$ and $\frac{1}{6}$ is found by multiplying the denominators 4 and 6 together to find a common denominator of 24. This method can always be used to find a common denominator for two fractions, although the common denominator thus found is not always the *least* common denominator. In the second way, which is shown at B, the same example is worked by using a common denominator of 12 instead of 24. In this case, 12 is called the *least common denominator* and was found by inspection. Make sure pupils understand the meaning of *factor*. In working the exercises on page 61, the pupil can always find a common denominator by multiplying the two denominators together. In some cases, however, he will find by inspection that a smaller common denominator can be used. The model example on page 62 should be most carefully studied, since it explains the most efficient method yet devised for finding the least common denominator of two or more fractions in more difficult cases. This method is to be used in exercises, such as ex. 2–4 on this page, in which the common denominator is not easily found by inspection. Exercises like those on page 62 are representative of the most difficult cases in the addition of fractions that are now taught at this grade level. In fact, they are more difficult than those usually met today in everyday applications. A few such exercises are given here, however, since they sometimes occur on state, county, or civil service examinations.

Workbook Reference: Arithmetic Workshop, Book 7, page 19

Key: Page 61 **2.** $12\frac{1}{12}$; $13\frac{1}{20}$; $16\frac{5}{24}$; $11\frac{8}{15}$; $13\frac{5}{24}$; $11\frac{11}{12}$; $12\frac{1}{15}$. **3.** $7\frac{7}{24}$; $2\frac{7}{20}$; $3\frac{7}{12}$; $5\frac{7}{24}$; $6\frac{7}{15}$; $9\frac{1}{24}$; $7\frac{19}{20}$. **4.** $3\frac{1}{12}$; $\frac{19}{20}$; $1\frac{7}{24}$; $1\frac{8}{15}$; $2\frac{7}{12}$; $\frac{17}{24}$; $3\frac{3}{40}$. **5.** $2\frac{19}{24}$; $2\frac{9}{20}$; $2\frac{5}{12}$; $4\frac{9}{20}$; $5\frac{1}{12}$; $\frac{13}{15}$; $1\frac{1}{15}$. **6.** $4\frac{23}{24}$; $5\frac{11}{40}$; $3\frac{11}{12}$; $1\frac{1}{24}$; $\frac{11}{24}$; $1\frac{7}{20}$; $2\frac{1}{24}$.

Page 62 **2.** $13\frac{5}{8}$; $14\frac{7}{48}$; $10\frac{19}{20}$; $6\frac{23}{48}$; $3\frac{7}{8}$; $9\frac{14}{15}$; $4\frac{13}{24}$. **3.** $16\frac{17}{24}$; $8\frac{9}{20}$; $16\frac{7}{8}$; $13\frac{21}{40}$; $17\frac{1}{20}$; $12\frac{13}{24}$; $14\frac{19}{24}$. **4.** $7\frac{17}{24}$; $10\frac{7}{8}$; $13\frac{19}{20}$; $8\frac{8}{15}$; $10\frac{23}{24}$; $8\frac{5}{8}$; $14\frac{13}{24}$.

Page 63

Aim: To reteach multiplication of a fraction and a whole number

Suggestions: In ex. 1 a fraction is multiplied by a whole number, while in ex. 2 a whole number is multiplied by a fraction. The model problem in each case shows the practical need for such work. In ex. 2 the answer represents cents, so attention is called to the fact that in most stores any fraction of a cent is called another cent. Exercises 3–8 contain abstract numbers, and all fractions in answers should be retained.

Key: **3.** $12\frac{3}{4}$; 15; 21; $5\frac{3}{5}$; $2\frac{7}{10}$. **4.** $6\frac{2}{3}$; $14\frac{1}{4}$; $11\frac{7}{10}$; $7\frac{1}{2}$; $2\frac{11}{12}$. **5.** $4\frac{1}{2}$; $23\frac{1}{2}$; $7\frac{1}{2}$; $3\frac{3}{5}$; $1\frac{5}{16}$. **6.** $8\frac{4}{5}$; $2\frac{1}{6}$; $10\frac{1}{12}$; $1\frac{4}{5}$; $8\frac{1}{10}$. **7.** $9\frac{3}{8}$; 14; $5\frac{1}{10}$; $2\frac{5}{8}$; $4\frac{7}{12}$. **8.** $7\frac{4}{5}$; $4\frac{1}{4}$; $6\frac{3}{4}$; $3\frac{3}{4}$; $2\frac{13}{16}$.

Page 64

Aim: To reteach multiplication of a mixed number and a whole number

Suggestion: The work here is only an extension of that given on the preceding page, with the extra steps of multiplying the whole numbers and adding the partial products.

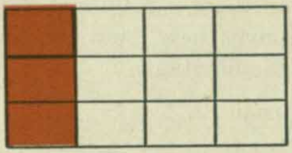
Workbook Reference: Arithmetic Workshop, Book 7, page 20

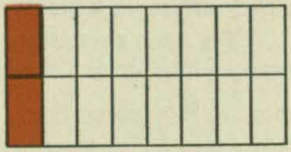
Key: **3.** 33; $103\frac{3}{5}$; $30\frac{1}{4}$; \$.92. **4.** $30\frac{3}{5}$; $83\frac{1}{3}$; $82\frac{1}{2}$; \$3.23$\frac{3}{4}$ or \$3.24. **5.** $21\frac{7}{8}$; $97\frac{1}{2}$; $114\frac{3}{4}$; \$3.83$\frac{1}{3}$ or \$3.84. **6.** $34\frac{2}{3}$; $83\frac{1}{8}$; 81; \$1.29$\frac{3}{8}$ or \$1.30.

Pages 65–66

Aims: To reteach multiplication of two fractions and to show how cancellation can shorten the work

Suggestions: When the multiplier is a whole number, multiplication means repeated addition. Thus, 3×4 means $4 + 4 + 4$, and $3 \times \frac{1}{2}$ means $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$. But when the multiplier is a fraction, multiplication means finding a fractional part of the multiplicand. Thus, $\frac{1}{2} \times 8$ means $\frac{1}{2}$ *of* 8 and $\frac{1}{3} \times \frac{1}{4}$ means $\frac{1}{3}$ *of* $\frac{1}{4}$. The diagrams below can be used to explain the multiplication of two fractions. *A* illustrates $\frac{1}{3}$ of $\frac{1}{4}$ $(\frac{1}{3} \times \frac{1}{4})$; *B* illustrates $\frac{1}{2}$ of $\frac{1}{8}$ $(\frac{1}{2} \times \frac{1}{8})$; and *C* illustrates $\frac{2}{3}$ of $\frac{3}{4}$ $(\frac{2}{3} \times \frac{3}{4})$. The explanation leads up to the rule: *To multiply a fraction by a fraction, first multiply the numerators; then multiply the denominators.*


A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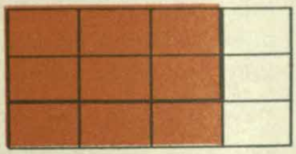

C

The work of multiplying fractions can be simplified by using *cancellation*, as shown in the model examples on page 65. When you cancel, you divide the numerator of one fraction and the denominator of the other fraction by the same number. You have really divided the numerator and the denominator of the final answer by the same number and applied the basic principle of fractions stated on page 52 of the text. If pupils understand the mathematical principle which justifies the process of cancellation, they are far less likely to try to use cancellation in situations where it is incorrect to do so. It should be emphasized that the kind of cancellation used in the multiplication of fractions cannot be used in the addition and subtraction of fractions. For example, in $\frac{3}{4} + \frac{2}{3}$ the 3's cannot be cancelled; likewise, 2 cannot be cancelled from the numerator 2 and the denominator 4. Page 66 gives practice on the work taught on pages 63 and 65.

Key: Page 65 **4.** $\frac{7}{16}$ in.; $\frac{3}{8}$ in.

Page 66 **1.** $10 \times \frac{3}{4}$ mi. $= 7\frac{1}{2}$ mi. **2.** $\frac{2}{3} \times \frac{9}{10}$ mi. $= \frac{3}{5}$ mi. **3.** $\frac{1}{2}$ mi.; $\frac{3}{5}$ mi.; $\frac{3}{8}$ mi. **4.** $\frac{9}{16}$; $4\frac{1}{2}$; $\frac{1}{4}$; $1\frac{4}{5}$; $2\frac{1}{4}$. **5.** $\frac{5}{16}$; $6\frac{2}{3}$; $\frac{2}{5}$; $\frac{3}{8}$; $\frac{3}{8}$. **6.** $\frac{3}{16}$; $\frac{5}{12}$; $1\frac{2}{3}$; $3\frac{2}{3}$; $10\frac{1}{2}$. **7.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{2}$; $\frac{3}{4}$; $\frac{1}{64}$. **8.** 4; $1\frac{1}{2}$; $\frac{4}{15}$; $\frac{1}{32}$; $10\frac{1}{2}$.

Page 67

Aim: To reteach multiplication of a mixed number by a fraction or a mixed number

Suggestion: In the model examples in ex. 1 and 2, remind the pupils that each mixed number is first changed to an improper fraction. Then cancellation is used in each example.

Key: **3.** 4; $1\frac{1}{2}$; $2\frac{1}{4}$; $1\frac{1}{4}$. **4.** $2\frac{1}{10}$; $1\frac{1}{2}$; 5; $1\frac{3}{4}$. **5.** $2\frac{1}{4}$; 12; $2\frac{4}{5}$; 1. **6.** 15; $3\frac{1}{2}$; 4; $3\frac{1}{2}$. **7.** 4; $7\frac{1}{2}$; $\frac{3}{4}$; $2\frac{1}{5}$. **8.** 4; $9\frac{1}{10}$; $\frac{3}{4}$; $2\frac{1}{3}$. **9.** $4\frac{3}{8}$; 6; $\frac{7}{12}$; 2. **10.** $4\frac{2}{3}$; $7\frac{1}{2}$; $1\frac{3}{4}$; $2\frac{1}{2}$. **11.** $4\frac{1}{6}$; 2; 6; $\frac{9}{10}$. **12.** 6; $5\frac{5}{6}$; 3; $\frac{3}{5}$. **13.** $1\frac{4}{5}$; $22\frac{1}{2}$; $\frac{5}{9}$; $1\frac{1}{2}$.

Page 68

Aim: To sum up all types of multiplication retaught on pages 63–67

Suggestion: There are probably more exercises on this page than are needed by most pupils. You could assign the girls all exercises with odd numbers and the boys all those with even numbers.

Workbook Reference: Arithmetic Workshop, Book 7, page 22

Key: **1.** $4\frac{7}{8}$ mi.; $6\frac{1}{2}$ mi.; $8\frac{1}{8}$ mi. **2.** 5¢; $12\frac{1}{2}$¢; $1\frac{2}{3}$¢; 15¢. **3.** 18¢. **4.** $18\frac{3}{4}$ gal. **5.** 200 lb. **6.** $11\frac{1}{2}$; 15; $\frac{5}{16}$; 39. **7.** 9; $16\frac{2}{3}$; $\frac{1}{3}$; $1\frac{1}{2}$. **8.** $17\frac{1}{2}$; 1; $\frac{5}{18}$; 9. **9.** 24; 6; $\frac{1}{4}$; $17\frac{1}{2}$. **10.** $4\frac{4}{5}$; 29; $\frac{4}{5}$; $10\frac{1}{2}$. **11.** $4\frac{3}{8}$; $\frac{9}{32}$; $\frac{1}{5}$; $2\frac{7}{9}$. **12.** $7\frac{1}{5}$; $5\frac{2}{5}$; $\frac{1}{2}$; $14\frac{1}{4}$. **13.** $19\frac{1}{2}$; 3; $\frac{8}{25}$; $73\frac{1}{3}$. **14.** 2; $73\frac{3}{4}$; $1\frac{4}{5}$; $19\frac{1}{2}$. **15.** $14\frac{2}{3}$; 5; $8\frac{1}{4}$; $\frac{5}{6}$. **16.** $1\frac{1}{6}$; $1\frac{1}{2}$; $27\frac{1}{2}$; $10\frac{1}{2}$. **17.** $1\frac{7}{8}$; 22; $106\frac{2}{3}$; $10\frac{1}{2}$. **18.** 22; $8\frac{3}{4}$; $1\frac{3}{8}$; $\frac{3}{8}$.

Page 69

Aims: To present the first set of improvement tests in the multiplication of whole numbers and to review arithmetical terms

Suggestions: At the top of the page a set of three improvement tests in multiplication is given. The exercises in each test must first be copied on a sheet of paper before the test is given. In copying, the pupils should spread out the examples so that there will be ample space for working each one. The time allowance of $4\frac{1}{2}$ minutes for a test is the time allowed *after* the test begins. This time does not include the time spent in copying. The multiplication tests should be given, scored, and recorded in the same manner as the tests in addition.

Since success in arithmetic depends upon understanding the language of arithmetic, it is important for the pupils to show that they understand the meanings of such words and phrases as those given in ex. 4–35.

Key: **1.** 142,105; 172,601; 174,986; 217,913; 354,432; 403,920. **2.** 113,274; 293,656; 103,197; 226,008; 682,767; 95,340. **3.** 706,266; 132,928; 134,784; 66,209; 458,304; 300,150.

Pages 70–71

Aim: To reteach problems with several steps by giving a model for working such problems

Suggestions: Go over the model problem with the pupils and find out whether there are any questions about the procedure. Give the following problem as a class practice exercise for all pupils: "Ann and Peggy made $15.00 by making and selling candy. They divided the money equally. Peggy put all but $\frac{1}{3}$ of her share in the savings bank, where she already had $18.65. Then how much did Peggy have in the bank?" Assign pages 70 and 71 as a written lesson to be handed in and corrected. Check very carefully the form of the work as well as the answers.

Key: 3. $\frac{7}{8}$; $\frac{3}{4}$; $\frac{5}{16}$; $\frac{3}{4}$; $\frac{1}{5}$; $\frac{1}{2}$. **4.** $5\frac{2}{3}$; 4; $3\frac{1}{8}$; $2\frac{1}{2}$; $3\frac{3}{4}$; $2\frac{1}{2}$; 4; $9\frac{1}{2}$; $4\frac{3}{4}$. **6.** $\frac{3}{2}$; $\frac{11}{12}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{5}{8}$; $\frac{7}{16}$. **7.** $\frac{9}{16}$. **8.** $3\frac{5}{6}$; $\frac{5}{12}$; $5\frac{1}{6}$; 12. **9.** $7\frac{1}{12}$; $1\frac{1}{5}$; $1\frac{31}{50}$; $1\frac{1}{20}$. **10.** $5\frac{1}{4}$; $1\frac{5}{24}$; $\frac{4}{15}$; $\frac{3}{4}$. **11.** 30; $3\frac{3}{4}$; 9; $\frac{3}{32}$. **12.** $1\frac{1}{5}$; 9; 12; 7. **13.** $6\frac{5}{8}$; $\frac{3}{8}$; $21\frac{1}{3}$; 300. **14.** $1\frac{3}{4}$; 6; $1\frac{7}{8}$; $\frac{1}{2}$. **15.** $7\frac{1}{2}$; $2\frac{1}{4}$; $\frac{1}{2}$; 7. **16.** 42; $1\frac{7}{12}$; $\frac{1}{8}$; $12\frac{1}{2}$. **17.** $2\frac{1}{4}$; $\frac{1}{12}$; $6\frac{5}{6}$; $2\frac{1}{2}$. **18.** $4\frac{1}{10}$; $\frac{3}{20}$; $3\frac{7}{10}$; $4\frac{13}{20}$. **19.** $3\frac{3}{4}$; $\frac{3}{8}$; $6\frac{7}{8}$; $2\frac{9}{16}$.

Page 79

Aim: To present Problem Test 2

Workbook Reference: Arithmetic Workshop, Book 7, page 25

Key: 1. 8 × 10¢ = 80¢; 80¢ − 59¢ = 21¢. **2.** $93\frac{1}{2}$ lb. + $91\frac{3}{4}$ lb. + $89\frac{1}{4}$ lb. = $274\frac{1}{2}$ lb.; $274\frac{1}{2}$ lb. ÷ 3 = $91\frac{1}{2}$ lb. **3.** (1) 9 is 3 times 3, so 9 pencils will cost 3 times as much as 3 pencils; 3 × 20¢ = 60¢; (2) \$1.00 ÷ \$.20 = 5 (times); 5 × 3 = 15 (pencils). **4.** $8\frac{1}{4}$ lb. ÷ $\frac{3}{4}$ lb. = 11 (boxes); 11 × \$.75 = \$8.25; \$8.25 − \$1.20 = \$7.05; \$7.05 ÷ 3 = \$2.35. **5.** 82 mi. ÷ $2\frac{1}{2}$ = $32\frac{4}{5}$ mi. **6.** $1\frac{1}{2}$ yd. = $4\frac{1}{2}$ ft., 2 ft. 8 in. = $2\frac{2}{3}$ ft., $4\frac{1}{2}$ ft. − $2\frac{2}{3}$ ft. = $1\frac{5}{6}$ ft.; or $1\frac{1}{2}$ yd. = 4 ft. 6 in., 4 ft. 6 in. − 2 ft. 8 in. = 1 ft. 10 in. **7.** 72 qt. − 15 qt. = 57 qt.; 57 × \$.55 = \$31.35; 15 × \$.40 = \$6.00; \$31.35 + \$6.00 = \$37.35. **8.** From 7 A.M. Wednesday until 9 A.M. Thursday is 26 hr.; 13 min. ÷ 26 = $\frac{1}{2}$ min. **9.** 12 × \$340 = \$4080; 52 × \$75 = \$3900; \$4080 − \$3900 = \$180 more earned by Miss West. **10.** 265 mi. ÷ 14 = $18\frac{13}{14}$ mi., or about 19 mi.

Page 80

Aim: To provide a diagnostic test with suggested practice pages

Suggestions: This diagnostic test includes all the work on common fractions. You may find that some pupils need a re-explanation of one or more kinds of exercises as well as more practice. Besides the suggested practice pages, there are more sets of fraction examples on pages 328–333.

Workbook Reference: Arithmetic Workshop, Book 7, page 26

Key: 1. $9\frac{1}{2}$; $14\frac{1}{2}$; $2\frac{5}{6}$; $11\frac{3}{16}$; $8\frac{3}{10}$. **2.** $6\frac{5}{12}$; $14\frac{3}{4}$; $9\frac{23}{24}$; $15\frac{1}{4}$; $14\frac{14}{15}$. **3.** $2\frac{2}{3}$; $4\frac{3}{10}$; $5\frac{5}{16}$; $3\frac{3}{4}$; $3\frac{1}{8}$. **4.** $4\frac{3}{8}$; $\frac{4}{5}$; $5\frac{2}{3}$; $1\frac{5}{6}$; $2\frac{11}{16}$. **5.** $12\frac{3}{4}$; 64; $3\frac{3}{4}$. **6.** $\frac{25}{36}$; $\frac{5}{8}$; $\frac{3}{4}$. **7.** 1; $4\frac{2}{3}$; $2\frac{7}{9}$. **8.** 6; 33; $10\frac{1}{2}$. **9.** 27; $1\frac{1}{8}$; $\frac{9}{10}$. **10.** $2\frac{1}{4}$; $6\frac{2}{5}$; 25. **11.** $\frac{1}{16}$; $\frac{2}{3}$; 9. **12.** $\frac{4}{9}$; 3; $\frac{7}{8}$.

Chapter 3

Aims of Chapter 3. The major aims of Chapter 3 are to:

1. Review and reteach the meaning of decimals.
2. Review and reteach place value in connection with decimals.
3. Introduce the meaning and use of exponents.
4. Analyze our number system and write numbers using bases other than ten.
5. Summarize the use of Roman numerals.
6. Reteach the changing of decimals to fractions and vice versa.
7. Reteach addition and subtraction of decimals.
8. Reteach the multiplication of decimals by whole numbers.
9. Reteach the multiplication of decimals by decimals.
10. Reteach division of decimals by whole numbers.
11. Reteach rounding off decimal quotients.
12. Reemphasize multiplication and division by 10, 100, and 1000.
13. Reteach division of decimals by decimals.
14. Reteach zero in the quotient.
15. Teach relationship of the standard time zones with practical problems on travel by train.
16. Provide review of the work in Chapters 1 and 2.

Pages 81–82

Aim: To explain and give practice on the time zones of the United States

Suggestions: Most pupils know something about time zones; therefore, this work is in the nature of a review of the names of the zones. But even adults, when asked what the time is in a different zone, have trouble converting the hour. Have pupils look at the map on page 81 and ask them the questions in ex. 1–3 on that page. Then pick out other cities on the map and ask questions about them. If you are likely to have any difficulty yourself, write out your extra questions with their answers so that there will be no confusion in class.

Key: Page 81 **1.** 8 P.M. **2.** 4 P.M.; 5 P.M. **3.** 10 A.M.; 11 A.M.; 12 noon; 9 A.M. Page 82 **1.** 3 hr.; 4 P.M. **2.** 8:30 P.M.; 6:30 P.M.; 7:30 P.M. **3.** 10 P.M. **4.** 1 P.M.; 10 A.M.; 12 noon; 1 P.M.; 11 A.M.; 10 A.M. **5.** 3:30 P.M.; 4:30 P.M.; 2:30 P.M.; 1:30 P.M. **6.** 2 P.M. **7.** 1 P.M.; 12 noon; 10 A.M. **8.** 9 A.M.; 11 A.M.; 12 noon. **10.** 11:30 A.M. **11.** 11:30 A.M.

Page 83

Aim: To give a practical project in which standard time zones are used

Suggestions: Either get pupils to bring timetables of transcontinental railroads and airlines or bring some yourself. Let the pupils find the abbreviations *ET*, *CT*, *MT*, and *PT* that are given in connection with the schedules of trains and airplanes. Ask each pupil to make a table like the one on page 83 showing a train or airplane trip across the United States. Have each one make two problems about the trip he has chosen.

Key: **1.** Eastern Time; Central Time; Mountain Time; Pacific Time. **2.** Tuesday at 9 A.M.; Thursday at 11:20 A.M.; 2:20 P.M. **3.** 68 hr. 20 min.; 35 hr. 30 min.; 43 hr. 20 min.; 17 hr. 40 min. **4.** Backward; 1 hr.; 3 times. **5.** Ahead 1 hr. **6.** 3 hr. slow. **7.** The difference in time between California and New York is 3 hr. If a plane leaves California at 8 A.M. (PT), it is the same as leaving at 11 A.M. (ET). From 11 A.M. to 3:35 P.M. is 4 hr. 35 min. **8.** The difference in time between Chicago and Los Angeles is 2 hr. If the train leaves Chicago at 7 P.M. (CT), it is the same as leaving at 5 P.M. (PT). $39\frac{1}{2}$ hr. after 5 P.M. on Wednesday is 8:30 A.M. on Friday.

Pages 84–85

Aim: To review and reteach the meaning of decimals

Suggestions: Discuss with the class the writing of common fractions as decimals when the denominators are 10, 100, 1000, and so on. Read some fractions, such as "thirteen thousandths," "five hundredths," and have different pupils write them on the chalkboard as common fractions and as decimal fractions. Bring out the fact that there are as many places to the right of the decimal point as there are zeros in the denominator of the common fraction. Give some class practice in reading decimals which you write on the chalkboard. You can use ex. 2–4 on page 85 or make up new ones of the same type. Have pupils write in decimal form what you read aloud. Use ex. 5–9 for this class exercise.

Key: Page 84 **6.** 4 tenths; 79 hundredths; 176 thousandths; 6 hundredths; 1 thousandth; 1 and 46 hundredths; 5 and 385 thousandths; 2 and 9 thousandths. **7.** 8 tenths; 4 tenths; 19 hundredths; 74 hundredths; 1 and 86 hundredths; 29 and 1 tenth; 7 and 4 thousandths; 3 and 204 thousandths. **8.** 1; 6 tenths; 5 hundredths; 69 hundredths; 6 and 25 hundredths; 30 and 6 tenths; 1 and 25 thousandths; 5 and 875 thousandths.

Page 85 **2.** 3 thousand, 725 ten-thousandths; 18 thousand, 875 hundred-thousandths; 879 thousand, 43 millionths; 2 thousand, 793 and 46 thousand, 718 hundred-thousandths. **3.** 932 ten-thousandths; 614 hundred-thousandths; 405 thousand, 916 millionths; 719 and 322 thousand, 761 millionths. **4.** 8 ten-thousandths; 45 hundred-thousandths; 600 thousand, 172 millionths; 400 and 260 thousand, 4 millionths. **5.** .6354; .571824; .003; .00718; 5.86; 9.2847. **6.** .0216; .039476; .027; .48175; 3.79; 8.4309. **7.** 2.000714. **8.** .5612. **9.** 18.425.

Page 86

Aims: To review, reteach, and extend place value; to introduce the meaning and use of exponents

Suggestions: Review some of the work on place value with whole numbers. This type of exercise has been used since Grade 3 and should give no trouble. Ex. 1 shows how the value of 7 changes depending upon the place in which it stands. Relate the value to ones place each time to bring out the means of determining place value. Compare the place values of relative positions in order to generalize that any place value is ten times the place value at its right and one tenth the place value at its left.

In developing the exponent as a convenient means of showing how many times 10 is a factor of a product, draw attention to the relationship of the number of zeros in the product to the exponent. Arrange in order the place values of whole numbers expressed as powers of ten and invite extension to $10^0 = 1$. Mention the fact that the Roman numeral system does not have place value.

Key: **4.** $10^5 = 10 \times 10 \times 10 \times 10 \times 10$, or 100,000; $5^2 = 5 \times 5$, or 25; $7^3 = 7 \times 7 \times 7$, or 343; $4^4 = 4 \times 4 \times 4 \times 4$, or 256. **6.** $485 = (4 \times 10^2) + (8 \times 10^1) + 5$; $6946 = (6 \times 10^3) + (9 \times 10^2) + (4 \times 10^1) + 6$; $9247 = (9 \times 10^3) + (2 \times 10^2) + (4 \times 10^1) + 7$; $23{,}843 = (2 \times 10^4) + (3 \times 10^3) + (8 \times 10^2) + (4 \times 10^1) + 3$.

Page 87

Aims: To present the first set of improvement tests in division and to give a mixed review of the computation of Chapters 1 and 2

Suggestions: A set of improvement tests in division is given at the top of this page. In giving Test 1a, first have the pupils copy the exercises on a sheet of paper, spreading them out over the paper in order to provide ample room for working each exercise. The time assigned for this test, which is 5 min., is the time for giving the test *after* it has been copied. The method of giving, scoring, and recording this test is the same as that for the improvement tests in addition. See page 344 of the text.

You may want to assign ex. 7–15 as homework. All the pupils should work these exercises correctly. Anyone who misses more than 5 exercises should be given extra help and practice.

Key: **1.** 528; 931 R8; 224. **2.** 583 R11; 446; 879. **3.** 959; 456 R9; 227. **4.** 757; 731 R16; 487. **5.** 993 R28; 230; 662. **6.** 387 R14; 889; 785. **7.** \$17.00; 6284; $6\frac{1}{4}$. **8.** 1,384,103; $2\frac{1}{16}$; $17\frac{1}{2}$. **9.** 596; 52,439,730; $1\frac{7}{8}$. **10.** 6006; \$26.77; $4\frac{1}{2}$. **11.** 200 R59; 965; $3\frac{1}{2}$. **12.** 79 R77; $1\frac{2}{3}$; 2. **13.** 6,789,930; \$30.00; $3\frac{1}{3}$. **14.** 5308; 513 R4; $4\frac{5}{12}$. **15.** 407 R11; $2\frac{5}{24}$; 10.

Page 88

Aim: To summarize the use of the Roman numerals

Suggestions: Have each pupil make his own chart and use it to count by ones, by fives, by tens, and by twenty-fives. Try to guide pupils to generalize that the

4.

Base Ten	Base Five	Base Ten	Base Five
1	1	14	24
2	2	15	30
3	3	16	31
4	4	17	32
5	10	18	33
6	11	19	34
7	12	20	40
8	13	21	41
9	14	22	42
10	20	23	43
11	21	24	44
12	22	25	100
13	23	26	101

5.

Base Ten	Base Five	Base Ten	Base Five	Base Ten	Base Five
5	10	80	310	155	1110
10	20	85	320	160	1120
15	30	90	330	165	1130
20	40	95	340	170	1140
25	100	100	400	175	1200
30	110	105	410	180	1210
35	120	110	420	185	1220
40	130	115	430	190	1230
45	140	120	440	195	1240
50	200	125	1000	200	1300
55	210	130	1010		
60	220	135	1020		
65	230	140	1030		
70	240	145	1040		
75	300	150	1100		

7. 3 fives and 1 one; 1 five and 4 ones; 4 fives and 2 ones; 1 twenty-five, 2 fives, and 1 one; 2 twenty-fives, 4 fives, and 0 ones. **8.** 3 twenty-fives, 4 fives, and 2 ones; 2 one hundred twenty-fives, 1 twenty-five, 3 fives, and 4 ones; 1 one hundred twenty-five, 3 twenty-fives, 2 fives, and 0 ones; 2 thousands, 8 hundreds, 5 tens, and 9 ones; 8 thousands, 3 hundreds, 0 tens, and 0 ones. **9.** 2 sevens and 1 one; 1 thirty-six, 5 sixes, and 3 ones; 6 sixty-fours, 2 eights, and 7 ones; 1 two and 1 one; 1 eight, 0 fours, 1 two, and 0 ones. **11.** $(3 \times 5) + 1 = 15 + 1 = 16$; $(1 \times 5) + 4 = 5 + 4 = 9$; $(4 \times 5) + 2 = 20 + 2 = 22$; $(1 \times 5^2) + (2 \times 5) + 1 = (1 \times 25) + 10 + 1 = 25 + 10 + 1 = 36$; $(2 \times 5^2) + (4 \times 5) + 0 = (2 \times 25) + 20 + 0 = 50 + 20 + 0 = 70$. **12.** $(3 \times 5^2) + (4 \times 5) + 2 = (3 \times 25) + 20 + 2 = 75 + 20 + 2$

$= 97$; $(2 \times 5^3) + (1 \times 5^2) + (3 \times 5) + 4 = (2 \times 125) + (1 \times 25) + 15 + 4 = 250 + 25 + 15 + 4 = 294$; $(1 \times 5^3) + (3 \times 5^2) + (2 \times 5) + 0 = (1 \times 125) + (3 \times 25) + 10 + 0 = 125 + 75 + 10 + 0 = 210$; $(2 \times 5^3) + (0 \times 5^2) + (4 \times 5) + 4 = (2 \times 125) + 20 + 4 = 250 + 20 + 4 = 274$; $(3 \times 5^3) + (0 \times 5^2) + (0 \times 5) + 3 = (3 \times 125) + 3 = 375 + 3 = 378$. **13.** $(1 \times 6^2) + (2 \times 6) + 5 = (1 \times 36) + 12 + 5 = 36 + 12 + 5 = 53$; $(2 \times 7) + 1 = 14 + 1 = 15$; $(1 \times 6^2) + (5 \times 6) + 3 = (1 \times 36) + 30 + 3 = 36 + 30 + 3 = 69$; $(1 \times 2) + 1 = 2 + 1 = 3$; $(2 \times 4^2) + (1 \times 4) + 3 = (2 \times 16) + 4 + 3 = 32 + 4 + 3 = 39$. **14.** $(1 \times 2^3) + (1 \times 2^2) + (1 \times 2) + 1 = 8 + 4 + 2 + 1 = 15$; $(1 \times 2^3) + (0 \times 2^2) + (1 \times 2) + 0 = 8 + 0 + 2 + 0 = 10$; $(2 \times 6^3) + (1 \times 6^2) + (3 \times 6) + 4 = (2 \times 216) + (1 \times 36) + 18 + 4 = 432 + 36 + 18 + 4 = 490$; $(2 \times 3^3) + (1 \times 3^2) + (2 \times 3) + 2 = (2 \times 27) + (1 \times 9) + 6 + 2 = 54 + 9 + 6 + 2 = 71$; $(3 \times 4^3) + (2 \times 4^2) + (2 \times 4) + 1 = (3 \times 64) + (2 \times 16) + 8 + 1 = 192 + 32 + 8 + 1 = 233$.

Page 96

Aim: To reteach the multiplication of a decimal by a whole number

Suggestions: To show pupils that the rule for placing the decimal point in the product is correct, have several pupils add rather than multiply to find the answer. Give the example 6 × 8.31 and let some pupils write 8.31 six times in a column and add. Also have pupils estimate answers before multiplying. In the model example, 2 × 11.7, have them think "2 × 11 = 22, so the answer should be somewhat larger than 22." This estimate shows that the decimal point in the problem should be at the right of 23.

Key: **3.** 1.908; 47.6; 85.2; 66.96; .063; .125. **4.** .078; 38.28; .096; .020; 3.141; 30.12. **5.** .945; .095; 4.557; 462.0; 170.18; 4795.3. **6.** 1.085; 650.16; .384; 2478.0; 1.776; 2278.5.

Pages 97–98

Aim: To reteach the multiplication of a decimal by a decimal

Suggestions: Before assigning written work on pages 97 and 98, give some class practice on estimating answers in the multiplication of a decimal by a decimal. Go over ex. 3 and 4 on page 97, asking what the estimated answers would be. Require answers like "somewhat more than 100" or "a little more than 99," and so on. In ex. 1 on page 97 the term *knots* is used. A knot is a speed of 1 sea mile per hour. 1 sea mile = 1.15 land miles. It is thus seen that a sea mile is longer than a land mile.

Workbook Reference: Arithmetic Workshop, Book 7, page 31

Key: Page 97 (The estimates are in parentheses.) **3.** (100) 103.32; (99) 104.16; (280) 309.52; (80) 86.645. **4.** (48) 50.530; (56) 52.780; (240) 257.92; (54) 56.2005.

Page 98 **1.** 391.800 mi. or 391.8 mi. **2.** 119.60 mi. or 119.6 mi. **3.** \$1.82. **4.** \$11.25. **5.** The second answer should be 7.3125; the fourth, 518.16. **6.** .388;

.585; .437; 2.8542. **7.** .266; 5.865; 80.04; 1.2054. **8.** 2.438; 18.69; .4644; 350.90. **9.** .1647; 4.968; 10.72; 1.0396. **10.** 3.712; .672; .874; 3.6396. **11.** 68.08; 89.28; 42.78; 15.130. **12.** 30.07; 9.50; .1162; 64.206. **13.** 6.889; 2.538; 3.128; .22560. **14.** 1.064; .6375; 25.90; 130.41. **15.** .2556; 5.846; .741; 20.202.

Page 99

Aims: To present the second set of improvement tests in addition and to review the vocabulary of arithmetic

Suggestions: The second set of improvement tests in addition, which is given on this page, is to be administered in the same manner as the first set of such tests, which was given on page 47 of the text. Remember that an improvement test in addition is not to be copied. Instead, the answers are to be written on folded paper, as explained on page 48. Make the vocabulary study an oral exercise with a pupil going to the chalkboard to write an example of each word or group of words. In the case of *estimate*, he should give an illustration of an estimate in an example such as 4.3×6.1, where the answer would be somewhat more than 24. For *time zone* he should be able to describe the time zones in the United States and give their names.

Key: **1.** 2668; 3852; 4016; 3467; 4245; 4357. **2.** 5159; 3766; 2777; 3093; 3169; 4144. **3.** 3344; 3056; 3088; 4327; 4393; 3865.

Page 100

Aim: To reteach the zero cases in the multiplication of decimals

Suggestions: Have several exercises done at the chalkboard in both the decimal form and the common fraction form. Use the exercises $.06 \times .09$, $.03 \times .015$, $.09 \times .007$, and so on. Use estimating for ex. 6–9 and have the exercises done orally in class. Here are a few more such exercises if you need them:

$.4 \times 1.36 = 5.44$ or $.544$ or $.0544$
$.8 \times .76 = .608$ or 6.08 or 60.8
$.9 \times .013 = .117$ or 11.7 or $.0117$
$.12 \times .16 = .0192$ or $.192$ or 1.92
$.31 \times .15 = 46.5$ or $.465$ or $.0465$

Workbook Reference: Arithmetic Workshop, Book 7, page 32

Key: **3.** .015; .0072; .00565; .0213. **4.** .084; .0027; .0048; .0016. **5.** .006; .075; .00024; .0189. **6.** 2.943. **7.** .094. **8.** .0399. **9.** .0072.

Pages 101–103

Aim: To reteach the division of a decimal by a whole number when there is no remainder, and the method of avoiding a remainder in such division by annexing zeros in the dividend

Suggestions: All the exercises on page 101 come out without a remainder, and the decimal point in the quotient is placed above the decimal point in the dividend. On page 102 the annexing of zeros in the dividend to avoid fractions in the quotients is presented. Some pupils may have difficulty annexing zeros to a whole number. An example that pupils will know from everyday life is \$10 or \$10.00, both meaning the same thing. So in a whole number a decimal point may be placed at the right of ones place, and any number of zeros may then be annexed. Ex. 4 and 5 on page 102 should have careful study since they show that when a 3-place decimal represents money, such as \$.051, the 1 in \$.051 represents $\frac{1}{10}$ of a cent.

Workbook Reference: Arithmetic Workshop, Book 7, page 34

Key: Page 101 **2.** 48.3 + 46.5 + 50.1 + 47.8 + 48.9 + 51.6 + 50.5 = 343.7 (lb.); 343.7 ÷ 7 = 49.1 (lb.). **3.** 3.9; 23.8; 9.46; 1.146; 4.3. **4.** .92; .934; 88.8; .3278; 1.25. **5.** 7.6; .437; .358; .219; 42.6. **6.** .17; .147; .94; 42.6; 7.3. **7.** 4.9; 28.9; 8.3; 11.8; .29. **8.** 8.4; 8.61; .71; 5.13; 40.8. **9.** .67; .835; 5.9; .232; .72.

Page 102 **2.** 10.25; 11.75; 2.88; 3.75; 4.6. **3.** 9.5; 16.8; 11.25; 2.96; 37.5. **6.** $7\frac{1}{2}$¢, \$.07$\frac{5}{10}$, \$.07$\frac{1}{2}$; \$4.25$\frac{5}{10}$, \$4.25$\frac{1}{2}$; $8\frac{7}{10}$¢, \$.08$\frac{7}{10}$; $16\frac{2}{5}$¢, \$.16$\frac{4}{10}$, \$.16$\frac{2}{5}$; \$11.12$\frac{5}{10}$, \$11.12$\frac{1}{2}$; $73\frac{1}{5}$¢, \$.73$\frac{2}{10}$, \$.73$\frac{1}{5}$; \$.08$\frac{5}{10}$, \$.08$\frac{1}{2}$, \$.085.

Page 103 **1.** \$2.37$\frac{1}{2}$; \$2.375. **2.** \$.87$\frac{4}{5}$; \$.878. **3.** \$.17$\frac{1}{4}$; \$.1725; yes. **4.** \$.18$\frac{3}{4}$; \$.1875; yes. **5.** \$1.0625; \$1.1775; \$.065; \$.123; \$.1575; \$.132. **6.** \$3.185; \$2.1025; \$.1675; \$.757; \$.366; \$.125. **7.** 7.4; 31.75; 99.25; 53.875; 39.75. **8.** 4.5; 57.5; 31.125; 88.8; 116.4. **9.** 19.5; 28.4; 107.2; 15.25; 3.0725. **10.** 15.5; 13.125; 1.2825; 61.75; 31.6. **11.** 11.5; 92.5; 94.8; 58.5; 5.1125. **12.** .65; 7.55; 8.26; 2.25; 7.66. **13.** .46; 3.325; 9.525; .65; 6.24.

Pages 104–105

Aim: To reteach rounding off decimals

Suggestions: In class place some mixed decimals on the chalkboard and ask pupils to round off to the nearest tenth or hundredth or thousandth. Here are some extra exercises to use — for nearest tenth: 6.34, 8.29, 7.51, 2.75, 1.98, 4.03; for nearest hundredth: 1.246, 8.017, 3.995, 7.621, 29.096, 15.953; for nearest thousandth: .5164, 3.0876, 2.7774, .0097, .0299. Point out in connection with ex. 11 on page 105 that nearest cent is the same as nearest hundredth of a dollar.

Workbook Reference: Arithmetic Workshop, Book 7, pages 35 and 36

Key: Page 104 **5.** 4.8; 1.1; 3.0; 71.9; 2.1; 1.0; 1.4; 6.0; 3.8. **6.** .72; .28; 3.46; 1.40; 6.07; 4.30; 7.12.

Page 105 **1.** 1.6; .1; 19.5; 1.1; .7. **2.** 1.8; 4.5; 9.8; .6; .5. **3.** 5.6; 1.3; 8.8; .2; 1.3. **4.** 17.3; 2.9; 11.2; .6; 7.5. **5.** .98; .05; .02; 1.12; .57. **6.** .44; .04; .26; 1.12; .58. **7.** .96; .17; 1.19; .18; 1.42. **8.** .10; .06; 7.51; .20; .08. **9.** 183.33 mi. **10.** 17.6 mi. **11.** \$.34; \$.40.

Pages 106–107

Aims: To reteach changing fractions to decimals and to extend the work to include repeating decimals and also the comparison of certain common fractions with decimals

Suggestions: On page 51 of the text it was pointed out in ex. 1 and 3 that a common fraction can have three different meanings. One of these meanings is that a fraction can indicate division; that is, $\frac{5}{8}$ means $5 \div 8$. This meaning of a fraction is used when you change fractions to decimals. On page 106 the explanations in ex. 1 and 2 should have careful study. In ex. 5 the fractions and their decimal equivalents should be memorized. On page 107 the repeating decimals given in ex. 7 represent an interesting arithmetical recreation; the answers to many of the fractions in ex. 8 and 9 are also interesting repeating decimals.

Workbook Reference: Arithmetic Workshop, Book 7, page 37

Key: Page 106 **4.** .83; .71.

Page 107 **1.** .6; .875; .25; .375; .65; .8; .76; .45; .3; .44; .85. **2.** .3; .3; .2; .8; .4; .9; .6; .7; .8; .3; .4. **3.** .13; .14; .89; .43; .17; .56; .87; .36; .55; .74; .73. **4.** .76; $\frac{3}{8}$; .55; .26. **5.** $\frac{4}{5}$; $\frac{5}{8}$; $\frac{17}{20}$; $\frac{9}{10}$. **6.** $\frac{1}{4}$; .822; .26; $\frac{18}{25}$. **8.** .888888; .666666; .111111; .571428; .363636; .162162; .121212; .270270; .161616; .703703; .027777. **9.** .333333; .555555; .285714; .833333; .513513; .370370; .733333; .722222; .515151; .545454; .010101.

Page 108

Aim: To provide a written review of previous computation

Suggestions: The whole page of exercises need not be assigned to all pupils. You might assign only part of ex. 1–5 and all the fraction work or any combination you see fit. Be sure that every pupil does at least one of each different kind of example.

Key: **1.** 2679; 2205; 2132; 1901; 2542; 1636; 2789. **2.** 6.03; 74.8; 1.790; 10.61; 1.046; 8.01; 96.9. **3.** $30\frac{1}{4}$; $106\frac{5}{6}$; $89\frac{1}{4}$; $100\frac{3}{5}$; $48\frac{11}{12}$; $43\frac{1}{10}$; $70\frac{1}{5}$. **4.** .124; 27.5; 1.24; 86; 3006. **5.** .82; .145; 23.7; 49; 994. **6.** $\frac{1}{4}$; 9; 132; $11\frac{1}{8}$. **7.** $1\frac{5}{8}$; $2\frac{1}{2}$; $2\frac{1}{2}$; $6\frac{3}{8}$. **8.** $\frac{4}{25}$; $\frac{5}{6}$; $1\frac{11}{16}$; $1\frac{1}{12}$. **9.** $1\frac{1}{12}$; 11; $1\frac{1}{5}$; $5\frac{11}{12}$. **10.** $\frac{1}{8}$; $31\frac{1}{2}$; 4; $10\frac{7}{8}$. **11.** $\frac{1}{6}$; $4\frac{1}{2}$; $6\frac{1}{6}$; $3\frac{1}{3}$. **12.** 9; $7\frac{1}{2}$; $3\frac{3}{4}$; $3\frac{1}{2}$. **13.** 18; 60; $2\frac{2}{9}$; $1\frac{1}{2}$. **14.** $9\frac{3}{5}$; 24; $4\frac{5}{8}$; $9\frac{1}{10}$.

Page 109

Aim: To give a set of improvement tests in subtraction

Suggestions: The second set of improvement tests in subtraction is given on this page. Since these are subtraction tests, the answers are to be written on folded paper. See page 48 of the text. Have pupils get out the graphs they made of their scores on Subtraction Tests 1a, 1b, and 1c and check on their improvement.

Key: **1.** \$70.99; \$78.89; \$61.02; \$48.95; \$47.09. **2.** \$40.10; \$75.62; \$62.57; \$50.94; \$46.74. **3.** \$13.98; \$55.80; \$17.21; \$72.33; \$29.89. **4.** \$30.42; \$31.69; \$37.68; \$50.44; \$12.59. **5.** \$16.78; \$5.42; \$2.63; \$20.86; \$48.37. **6.** \$13.20; \$50.82; \$77.53; \$56.06; \$22.54. **7.** \$44.02; \$58.69; \$11.79; \$38.68; \$28.68. **8.** \$27.54; \$42.16; \$27.66; \$10.77; \$66.68. **9.** \$69.69; \$51.34; \$14.53; \$27.05; \$13.35.

Page 110

Aim: To review multiplication and division by 10, 100, and 1000

Suggestion: Go over this page in class and make up more examples in multiplication or division with 10, 100, or 1000. Have pupils learn the two rules.

Key: **2.** (By 10) \$12.80, \$132.90, 64, 9, 87, .2, \$31.75, 272.045; (by 100) \$128, \$1329, 640, 90, 870, 2, \$317.50, 2720.45; (by 1000) \$1280, \$13,290, 6400, 900, 8700, 20, \$3175, 27,204.5. **4.** (By 10) 4.8, 32.1, .64, \$13.40, \$2.30, \$123.70, \$142.735; (by 100) .48, 3.21, .064, \$1.34, \$.23, \$12.37, \$14.2735; (by 1000) .048, .321, .0064, \$.134, \$.023, \$1.237, \$1.42735.

Page 111

Aim: To review division by a decimal when there is no remainder

Suggestions: Go over ex. 1 and 2 in class to make sure that all pupils know how to proceed. Have two or three exercises from rows 3–6 put on the chalkboard so that you can clarify the procedure for any pupils who might have difficulty.

Workbook Reference: Arithmetic Workshop, Book 7, page 39

Key: **3.** 56; 49; 13; 18; 14. **4.** 36; 39; 12; 13; 41. **5.** 58; 97; 12; 16; 37. **6.** 8 wk.

Page 112

Aim: To reteach division by a decimal when there is no remainder by using the caret

Suggestions: If the use of the caret is avoided in your school, rewrite the model example as in B and proceed with the division as on page 111.

Ex. 1 shows how to divide by a decimal without rewriting the example with the divisor as a whole number. The work is accomplished by making use of the caret, as explained in ex. 1. When examples with decimal divisors are worked by rewriting the example so that the divisor will be a whole number, as was done on page 111, there is always the possibility that an error may be made in copying the numbers. Likewise, the rewriting requires extra time. The possibility of error is avoided, and the extra time is saved, if the work is done as shown on page 112.

Workbook Reference: Arithmetic Workshop, Book 7, page 41

Key: **2.** 3.61; 2.40; 52.89; 13.26. **3.** .88; 5.36; 165.94; 1.60. **4.** 25.11; 15.77; 15.80; 3.82. **5.** .74; 2.95; 7.68; 20.97. **6.** .65; 29.76; 7.32; 57.93.

Page 113

Aim: To present a project which uses the rounding off of quotients where divisors are decimals

Suggestion: Page 113 gives more of the work done on pages 111 and 112 in the form of a project and abstract exercises. It sums up the most difficult topic in the division of decimals.

Workbook Reference: Arithmetic Workshop, Book 7, page 40

Key: **1.** 37.7 mi. **2.** 19.1 mi. **3.** 38.9 mi. **4.** 231.8; 6.9; 2.1; 54.2. **5.** 42.8; 274.1; 3.0; 1.0. **6.** .6; .3; .6; 181.7. **7.** 3.4; 6.3; 17.5; 1.4.

Pages 114–115

Aim: To extend work on distance-rate-time problems

Suggestions: The formulas at the bottom of page 115 should be carefully explained. These formulas are really the outcome of the pupils' experience in solving the problems on page 115. They can be used to solve distance-rate-time problems. Have the pupils solve some problems using the formulas.

Key: Page 114 **1.** .2 mi. **2.** 1.0 mi.; 2.0 mi.; 3.0 mi. **3.** 12 mi. **4.** .19 mi. per min.

Page 115 **1.** 13 mi.; $16\frac{1}{4}$ mi.; $19\frac{1}{2}$ mi. **2.** 95 mi. ÷ $2\frac{1}{2}$ = 38 mi. **3.** 2 hr. **4.** 945 mi. ÷ 3.25 = 290.76 mi., or 290.8 mi. **5.** $4\frac{1}{4}$ hr. = 4.25 hr.; 500 ÷ 4.25 = 117.647 (mi.), or 117.65 (mi.). **6.** 280 ÷ 35 = 8 (hr.); 8 hr. + 1 hr. = 9 hr. **7.** 8193 ÷ 30 = 273.1 (mi.). **8.** (1) 3 hr. 36 min. = 3.6 hr., 3.6 × 55 mi. = 198 mi.; (2) 374 mi. ÷ 55 mi. = 6.8 (hr.); 6.8 hr. = 6 hr. 48 min. **9.** From 2:10 P.M. to 10:55 P.M. is 8 hr. 45 min.; 8 hr. 45 min. = 8.75 hr.; 329.9 mi. ÷ 8.75 = 37.70 mi., which is called 37.7 mi. **10.** (1) 355 mi. ÷ 60 = $5\frac{11}{12}$ mi.; (2) $5\frac{11}{12} \times 5280$ ft. = 31,240 ft.; 31,240 ft. ÷ 60 = $520\frac{2}{3}$ ft.

Pages 116–117

Aim: To reteach and give practice on the zero difficulties in quotients where the example has a decimal divisor or dividend or both

Suggestions: This type of problem is often difficult for pupils to grasp. Problems such as ex. 2 on page 116 and ex. 1–3 on page 117 help pupils to understand what kind of answer to expect. For example, if the divisor is a decimal such as \$.05, and the dividend is \$16, ask pupils whether the answer will be a larger number or a smaller number than the number in the dividend. Since \$.05 is contained in \$1 twenty times, how many times is it contained in \$2? in \$3? in \$5? in \$10? in \$16? This will help to establish the idea that when the divisor is a decimal less than 1, the quotient will be larger than the dividend.

Workbook Reference: Arithmetic Workshop, Book 7, page 42

Key: Page 116 **2.** 7845 fares. **3.** .08; 500; .042; 1900; 400. **4.** .09; 800; .007; 4800; 3000. **5.** 120; 60; 320; .09; 12,000. **6.** 70; 90; .039; 2400; 1100. **7.** .06; 1800; 720; .09; 400. **8.** .07; 900; 250; 1200; 4000.

Page 117 **1.** \$6.60 ÷ 240 = \$.0275, or $2\frac{3}{4}$¢. **2.** \$16 ÷ \$.05 = 320 (papers). **3.** (1) \$2 ÷ 500 = \$.004, or $\frac{2}{5}$¢; (2) 30 × $\frac{2}{5}$¢ = 12¢. **4.** .8; .08; 80; 80; 800. **5.** .05; 5; 50; 500; .5. **6.** 9; 90; .9; 900; 90. **7.** 40; .4; .4; 400; 4. **8.** .6; .6; 60; 60; 600. **9.** .7; 7; 70; 700; .7. **10.** 860.; 1380.; 7300.; 42,500. **11.** .037; .053; .091; 1070. **12.** 940.; 6200.; 208,000.; 214,900. **13.** .075; 490.; 90.; .0373.

Page 118

Aim: To sum up the four processes with decimals that were retaught in Chapter 3

Suggestions: This review is in preparation for the diagnostic test on page 120. You should check carefully all mistakes made on these review exercises and take up those mistakes with the individual pupils, reteaching topics if necessary.

Key: **1.** 2 and 5 tenths, $2\frac{1}{2}$; 7 and 25 hundredths, $7\frac{1}{4}$; 4 and 375 thousandths, $4\frac{3}{8}$; 11 and 7 tenths, $11\frac{7}{10}$; 9 tenths, $\frac{9}{10}$; 15 hundredths, $\frac{3}{20}$. **2.** .6; .45; .875; .3; .6875; .19; .033; .28. **3.** .37; .55; .15; .22; .87; .81; .71; .63. **4.** CMXLVII; MDCCCXXXII; MMCDLXV; $\overline{\text{XXXV}}$DCCCXLVI. **5.** 18.11; 14.250. **6.** 28.163; \$195.05. **7.** .144; .030; .02021; 87.00; .00096; 126.80. **8.** 7.6; 2.1375; 38.75; .9; .043. **9.** 9.5; 73.5; 32.8; 1.7625; 12.75. **10.** 11.6; 1.3; 54.1; 9.1; 1.6. **11.** .07; .55; .10; 22.42; 8.19. **12.** .03; .41; .41; 8.91; 14.02.

Page 119

Aim: To present Problem Test 3

Workbook Reference: Arithmetic Workshop, Book 7, page 43

Key: **1.** 5.6 hr. **2.** 62 + 59 + $57\frac{1}{2}$ = $178\frac{1}{2}$ (in.); $178\frac{1}{2}$ in. ÷ 3 = $59\frac{1}{2}$ in. **3.** 15 is 5 times as many as 3; 5 × 10¢ = 50¢. **4.** \$2.50 − \$1.96 = \$.54; \$.54 ÷ 2 = \$.27. **5.** $2\frac{1}{2} + 1\frac{3}{4} + 2\frac{1}{4} + 2\frac{3}{4} + 5\frac{1}{4} = 14\frac{1}{2}$ (hr.); $14\frac{1}{2}$ × \$.60 = \$8.70; \$8.70 ÷ 5 = \$1.74. **6.** 26 × \$.62 = \$16.12; \$16.12 − \$10.65 = \$5.47. **7.** 27 in. = $\frac{3}{4}$ yd.; 1 yd. 9 in. = $1\frac{1}{4}$ yd.; $\frac{3}{4}$ × \$1.25 = \$.93$\frac{3}{4}$, which is called \$.94; $1\frac{1}{4}$ × \$.50 = \$.62$\frac{1}{2}$, which is called \$.63; \$.94 + \$.63 = \$1.57. **8.** (1) 3 × \$1.45 = \$4.35; \$4.35 + \$2.39 = \$6.74; \$10.00 − \$6.74 = \$3.26; (2) 1 penny, 1 quarter, and 3 one-dollar bills. **9.** $1\frac{1}{4}$ cups flour, $\frac{1}{4}$ cup butter. **10.** 4 bu. = 16 pk.; 16 pk. − 10 pk. = 6 pk.; 10 × \$.75 = \$7.50; 6 × \$.60 = \$3.60; \$7.50 + \$3.60 = \$11.10.

Page 120

Aim: To give a diagnostic test with page references for remedial work

Workbook Reference: Arithmetic Workshop, Book 7, page 44

Key: **1.** .875; .8; .5625; .56; .85. **2.** .08; .55; .91; .46; .26. **3.** 17.45; 15.125; 570.7; \$60.74. **4.** 1.7; 192.9; .375; \$7.06. **5.** 192.60; 3.3705; .0639; 16.200. **6.** 3.875; .68; .35; .31. **7.** .078; .031; .047; .076. **8.** .08; 730; 2900; 1590. **9.** 24.1; 75.2; 1.7; 3.1.

Chapter 4

Aims of Chapter 4. The major aims of Chapter 4 are to:

1. Review the meaning of per cent.
2. Review the reading and writing of per cents.
3. Reteach the finding of a per cent of a number.
4. Teach the use of fractions in place of certain equivalent per cents.
5. Teach simple discount and the special vocabulary of discount.
6. Teach rounding off answers to the nearest cent when finding a per cent of a given number of dollars and cents.
7. Teach estimating answers where a per cent of a number is to be found.
8. Teach simple work on commission and the special vocabulary of commission.
9. Teach the use of a per cent ending in a fraction.
10. Teach the use of per cents over 100%.
11. Teach the meaning of special terms used in percentage.
12. Review the work on time zones and timetables in connection with air travel.
13. Review problem solving and extend it to applications of percentage.

The Teaching of Percentage. From the standpoint of computation, the subject of percentage is merely another application of decimals. Thus there are no new operations to be learned as there are in a subject like common fractions. The knowledge of decimals which the pupil has already acquired will serve all his needs in percentage. In fact, most of the actual work that the pupil will do in percentage will involve easier computation with decimals than much of that which he has already done.

From the standpoint of language, however, percentage is *entirely new*. In fact, the teaching of percentage is largely a series of language lessons. Percentage is new also with respect to its symbolism; that is, with respect to its method of expressing *hundredths* by means of the symbol %. Success in percentage requires that the pupil fully appreciate the fact that 37% is another way of writing .37 or $\frac{37}{100}$. Whenever a pupil sees 37%, he must understand that he can substitute .37 for it. Likewise, when he sees .37, he must be able to change it to 37%. The ability to make this rapid change from a decimal representing *hundredths* to the per cent symbolism, and vice versa, is one of the important skills that must be acquired early in the study of percentage.

In teaching percentage you must clearly understand that your task is not to teach pupils to *compute* with decimals that represent per cents. You must teach them to *interpret* per cents and to understand their significance. Such interpretations are frequently necessary in situations where no computation at all is required. For example, in reading a newspaper you are often required to interpret a statement such as "All coats are to be sold at 25% off" or "The population has increased 5%"; yet in these instances you make no computations. In fact, for many people, percentage is usually applied in *interpreting* what they read.

Pages 121–123

Aim: To present a project on travel by air, including standard time zones and the cost of tickets

Suggestions: Review the work on pages 81 and 82 of the text, where an explanation is given of standard time zones and the abbreviations *PT*, *MT*, and so on. The cost of the tickets on page 123 requires the inclusion of a 5% tax. Pupils have studied how to find a per cent of a number in Grade 6, so the work on this page introduces a review of percentage.

Such special fares for air travel as the Family Plan vary from time to time. If possible, have pupils find out from ticket offices or newspapers the special fares that are currently available.

Key: Page 121 **1.** Pacific Time, Central Time, Eastern Time; Mountain. **2.** 10:00 A.M.; 1:00 P.M. **3.** 3:25 P.M., 4:00 P.M.; 1:25 P.M.; 4:25 P.M. **4.** 5:50 P.M.; 2:50 P.M.; 4:50 P.M.

Page 122 **1.** The difference in time between Los Angeles and Detroit is 3 hr.; if a plane leaves Los Angeles at 10:00 A.M. (PT), the time in Detroit is 1:00 P.M.; from 1:00 P.M. to 5:50 P.M. is 4 hr. 50 min. **2.** (1) 4 hr. 50 min. − 35 min. = 4 hr. 15 min.; (2) 4 hr. 15 min. = 4.25 hr.; 1983 mi. ÷ 4.25 = 466.5 mi., which is called 467 mi. **3.** When a plane leaves Los Angeles at 5:00 P.M. (PT), the time in Detroit is 8:00 P.M.; 4 hr. 50 min. after 8:00 P.M. is 12:50 A.M. of the following day. **4.** (1) If a plane leaves Detroit at 11:35 P.M. (ET), the time in Los Angeles is 8:35 P.M. (PT); from 8:35 P.M. to 1:50 A.M. is 5 hr. 15 min.; (2) no; (3) 5 hr. 15 min. − 4 hr. 50 min. = 25 min. **5.** (1) If a plane leaves Los Angeles at 9:00 A.M. (PT), the time in Washington is 12:00 noon; from 12:00 noon to 4:30 P.M. is 4 hr. 30 min.; (2) 4 hr. 30 min. = 4.5 hr.; 2300 mi. ÷ 4.5 = 511.1 mi., which is called 511 mi. **6.** If a plane leaves Boston at 8:30 P.M. (ET), the time in San Francisco is 5:30 P.M.; 6 hr. 45 min. after 5:30 P.M. on Monday is 12:15 A.M. on Tuesday. **7.** If a plane leaves New York at 9:00 A.M. (ET), the time in Phoenix is 7:00 A.M.; 7:00 A.M. to 12:50 P.M. is 5 hr. 50 min.; from 10:05 A.M. to 10:45 A.M. is 40 min.; 5 hr. 50 min. − 40 min. = 5 hr. 10 min.

Page 123 **1.** \$161.91. **2.** $\frac{2}{3}$ × \$161.91 = \$107.94; 2 × \$107.94 = \$215.88; $\frac{1}{2}$ × \$161.91 = \$80.96; \$161.91 + \$215.88 + \$80.96 = \$458.75. **3.** .05 × \$120.30 = \$6.0150, which is called \$6.02; \$120.30 + \$6.02 = \$126.32. **4.** 3 × \$126.32 = \$378.96; $\frac{1}{2}$ × \$126.32 = \$63.16; \$378.96 + \$63.16 = \$442.12. **5.** 3 × \$.73 = \$2.19; 5 × \$.73 = \$3.65; 8 × \$.73 = \$5.84; 10 × \$.73 = \$7.30. **6.** (1) .05 × \$186.90 = \$9.35; \$186.90 + \$9.35 = \$196.25; (2) .05 × \$145.10 = \$7.26; \$145.10 + \$7.26 = \$152.36; (3) \$196.25 − \$152.36 = \$43.89.

Pages 124–125

Aim: To review the meaning of per cent and the language peculiar to percentage

Suggestions: The meaning of per cent is explained in ex. 1 on page 124. It should be made clear to the pupils that 83% is a new way of writing $\frac{83}{100}$ or .83.

Likewise, 7% is a new way of writing $\frac{7}{100}$ or .07. It will be well for you to write on the board the four different ways of writing 83%. These ways are as follows: $\frac{83}{100}$ = .83 = 83% = 83 per cent. Make clear to the pupils that any one of the four expressions may be exchanged for another. In this connection ex. 3 and 4 on page 124 are important. You should remember that any new method of writing or expressing a number becomes familiar to the pupil only by much practice in using it. Ex. 5–10 on page 124 represent informational uses of per cent. On page 125 the meaning of per cent is further illustrated through diagrams in which large squares are divided into 100 small squares. More work on diagrams of per cents may be assigned for a class exercise. Supply each pupil with a sheet of graph paper. Have the pupils mark off several large squares, each of which is 10 by 10 small squares. Ask them to color different per cents of each one.

Workbook Reference: Arithmetic Workshop, Book 7, page 45

Key: Page 124 **3.** 19%; 37%; 80%; 52%; 75%; 9%; 1%. **4.** 17%; 40%; 33%; 25%; 1%; 8%; 2%.

Page 125 **1.** 1%. **2.** 7%; 93 small squares; subtract 7 from 100; 93%. **3.** B: red, 28%; white, 72%. C: red, 50%; white, 50%. **5.** 75%; 59%; 50%. **6.** 90%; 75%.

Page 126

Aim: To review how to read and write per cents

Suggestions: This page can be a class exercise. To show what is wanted, write $\frac{39}{100}$ on the blackboard and have one pupil write the sign = followed by another equivalent form of this fraction, such as .39. Have a second pupil proceed in the same way, writing a second = sign followed by a still different form of the fraction. Continue until all four forms are written. Any order of these equivalents will be acceptable.

Key: **1.** .39, 39%, 39 per cent; .43, 43%, 43 per cent; .57, 57%, 57 per cent; .13, 13%, 13 per cent; .09, 9%, 9 per cent; .07, 7%, 7 per cent; .01, 1%, 1 per cent. **2.** 19%; 25%; 70%; 8%; 5%; 1%. **3.** 33%; 75%; 22%; 38%; 3%; 8%. **4.** .72; .11; .60; .15; .02; .06. **5.** .75; .43; .68; .08; .04; .01. **6.** .16; .09; .02. **7.** $\frac{3}{4}$. **8.** $\frac{1}{2}$. **9.** $\frac{1}{10}$. **10.** $\frac{1}{4}$. **11.** $\frac{9}{10}$. **12.** $\frac{4}{5}$. **13.** 50% each.

Page 127

Aim: To present the second set of improvement tests in multiplication and the second set in division

Suggestion: In giving the improvement tests on this page, remember to have each test copied on paper before the test begins. The time assignment for each test is the time actually allowed for taking it *after* the copying has been completed.

Key: **1.** 261,348; 286,902; 179,014; 12,954; 214,912; 297,360. **2.** 191,488; 65,610; 511,602; 92,106; 156,137; 726,680. **3.** 90,402; 266,975; 366,268; 69,641;

106,708; 103,040. **4.** 824; 809 R13; 264. **5.** 804; 766 R9; 434. **6.** 263 R18; 306; 868. **7.** 895; 352 R16; 862. **8.** 583 R11; 364; 147. **9.** 706 R6; 524; 994.

Pages 128–129

Aim: To reteach how to find a per cent of a number

Suggestions: On these pages the pupil is shown that to find 62% of a number, he can change 62% to .62 and multiply the number by .62. This multiplication by a decimal is work he already knows how to do, so finding a per cent of a number becomes a very simple process. The problem work at the top of page 129 gives further practice on the language of percentage.

Workbook Reference: Arithmetic Workshop, Book 7, page 46

Key: Page 128 **2.** 36. **3.** .64; 96. **4.** .30; 96. **5.** .02; 12. **6.** 21; 24; 12; $38.00. **7.** 9; 10; 21; $48.75. **8.** 38; 3; 23; $27.50. **9.** 42; 36; 20; $70.00. **10.** 14; 14; 6; $93.00. **11.** 42; 24; 63; $15.60.

Page 129 **1.** .92 × 25 = 23 (problems). **2.** 22 girls. **3.** 19 answers; 1 answer. **4.** 1548 pupils. **5.** 3 oranges. **6.** 90 chicks. **7.** 9; $14.00; $320.00. **8.** 49; $13.00; $1098.00. **9.** 12; $22.00; $279.00. **10.** 21; $57.00; $2520.00. **11.** 41; $6.00; $1560.00. **12.** 18; $21.00; $552.00. **13.** 68; $29.00; $2640.00. **14.** 3; $45.00; $1192. **15.** 36; $6.00; $245.00. **16.** 9; $36.00; $240.00. **17.** 6; $6.00; $327. **18.** 12; $63.00; $255.00.

Page 130

Aim: To review the meaning of 100% and its applications in everyday life

Suggestions: The expression "100%" is a good illustration of the language element in percentage. As ordinarily used, 100% is the equivalent of the word *all*. The pupils may be asked to bring to class illustrations of the use of 100% as given in ex. 7–12.

Workbook Reference: Arithmetic Workshop, Book 7, page 48

Page 131

Aim: To show how the whole, or 100%, is made up of parts that total 100%

Suggestion: Give considerable attention to work of this kind. Begin by giving problems such as "42% of this class are boys. What per cent are girls?" "53% of the pupils in a class had the flu. What per cent did not have the flu?" Then ask each pupil to make up one good problem of this kind.

Key: **2.** 90%. **3.** 10%. **4.** 75%. **5.** 80%. **6.** 10% + 30% = 40%; 100% − 40% = 60%. **7.** 50% + 20% = 70%; 100% − 70% = 30%.

Pages 132–136

Aim: To teach the fractional equivalents of certain frequently used per cents

Suggestions: Notice that page 132 uses only 25% and that page 133 brings in the other two very easy per cents, 50% and 75%, and their equivalent fractions. Since these pages are a review, some pupils will already know these short cuts. Page 134 gives practical problems using 25%, 50%, and 75%. Pupils soon realize why these methods are called "short cuts." Pages 135 and 136 present other common per cents and their equivalent fractions. These short-cut equivalents should be learned because they will be used often.

Workbook Reference: Arithmetic Workshop, Book 7, page 49

Key: Page 132 **1.** 7876 mi. **3.** \$5; \$20; \$25. **4.** \$9; \$11; \$30. **5.** \$4.50; \$12.50; \$50. **6.** \$8; \$6.25; \$120.

Page 133 **3.** 1; 27; 7; 30. **4.** 1; 11; 15; 9. **5.** 6; 13; 8; 9. **6.** 3; 21; 7; 9. **7.** \$23.25. **8.** $\frac{1}{4} \times \$50 = \$12.50$; $\frac{1}{4} \times \$38.60 = \$9.65$; $\$12.50 + \$9.65 = \$22.15$. **9.** 69; \$32.50; \$5.75. **10.** 19; \$107.50; \$12.18. **11.** 49; \$120; \$5.09. **12.** 17; \$155.50; \$28.11. **13.** 72; \$44.50; \$12.50. **14.** 44; \$225; \$3.15.

Page 134 **1.** $\frac{1}{4} \times \$.84 = \$.21$; $45 \times \$.21 = \$9.45$. **2.** $\frac{1}{4} \times 680 = 170$ (new members). **3.** (1) $\frac{3}{4} \times \$50.00 = \$37.50$; (2) $\frac{3}{4} \times \$45.80 = \$34.35$; (3) $\$45.80 - \$34.35 = \$11.45$. **4.** $\frac{1}{2} \times \$425 = \$212.50$; $12 \times \$212.50 = \$2550.00$. **5.** $\frac{1}{4} \times \$2680 = \$670$. **6.** $\frac{1}{2} \times 638 = 319$ (pupils). **7.** $\frac{1}{2} \times 976 = 488$ (pupils).

Page 135 **4.** 15; 12; 20. **5.** 14; 9; 25. **6.** 12; 14; 2. **7.** 6; 5; 7. **8.** 45; 180; 106. **9.** 18; 23; 301. **10.** 52; 24; 96. **11.** 69; 125; 75.

Page 136 **1.** \$.58; \$57; \$20.44. **2.** \$2.73; \$260; \$8.90. **3.** \$.32; \$38; \$23.15. **4.** \$2.07; \$57; \$12.85. **5.** \$.70; \$43; \$6.50. **6.** 90%; $37\frac{1}{2}\%$; $37\frac{1}{2}\%$; $62\frac{1}{2}\%$. **7.** 50%; $66\frac{2}{3}\%$; $83\frac{1}{3}\%$; 40%. **8.** 34%; 65%; $66\frac{2}{3}\%$; $87\frac{1}{2}\%$. **9.** $\frac{1}{8} \times \$330 = \$41.25$. **10.** $\frac{1}{3} \times \$1.98 = \$.66$. **11.** $\frac{2}{3} \times \$525 = \$350$. **12.** $\frac{1}{3} \times \$19.95 = \$6.65$. **13.** $\frac{7}{8} \times \$50 = \$43.75$. **14.** $\frac{1}{6} \times \$108 = \$18$.

Page 137

Aim: To teach discount in the form in which it is used in retail stores, and the terms peculiar to this topic

Suggestions: This page presents the subject of discount as it applies to the retail customer. For this reason discounts offered at "sales" are emphasized. Make sure that the pupils understand and use properly such terms as *discount, marked price*, and *net price*. In connection with this work, encourage the pupils to bring to class advertisements of sales on which discounts are allowed. You should have a folder of such advertisements in which you keep good examples of discount which you have found in newspapers and magazines.

Workbook Reference: Arithmetic Workshop, Book 7, page 50

Key: **1.** (1) $\frac{1}{3} \times \$39.00 = \$13.00$, $\$39.00 - \$13.00 = \$26.00$; (2) $\frac{1}{3} \times \$27.75 = \$9.25$, $\$27.75 - \$9.25 = \$18.50$; (3) $\frac{1}{4} \times \$12.80 = \$3.20$, $\$12.80 - \$3.20 = \$9.60$; (4) $\frac{1}{4} \times \$4.00 = \$1.00$, $\$4.00 - \$1.00 = \$3.00$; (5) $\frac{1}{4} \times \$7.00 = \$1.75$, $\$7.00 - \$1.75 = \$5.25$; (6) $\frac{1}{4} \times \$9.00 = \$2.25$, $\$9.00 - \$2.25 = \$6.75$; (7) \$26.00

+ \$18.50 + \$9.60 + \$3.00 + \$5.25 + \$6.75 = \$69.10. **3.** (1) .15 × \$35 = \$5.25; (2) \$35.00 − \$5.25 = \$29.75. **4.** \$10.00 + \$18.00 + \$2.00 = \$30.00; $\frac{1}{5} \times \$30.00 = \$6.00$; \$30.00 − \$6.00 = \$24.00. **5.** (1) (a) $\frac{1}{10} \times \$24.50 = \$2.45$; (b) \$24.50 − \$2.45 = \$22.05; (2) (a) $\frac{3}{8} \times \$12.00 = \$4.50$; (b) \$12.00 − \$4.50 = \$7.50; (3) (a) $\frac{1}{5} \times \$11.35 = \$2.27$; (b) \$11.35 − \$2.27 = \$9.08. **6.** (1) (a) $\frac{3}{10} \times \$18.00 = \$5.40$; (b) \$18.00 − \$5.40 = \$12.60; (2) (a) $\frac{1}{8} \times \$15.60 = \$1.95$; (b) \$15.60 − \$1.95 = \$13.65; (3) (a) $\frac{1}{2} \times \$18.98 = \$9.49$; (b) \$18.98 − \$9.49 = \$9.49. **7.** (1) (a) $\frac{1}{4} \times \$19.00 = \$4.75$; (b) \$19.00 − \$4.75 = \$14.25; (2) (a) $\frac{1}{3} \times \$15.75 = \$5.25$; (b) \$15.75 − \$5.25 = \$10.50; (3) (a) $\frac{2}{5} \times \$10.50 = \$4.20$; (b) \$10.50 − \$4.20 = \$6.30. **8.** (1) (a) $\frac{1}{5} \times \$14.75 = \$2.95$; (b) \$14.75 − \$2.95 = \$11.80; (2) (a) $\frac{1}{8} \times \$20.00 = \$2.50$; (b) \$20.00 − \$2.50 = \$17.50; (3) (a) $\frac{1}{10} \times \$16.80 = \$1.68$; (b) \$16.80 − \$1.68 = \$15.12.

Page 138

Aim: To teach the rounding off of answers correct to the nearest cent

Suggestions: When the answer to a problem involving money contains more than two decimal places, such as \$7.4375, it is customary to round off this number correct to the nearest cent. This practice is explained on the first half of page 138 and is followed by a rule. Notice that the figure to watch is the one in the third decimal place. The figures in the fourth and fifth decimal places do not affect the rounding off. Call the pupils' attention to the statement below the boxed rule on page 138. Before assigning the written work on page 138, give some oral work on rounding off numbers. Use these numbers to be rounded off to the nearest cent: \$1.068; \$3.799; \$8.1253; \$9.4989; \$.7632; \$.1960; \$5.3125; \$.8734; \$1.9999; \$12.0988.

Workbook Reference: Arithmetic Workshop, Book 7, page 52

Key: **2.** \$4.19; \$.80; \$2.17. **3.** \$1.31; \$.61; \$2.16. **4.** \$2.34; \$.12; \$2.88. **5.** \$1.27; \$4.58; \$9.95. **6.** \$2.26; \$5.03; \$2.22.

Page 139

Aim: To teach how to estimate the answer in finding a per cent of a number

Suggestions: The work on this page is closely related to the short cuts in percentage given on pages 132–136. If you wish to estimate 49% of a number, you know that the answer is very close to 50% of the number, which is the same as one half of the number. Therefore, if you find one half of the number, the answer will serve as an estimate of 49% of the number. When you estimate a given per cent of a number, the given per cent should be close to some per cent that you can find by a short cut, such as 25%, $33\frac{1}{3}\%$, $37\frac{1}{2}\%$, and so on. Give the following oral practice: "What short cut would you use in *estimating* with these per cents? 13%; 19%; 11%; 24%; 34%; 67%; 61%; 76%; 85%; 36%." Expect this answer for the first one: "13% is near $12\frac{1}{2}\%$ or $\frac{1}{8}$."

Workbook Reference: Arithmetic Workshop, Book 7, page 53

Page 146 **2.** 1.85; 3.27; 4.06; 5.03; 15.56. **3.** 3.24; 2.15; 7.00; 8.00; 13.23. **4.** \$214.52; \$3510; \$413.88. **5.** \$652.50; \$553.50; \$1224.55. **6.** \$1937.50; \$715; \$189.38. **7.** \$390.08; \$1754.50; \$1051.07. **8.** (1) \$45 ÷ 3 = \$15; (2) 2 × \$45 = \$90; (3) \$90 ÷ 3 = \$30. **9.** (1) 2 × \$45 = \$90, \$45 + \$90 = \$135; (2) \$135 ÷ 3 = \$45. **10.** No; \$135; \$180.

Page 147

Aim: To teach some new technical expressions in percentage

Suggestions: The work on this page is really a lesson in the language of percentage and requires careful teaching. Give some oral class work to make sure that the technical expressions are understood. Take the simple number \$20 and ask what it will be if it increases 25%, if it becomes 25% more than originally, if it is decreased by 25%. Do the same thing with other easy numbers until you feel sure that all pupils understand these terms.

Workbook Reference: Arithmetic Workshop, Book 7, page 59

Key: **2.** 120; 36. **3.** 75; 27. **4.** $133\frac{1}{3}$; 32. **5.** 200; 120. **6.** 100% + 25% = 125%; 1.25 × 1820 = 2275 (pupils). **7.** "10% less than."

Page 148

Aim: To review the language, problems, and computation of percentage

Suggestion: This page prepares the pupils for the two tests which follow. Carefully check mistakes on this review so that they can be corrected and the topic can be retaught if necessary.

Key: **1.** 25% of the pupils. **2.** 200% as many people. **3.** 50%; 73%; 1%; 235%; 100%; 9%; 175%. **4.** 28%; 64%; 15%; 98%; 4%; 8%; 9%; 6%. **5.** .42; .055; .0975; 1.72; 12.75; 9.00. **6.** 81; 139; 1440.5. **7.** 13; 8.4; 186. **8.** 325; 378; 7098. **9.** 65%. **10.** \$.90. **11.** \$4.25; \$4.25. **12.** \$3.29. **13.** Estimate, \$55; exact answer, \$52.80. **14.** \$915.00. **15.** \$100; \$500; \$300.

Page 149

Aim: To present Problem Test 4

Suggestion: These problems review the work on percentage as well as other topics previously taught.

Key: **1.** .05 × \$87.75 = \$4.39; \$87.75 + \$4.39 = \$92.14. **2.** $\frac{1}{10}$ × \$6.85 = \$.685, which is called \$.69; .03 × \$6.85 = \$.2055, which is called \$.21; \$6.85 + \$.69 + \$.21 = \$7.75. **3.** \$33.50 + \$3.64 + \$1.80 = \$38.94; \$38.94 ÷ 2 = \$19.47. **4.** \$20 ÷ \$2.50 = 8 (mo.). **5.** 2860 mi. ÷ 153 = 18.69 mi., or 18.7 mi. **6.** \$2.50 + \$1.75 + \$.80 + \$1.00 + \$.15 + \$4.25 = \$10.45; \$.26 + \$1.33 + \$2.06 + \$.19 + \$.88 + \$1.65 + \$.74 = \$7.11; \$10.45 − \$7.11 = \$3.34. **7.** $4\frac{1}{2}$ × \$.45 = \$2.02$\frac{1}{2}$, which is called \$2.03. **8.** 30 yd. ÷ $1\frac{1}{2}$ yd. = 20 (pieces). **9.** $\frac{1}{5}$ × \$32.50 = \$6.50; \$32.50 − \$6.50 = \$26.00. **10.** 2470.3 mi. − 2191.6 mi. = 278.7 mi.

Page 150

Aim: To give a diagnostic test with page references for remedial work

Key: **1.** .25; .77; .06; .02. **2.** .065; .0925; .0575; .155. **3.** 2.80; 3.00; 1.85; 12.56. **4.** 18%; 27%; 8%; 30%. **5.** 25%; 50%; 37½%; 66⅔%. **6.** $1.50; $48.00; $156.20. **7.** $15; $118; $138. **8.** $35; $50; $72. **9.** $.80; $7.75; $12.24. **10.** $60; $192; $403. **11.** $.13; $1.13; $3.82. **12.** $68.75, $206.25; $29.70, $89.10; $3.42, $10.26. **13.** ($6) $6.60; ($27) $26.46; ($6) $5.94.

Chapter 5

Aims of Chapter 5. The major aims of Chapter 5 are to:

1. Teach important home and personal uses of arithmetic.
2. Teach the measurement and cost of electricity.
3. Teach the measurement and cost of gas.
4. Teach economy in buying.
5. Teach the meaning and cost of installment buying.
6. Teach household budgeting.
7. Teach the facts and figures about United States postal service.
8. Compare costs of different ways of shipping.
9. Teach the use and cost of telephone and telegraph services.
10. Review computation by means of improvement tests and other practice.
11. Review the work of the first half year.

Pages 151–153

Aim: To teach how electricity is measured and how its cost in running certain electric appliances is computed

Suggestions: Today electricity is so widely used in the home that knowing how to measure it and how to compute its cost is important. While electricity is studied in certain science classes, such work usually includes no instruction on the arithmetic involved in the use of electricity. To understand the measurement of electricity, it is important at the beginning to make clear the meaning of the terms *watt*, *watt-hour*, and *kilowatt-hour* and to recognize the abbreviations *W* (for *watt*) and *kw-hr* (for *kilowatt-hour*). Since these are new words to most pupils, it will help to show them that electric light bulbs are often marked 50W, 60W, 100W, and 150W and that each of these marks tells how many watt-hours of electricity are used by an electric bulb so marked if it burns for 1 hour. If a bulb marked 100W is burned for 10 hr., it will use 1000 watt-hours of electricity, which is the same as 1 *kilowatt-hour* (1 kw-hr). The exercises on page 152 should be supplemented by oral exercises of a similar kind.

Ex. 1 on page 153 gives the number of watts of electricity used per hour by several different kinds of appliances. To give further reality to this work, ask the pupils to look at any electric appliances they have at home and to find how they are marked. They may find, for example, that an electric iron is marked 1000W or 1100W, that a toaster is marked 900W, and that the motor on a vacuum cleaner is marked 210W or 300W. In many cases the word *watt* is used instead of the abbreviation *W*. A motor marked 300W is, of course, stronger than one marked 210W. Also point out that electric appliances that produce heat, such as toasters, electric stoves, and irons, use much more electricity per hour than appliances with electric motors such as electric fans, electric clocks, and washing machines. Work of this kind helps the pupils better to understand this technical subject.

Key: Page 151 **3.** (1) 20 × 150 = 3000 (watt-hours); (2) 3000 ÷ 1000 = 3 (kw-hr).

Page 152 **2.** .85 kw-hr; 1.75 kw-hr. **3.** (1) 6 × 40 = 240 (watt-hours); (2) 240 ÷ 1000 = .24 (kw-hr); (3) 5 × 60 = 300 (watt-hours), 300 ÷ 1000 = .3 (kw-hr); (4) 8 × 60 = 480 (watt-hours), 480 ÷ 1000 = .48 (kw-hr); (5) 20 × 60 = 1200 (watt-hours), 1200 ÷ 1000 = 1.2 (kw-hr). **4.** (1) 20 × 24 hr. = 480 hr., 75 × 480 = 36,000 (watt-hours), 36,000 ÷ 1000 = 36 (kw-hr); (2) 36 × \$.05 = \$1.80. **5.** 1000 ÷ 50 = 20 (hr.). **6.** (1) 100W + 1000W + 150W = 350W, 4 × 350 = 1400 (watt-hours), 1400 ÷ 1000 = 1.4 (kw-hr); (2) 1.4 × 3¢ = 4.2¢. **7.** (1) 80 × 900 = 72,000 (watt-hours), 72,000 ÷ 1000 = 72 (kw-hr); (2) 72 × \$.03 = \$2.16. **8.** (1) From 9:45 P.M. until 7:45 A.M. is 10 hr.; 10 × 100 = 1000 (watt-hours); (2) 1000 ÷ 1000 = 1 (kw-hr), 1 × 2½¢ = 2½¢.

Page 153 **1.** (1) 30 × 24 = 720 (hr.), 2 × 720 = 1440 (watt-hours); (2) 1440 ÷ 1000 = 1.44 (kw-hr). **2.** (1) 30 × 8 = 240 (hr.), 240 × 300 = 72,000 (watt-hours), 72,000 ÷ 1000 = 72 (kw-hr); (2) 72 × \$.03 = \$2.16. **3.** ½ × 5 = 2½ (hr.), 2½ hr. + 3 hr. = 5½ hr., 4 × 5½ = 22 (hr.); 22 × 300 = 6600 (watt-hours), 6600 ÷ 1000 = 6.6 (kw-hr). **4.** (1) 12 min. = ⅕ hr., 30 × ⅕ hr. = 6 hr., 6 × 900 = 5400 (watt-hours), 5400 ÷ 1000 = 5.4 (kw-hr); 15 min. = ¼ hr., 30 × ¼ hr. = 7½ hr., 7½ × 800 = 6000 (watt-hours), 6000 ÷ 1000 = 6 (kw-hr); 5.4 kw-hr + 6 kw-hr = 11.4 kw-hr; (2) 11.4 × 4¢ = 45.6¢. **5.** (1) 40 min. = ⅔ hr., 3 × ⅔ hr. = 2 hr., 2 × 700 = 1400 (watt-hours); (2) 1400 ÷ 1000 = 1.4 (kw-hr); (3) 1.4 × 4¢ = 5.6¢. **6.** 90 kw-hr; 300 kw-hr. **7.** (1) 30 min. = ½ hr., 4 × ½ hr. = 2 hr., 5 × 2 hr. = 10 hr., 10 × 250 = 2500 (watt-hours), 2500 ÷ 1000 = 2.5 (kw-hr); (2) 2.5 × 3¢ = 7.5¢. **8.** (1) 2 × 1320 = 2640 (watts), 30 × 2640 = 79,200 (watt-hours), 79,200 ÷ 1000 = 79.2 (kw-hr); (2) 79.2 × \$.02½ = \$1.98.

Page 154

Aim: To present a project on changing recipes

Suggestion: The exercises in changing recipes provide an excellent application of the multiplication of fractions. Ask each pupil to bring to class one recipe that uses at least three fractions or mixed numbers. Ask that each recipe be changed to another that uses half the amount; to one that uses one and a half times the amount.

Key: **1.** 1¼ cups. **2.** Beef: ¾ lb.; shortening: ⅛ cup; water: 2 cups; salt: ⅝ tsp.; dash of pepper; flour: 1 tbsp.; water: 1 tbsp.; onions: ½ cup; peppers: ⅛ cup; tomatoes: 1 cup. **3.** (1) Flour: ¾ cup; baking powder: 1 tsp.; salt: ¼ tsp.; sugar: ¼ cup; egg yolks: 1; milk: ¾ cup; butter: 3 tbsp.; chocolate: 1¼ squares; vanilla: ½ tsp.; egg whites: 1; (2) flour: 2¼ cups; baking powder: 3 tsp.; salt: ¾ tsp.; sugar: ¾ cup; egg yolks: 3; milk: 2¼ cups; butter: 9 tbsp.; chocolate: 3¾ squares; vanilla: 1½ tsp.; egg whites: 3. **4.** Flour: 3 cups; baking powder: 3¾ tsp.; sugar:

4½ tbsp.; salt: ¾ tsp.; nut meats: ¾ cup; eggs: 1½ (use 2 small eggs or 1 egg and the yolk of another egg); milk: 1½ cups; butter: 6 tbsp.

Page 155

Aim: To give a new set of improvement tests in subtraction

Suggestion: This set of improvement tests on subtraction is to be administered in the same way as the preceding tests on subtraction.

Key: **1.** 65,498; 22,649; 30,988; 54,759. **2.** 88,049; 9779; 56,873; 27,042. **3.** 68,304; 38,779; 27,795; 58,629. **4.** 73,626; 49,487; 17,099; 32,286. **5.** 24,456; 30,606; 38,543; 11,275. **6.** 47,319; 6515; 11,413; 16,710. **7.** 15,884; 16,275; 39,887; 48,865. **8.** 69,955; 40,924; 67,319; 28,641. **9.** 59,862; 65,788; 73,453; 22,356.

Pages 156–158

Aim: To teach how to read an electric meter and to figure the cost of using a given number of killowatt-hours of electricity

Suggestions: The work on page 156 shows how to read an electric meter. In ex. 1 the four dials of an electric meter are shown. Attention is called to the fact that on dials A and C the pointers move in the *counterclockwise* direction and the numbers are also arranged in the counterclockwise order. On dials B and D the pointers move in the *clockwise* direction, and the numbers are also arranged in that order. On page 157 more difficult cases of reading the dials are given. This page should have careful study.

In connection with this work on meters, it will be helpful to ask the pupils to read the electric meter in their homes, bringing to class a copy of the dials and the position of the pointer on each dial. In this set of diagrams brought in by pupils there should be some readings that cause difficulty. Take up these difficult ones with the class as a whole. Since the reading of an electric meter is somewhat technical and requires considerable time if it is done properly, you may prefer to omit this topic if your class is below average in ability.

In ex. 3 on page 158 rates for electricity in a typical city are given. Notice that the more electricity one uses, the cheaper it gets. Ask the pupils to find out about the electric rates in your community. Compare them with the rates given in ex. 3. In this connection it should be pointed out that the rates for electricity vary considerably throughout the country. In those parts of the country in which electricity is generated by burning coal, the rates are higher than in those areas in which it is generated by water power. In those sections of the country in which big dams have been built by the government, electricity is especially cheap.

Key: Page 156 **2.** 2784, 3260, 3645; 476 kw-hr; 385 kw-hr. **3.** 389 kw-hr. **4.** 326 kw-hr.

Page 157 **2.** 2879; 6491.

Page 158 **2.** $7.70. **3.** (1) 35 × $.05 = $1.75, 20 × $.04 = $.80; $.85 + $1.75 + $.80 = $3.40; (2) 110 = 15 + 35 + 50 + 10; 15 kw-hr cost $.85, 35 × $.05 = $1.75, 50 × $.04 = $2.00, 10 × $.03 = $.30; $.85 + $1.75 + $2.00 + $.30 = $4.90. **4.** (1) 40 = 15 + 25, 25 × $.05 = $1.25, $.85 + $1.25 = $2.10; (2) 180 = 15 + 35 + 50 + 80, 35 × $.05 = $1.75, 50 × $.04 = $2.00, 80 × $.03 = $2.40, $.85 + $1.75 + $2.00 + $2.40 = $7.00; (3) 220 = 15 + 35 + 50 + 100 + 20, 35 × $.05 = $1.75, 50 × $.04 = $2.00, 100 × $.03 = $3.00, 20 × $.02 = $.40, $.85 + $1.75 + $2.00 + $3.00 + $.40 = $8.00; (4) 375 = 15 + 35 + 50 + 100 + 175, 35 × $.05 = $1.75, 50 × $.04 = $2.00, 100 × $.03 = $3.00, 175 × $.02 = $3.50, $.85 + $1.75 + $2.00 + $3.00 + $3.50 = $11.10. **5.** $.34; $.49; $.21; $.70; $.80; $1.11. **6.** 3767 − 3684 = 83; 83 = 15 + 35 + 33; 15 kw-hr cost $.85; 35 × $.05 = $1.75; 33 × $.04 = $1.32; $.85 + $1.75 + $1.32 = $3.92.

Page 159

Aim: To teach how gas is measured and how to find the cost of a given number of cubic feet of gas

Suggestions: In ex. 3 on page 159, rates for gas in a typical city are given. Point out that, as with electricity, the more gas one uses, the cheaper it gets. Ask the pupils to find out about the gas rates in your community, and have them solve some examples using these rates. It should be pointed out that the rates for gas vary considerably throughout the country, and that it would not be surprising to find rates different from those in ex. 3.

Key: **2.** (1) 1800 ÷ 1000 = 1.8, 1.8 × $1.25 = $2.25; (2) 9400 ÷ 1000 = 9.4, 9.4 × $1.25 = $11.75; (3) 12,200 ÷ 1000 = 12.2, 12.2 × $1.25 = $15.25; (4) 10,400 ÷ 100 = 10.4, 10.4 × $1.25 = $13.00; (5) 8600 ÷ 1000 = 8.6, 8.6 × $1.25 = $10.75; (6) 2500 ÷ 1000 = 2.5, 2.5 × $1.25 = $3.125, or $3.13; (7) 8880 ÷ 1000 = 8.88, 8.88 × $1.25 = $11.10. **3.** (1) 900 = 200 + 700, 700 ÷ 100 = 7, 7 × $.10 = $.70, $2.00 + $.70 = $2.70; (2) 2400 = 200 + 1800 + 400, 1800 ÷ 100 = 18, 18 × $.10 = $1.80, 400 ÷ 100 = 4, 4 × $.08 = $.32, $2.00 + $1.80 + $.32 = $4.12; (3) 6500 = 200 + 1800 + 4500, 1800 ÷ 100 = 18, 18 × $.10 = $1.80, 4500 ÷ 100 = 45, 45 × $.08 = $3.60, $2.00 + $1.80 + $3.60 = $7.40; (4) 11,600 = 200 + 1800 + 9600, 1800 ÷ 100 = 18, 18 × $.10 = $1.80, 9600 ÷ 100 = 96, 96 × $.08 = $7.68, $2.00 + $1.80 + $7.68 = $11.48; (5) 55,300 = 200 + 1800 + 28,000 + 25,300, 1800 ÷ 100 = 18, 18 × $.10 = $1.80, 28,000 ÷ 100 = 280, 280 × $.08 = $22.40, 25,300 ÷ 100 = 253, 253 × $.07 = $17.71, $2.00 + $1.80 + $22.40 + $17.71 = $43.91; (6) 120,000 = 200 + 1800 + 28,000 + 70,000 + 20,000, 1800 ÷ 100 = 18, 18 × $.10 = $1.80, 28,000 ÷ 100 = 280, 280 × $.08 = $22.40, 70,000 ÷ 100 = 700, 700 × $.07 = $49.00, 20,000 ÷ 100 = 200, 200 × $.065 = $13.00, $2.00 + $1.80 + $22.40 + $49.00 + $13.00 = $88.20. **4.** 12,300 cu. ft.; 12,300 = 200 + 1800 + 10,300, 1800 ÷ 100 = 18, 18 × $.10 = $1.80, 10,300 ÷ 100 = 103, 103 × $.08 = $8.24, $2.00 + $1.80 + $8.24 = $12.04. **5.** 11,900 cu. ft.; 11,900 = 200 + 1800 + 9900, 1800 ÷ 100 = 18, 18 × $.10 = $1.80, 9900 ÷ 100 = 99, 99 × $.08 = $7.92, $2.00 + $1.80 + $7.92 = $11.72.

Page 160

Aim: To give practice in eliminating extra data that are not needed in solving problems

Suggestion: Go over the first two problems orally, asking individual pupils which numbers in the problems are not needed. Point out that the pupils are to use only the numbers necessary for the solution of the problem.

Workbook Reference: Arithmetic Workshop, Book 7, page 68

Key: **1.** "8:30 A.M." and "50 mi." not needed; 171 mi. ÷ 38 mi. = $4\frac{1}{2}$ (hr.). **2.** "39 in.," "$1.00," and "$.75" not needed; $3\frac{7}{8}$ yd. − $3\frac{1}{2}$ yd. = $\frac{3}{8}$ yd. **3.** "$42.50" and "$13.50" not needed; 2 × $2.75 = $5.50; $16.75 + $5.50 = $22.25. **4.** "52 games" and "39" not needed; $\frac{4}{5}$ × 40 = 32 (games). **5.** "$1.25" and "$12.16" not needed; $.38 + $.14 + $.25 + $.35 = $1.12. **6.** "$.50" and "5 yd." not needed; $1.20 ÷ 15 = $.08. **7.** "20 min.," "9 min.," and "40 min." not needed; $2\frac{1}{8}$ mi. − $1\frac{1}{4}$ mi. = $\frac{7}{8}$ mi.

Page 161

Aim: To teach the economy of buying supplies in large packages if the food can be used before it spoils

Suggestion: Have pupils secure the local prices of the six articles mentioned at the top of the page. Caution them to make certain that the article in the large package is exactly the same as the one in the small package. Then ask them to find the saving effected by buying the larger package.

Key: **1.** 2 × 23¢ = 46¢; 46¢ − 33¢ = 13¢. **2.** (1) 3 × 35¢ = 105¢, 105¢ − 97¢ = 8¢; (2) 2 × 29¢ = 58¢, 58¢ − 49¢ = 9¢; (3) 2 × 38¢ = 76¢, 76¢ − 67¢ = 9¢. **3.** 1 gal. = 8 pt., 8 × $1.15 = $9.20, $9.20 − $7.00 = $2.20. **4.** 5 × 69¢ = 345¢, 345¢ − 269¢ = 76¢. **5.** 35¢ ÷ 12 = $2\frac{11}{12}$¢, 29¢ ÷ 9 = $3\frac{2}{9}$¢; so the oranges are cheaper at 1 doz. for 35¢. **6.** (1) 4 qt. = 1 gal., 4 × $1.29 = $5.16, $5.16 − $4.42 = $.74; (2) 1 gal. and 2 qt.

Pages 162–163

Aim: To show, in problems that are suitable for pupils of Grade 7, the advantages and disadvantages of buying articles on the installment plan

Suggestions: Most of the pupils will know about "buying on time" because many of the articles in their homes, and the family car also, are probably bought on the installment plan. Point out in this connection that installment costs of articles are higher than cash prices. If an article is a necessary one which the family or business cannot get along without, installment buying is a necessity and a convenience. However, if an article is not at all necessary but is for entertainment or pleasure, then it is more economical to wait until the cash is saved before making the purchase. The class can have an interesting discussion along these lines. In Grade 8 the rate of interest charged for buying on the installment plan is studied.

Key: Page 162 **1.** (1) ⅕ × $375 = $75, 12 × $26.60 = $319.20, $75 + $319.20 = $394.20; (2) $394.20 − $375 = $19.20. **2.** (1) 6 × $34.51 = $207.06, $100 + $207.06 = $307.06; (2) 12 × $17.76 = $213.12, $100 + $213.12 = $313.12; (3) 18 × $12.18 = $219.24, $100 + $219.24 = $319.24; (4) $313.12 − $300 = $13.12. **3.** (1) $75 + $100 = $175; (2) $26.60 + $17.76 = $44.36. **4.** (1) 15 × $20 = $300, $25 + $300 = $325; (2) $325 − $295 = $30.

Page 163 **1.** (1) ⅓ × $2700 = $900; (2) (a) $1800 + $60 + $93 = $1953; (b) $1953 ÷ 12 = $162.75; (3) $2700 + $60 + $93 = $2853. **2.** (1) ⅓ × $3000 = $1000, $3000 − $1000 = $2000, $68 + $2000 = $2068, .05 × $2068 = $103.40, $2000 + $68 + $103.40 = $2171.40, $2171.40 ÷ 12 = $180.95; (2) $3000 + $68 + $103.40 = $3171.40.

Pages 164–166

Aims: To show what a budget is and to give budget problems that are meaningful to the pupils in Grade 7

Suggestions: The problems on budgeting apply work on percentage and give more practice on that topic. They also afford an excellent opportunity for an interesting class discussion relating to the various items that make up a budget and to the circumstances that make it desirable to modify a given budget. Ask pupils to make out a budget for themselves showing how they think their weekly allowance should be spent. This would be a budget for the week only, with items given in per cents. From these personal budgets of individuals, the class might make a budget that would be generally useful to all the pupils.

Workbook Reference: Arithmetic Workshop, Book 7, page 98

Key: Pages 164–165 **2.** (1) ⅕ × $4000 = $800; (2) $800 ÷ 12 = $66.66⅔, or about $67. **3.** Rent can be left out of the budget, but taxes and upkeep of the house will have to be added to the budget. **4.** (1) ¼ × $4000 = $1000; (2) $1000 ÷ 12 = $83.33⅓, or about $83. **5.** (1) .3 × $4000 = $1200; (2) They would need to reduce some other items by a total of 5%; (3) They might have reduced the reserve fund by 5% or have made small reductions in the allowances for clothing, advancement, and savings, these reductions amounting to a total of 5%. **6.** (1) .15 × $4000 = $600; (2) $600 ÷ 12 = $50; (3) Clothing is generally bought in the spring and in the fall; so $50 is not spent each month. Probably more is spent in the fall because winter clothing is more expensive than summer clothing. **7.** .08 × $4000 = $320; $320 ÷ 12 = $26.66⅔, or about $27. **8.** (1) .10 × $4000 = $400; (2) if $200 were spent for a vacation trip, only $200 would be left for all the other important items under *advancement*. The Browns cannot afford to spend as much as $200 for a vacation trip. **9.** (1) .15 × $4000 = $600; (2) 12 × $15 = $180, $125 + $180 = $305; $600 − $305 = $295. No, the payments do not use up the amount planned for savings; $295 is left for other savings. **10.** The reserve fund might be used, or even the savings. The other items, except rent and operating expenses, might also be reduced somewhat. **11.** 100%; 100%.

Page 166 **1.** Savings: .08 × \$3000 = \$240; shelter: $\frac{1}{4}$ × \$3000 = \$750; food: .30 × \$3000 = \$900; clothing: .12 × \$3000 = \$360; operating: .10 × \$3000 = \$300; advancement: .08 × \$3000 = \$240; reserve: .07 × \$3000 = \$210. **2.** 52 × \$80 = \$4160. Use budget for \$4200. Savings: .12 × \$4160 = \$499.20; shelter: $\frac{1}{5}$ × \$4160 = \$832; food: $\frac{1}{4}$ × \$4160 = \$1040; clothing: .14 × \$4160 = \$582.40; operating: .12 × \$4160 = \$499.20; advancement: .09 × \$4160 = \$374.40; reserve: .08 × \$4160 = \$332.80. **3.** \$499.20 − \$90 = \$409.20. This amount could be saved in the savings bank, in savings bonds, in a savings-and-loan association, etc. **4.** .28 × \$3600 = \$1008; yes. **5.** .10 × \$3600 = \$360; \$360 − \$325 = \$35; so the Balls saved \$35 less than they planned. **6.** (1) Savings: .15 × \$4800 = \$720; shelter: $\frac{1}{5}$ × \$4800 = \$960; food: .21 × \$4800 = \$1008; clothing: .14 × \$4800 = \$672; operating: .12 × \$4800 = \$576; advancement: .10 × \$4800 = \$480; reserve: .08 × \$4800 = \$384; (2) Savings: .15 × \$5400 = \$810; shelter: .18 × \$5400 = \$972; food: $\frac{1}{5}$ × \$5400 = \$1080; clothing: .15 × \$5400 = \$810; operating: .11 × \$5400 = \$594; advancement: .12 × \$5400 = \$648; reserve: .09 × \$5400 = \$486. **7.** 30% of \$3000 is \$900, and 20% of \$5400 is \$1080, which shows that although the per cent used is lower, the allowance for food is greater, because of the larger income. **8.** 100%.

Page 167

Aim: To give a new set of improvement tests in multiplication and also one in division

Suggestion: These improvement tests should be given in the same manner as the preceding improvement tests in multiplication and division.
Key: **1.** 227,290; 447,810; 830,514; 300,543; 533,106; 217,297. **2.** 493,344; 57,330; 250,107; 94,848; 71,421; 220,156. **3.** 35,750; 106,887; 369,930; 705,525; 366,224; 152,848. **4.** 36.3; 93.8; 67.2. **5.** 83.1; 75.5; 64.7. **6.** 94.0; 67.5; 43.2. **7.** 80.2; 89.6; 79.1. **8.** 89.5; 81.0; 62.7. **9.** 31.4; 63.2; 94.5.

Pages 168–171

Aims: To acquaint the pupils with the different services rendered by the United States Post Office Department and to give practical problems relating to these services

Suggestions: The rates charged for first class mail, for parcel post, and for other postal services are changed from time to time by Act of Congress. The rates given in these problems were correct at the time the book was published, and the problems should be worked using these rates. As a class exercise have the pupils inquire at the post office concerning the current rates and then work the problems again if any rates have been changed. Some pupils should check on first class rates, others on special delivery, others on registered mail, and still others on parcel post. Those pupils checking on parcel post should secure the latest regulations concerning the maximum size and weight of parcel post packages.

Key: Page 168 **1.** 5¢, 10¢, 15¢, 25¢, 35¢; 8¢, 16¢, 24¢, 40¢, 56¢. **2.** 3, 4. **3.** 15¢; 45¢; 30¢. **4.** 75¢; $1.00. **5.** (1) 8¢ + 30¢ = 38¢; (2) 2 × 8¢ = 16¢, 16¢ + 30¢ = 46¢; (3) 3 × 8¢ = 24¢, 24¢ + 30¢ = 54¢.

Page 169 **1.** $.60; $.75; $1.25; $1.00; $1.50. **2.** 8 × $.05 = $.40; $1.00 + $.40 = $1.40. **3.** 3 × $.08 = $.24, $.24 + $.75 + $.10 = $1.09. **4.** (1) 3 × $.05 = $.15, $.15 + $.75 = $.90; (2) 2 × $.05 = $.10, $.10 × $.60 = $.70; (3) 24 × $.05 = $1.20, $1.20 + $1.00 = $2.20. **5.** (1) 3 × $.05 = $.15, $.15 + $.75 = $.90; (2) 3 × $.05 = $.15, $.15 + $1.00 = $1.15; (3) 10 × $.05 = $.50, $.50 + $.75 = $1.25. **6.** (1) 2 × $.05 = $.10, $.10 + $.75 = $.85; (2) 2 × $.05 = $.10, $.10 + $1.00 = $1.10; (3) 4 × $.05 = $.20, $.20 + $1.50 = $1.70.

Pages 170–171 **2.** (1) 17″ is length; 8″ + 5″ + 8″ + 5″ = 26″, girth; 17″ + 26″ = 43″, which is under 100″; (2) 36″ is length; 18″ + 12″ + 18″ + 12″ = 60″, girth; 36″ + 60″ = 96″, which is under 100″; (3) 4′ is length; 1′ + 1½′ + 1′ + 1½′ = 5′, girth; 4′ + 5′ = 9′, or 108″, which is too large. **3.** (1) 42″ is length; 6″ + 4″ + 6″ + 4″ = 20″, girth; 42″ + 20″ = 62″, which is under 100″; (2) 40″ is length; 17″ + 16″ + 17″ + 16″ = 66″, girth; 40″ + 66″ = 106″, which is too large; (3) 6′ is length; ½′ + ¾′ + ½′ + ¾′ = 2½′, girth; 6′ + 2½′ = 8½′, or 102″, which is too large. **4.** 5 lb. − 1 lb. = 4 lb.; 4 × 6.9¢ = 27.6¢, which is called 28¢; 24¢ + 28¢ = 52¢. **5.** (1) 2 × $.35 = $.70, 6 × $.06 = $.36, $.35 + $.36 = $.71; (2) 8 × $.13 = $1.04, 6 × $.12 = $.72, $.51 + $1.04 + $.72 = $2.27; (3) 8 × $.05 = $.40, 26 × $.04 = $1.04, $.33 + $.40 + $1.04 = $1.77. **6.** 3 × $.08 = $.24, 1 × $.07 = $.07, $.39 + $.24 + $.07 = $.70; (2) 8 × $.19 = $1.52, 11 × $.18 = $1.98, $.64 + $1.52 + $1.98 = $4.14; (3) 3 × $.08 = $.24, 45 × $.07 = $3.15, 16 × $.06 = $.96, $.39 + $.24 + $3.15 + $.96 = $4.74. **7.** 8 × $.10 = $.80, $.45 + $.80 + $.30 = $1.55. **8.** 38 × $.05 = $1.90, $.10 + $1.90 = $2.00. **9.** 8 × $.13 = $1.04, 5 × $.12 = $.60, $.51 + $1.04 + $.60 = $2.15; $9.79 ÷ $2.15 = 4.55, or about 4.6 (times). **10.** (1) $.90 + [illegible]35 = $1.25; (2) 18 × $.35 = $6.30, $.90 + $6.30 = $7.20. **11.** 2 lb. = 32 oz.; 32 oz. − 4 oz. = 28 oz.; 28 oz. ÷ 4 oz. = 7 (times); 7 × $.42 = $2.94; $1.89 + $2.94 = $4.83.

Page 172

Aim: To compare different ways of shipping and their costs

Suggestions: This page gives other means of shipping things besides parcel post. The examples serve to show how shipping costs can be found by using the rates between certain cities. Since these rates differ for cities separated by varying distances, the pupils can find out new rates by choosing two cities in different parts of the country and then inquiring about the rates. Ex. 1 shows the range of possibilities. Divide the class into 5 groups and let each group be responsible for securing the facts about the topic assigned to it. The topics are railway express, air express, air freight, rail freight, and truck freight. Have each group make up two problems similar to ex. 3–5. Note in ex. 3 that the 6 packages sent together to the same destination are considered one shipment.

Key: **3.** 3 × $3.59 = $10.77, 3 × $2.57 = $7.71, $10.77 + $7.71 + $.30 = $18.78; $7.04 + $.30 = $7.34; $18.78 − $7.34 = $11.44. **4.** 300 lb. ÷ 100 lb. = 3 (times); 3 × $18.75 = $56.25. **5.** 4½ × $2.57 = $11.57.

Pages 173–174

Aim: To acquaint the pupils with telephone and telegraph services

Suggestions: The problems on these pages show the general principles on which telephone and telegraph charges are based. The table on page 173 gives rates from New York to four other cities. If it is possible for the pupils to secure this type of information about rates between their city and three or four other cities, let them make a table like that in ex. 2 on page 173 and use it in problems like ex. 3 and 4. Mention to the pupils that rates are further reduced between 9:00 P.M. and 4:30 A.M. on certain station-to-station phone calls on the U.S. mainland. Pupils can visit the telegraph office and secure rates in effect from that office to several large cities. They should ask the rate for 15 words and for each extra word. Night and day letter rates can also be found. With these rates, problems like 3–8 on page 174 can be prepared and solved by the pupils.

Key: Page 173 **3.** (1) 12 min. − 3 min. = 9 min., 9 × $.45 = $4.05, $2.85 + $4.05 = $6.90, 1.10 × $6.90 = $7.59; (2) 9 × $.35 = $3.15, $2.45 + $3.15 = $5.60, 1.10 × $5.60 = $6.16, $7.59 − $6.16 = $1.43; (3) 9 × $.35 = $3.15, $1.40 + $3.15 = $4.55, 1.10 × $4.55 = $5.005, which is called $5.01, $7.59 − $5.01 = $2.58. **4.** (1) 7 min. − 3 min. = 4 min., 4 × $.45 = $1.80, $1.75 + $1.80 = $3.55, 1.10 × $3.55 = $3.905, which is called $3.91; (2) 4 × $.60 = $2.40, $2.25 + $2.40 = $4.65, 1.10 × $4.65 = $5.115, which is called $5.12, $5.12 − $3.91 = $1.21; (3) 4 × $.60 = $2.40, $3.50 + $2.40 = $5.90, 1.10 × $5.90 = $6.49, $6.49 − $3.91 = $2.58.

Page 174 **3.** 20 − 15 = 5 (words); 5 × $.05 = $.25; $1.10 + $.25 = $1.35; 1.10 × $1.35 = $1.485, which is called $1.49. **4.** 19 − 15 = 4 (words); 4 × $.065 = $.26; $1.45 + $.26 = $1.71; 1.10 × $1.71 = $1.881, which is called $1.88. **5.** 27 − 15 = 12 (words); 12 × $.06 = $.72; $1.30 + $.72 = $2.02; 1.10 × $2.02 = $2.222, which is called $2.22. **6.** 60 − 50 = 10 (words); 10 ÷ 5 = 2 (times); 2 × $.07 = $.14; $1.05 + $.14 = $1.19; 1.10 × $1.19 = $1.309, which is called $1.31. **7.** 80 − 50 = 30 (words); 30 ÷ 5 = 6 (times); 6 × $.075 = $.45; $1.10 + $.45 = $1.55; 1.10 × $1.55 = $1.705, which is called $1.71. **8.** (1) 60 − 50 = 10 (words), 10 ÷ 5 = 2 (times), 2 × $.12 = $.24, $1.75 + $.24 = $1.99, 1.10 × $1.99 = $2.189, which is called $2.19; (2) 76 words count as 80 words, 80 − 50 = 30 (words), 30 ÷ 5 = 6 (times), 6 × $.12 = $.72, $1.75 + $.72 = $2.47, 1.10 × $2.47 = $2.717, which is called $2.72.

Page 175

Aim: To review computation with decimals and per cents

Key: **1.** 31.4; 1123.6. **2.** 6.04; 1.3343. **3.** 40.674; 1.616. **4.** 27.625; $383.18. **5.** 74.616; 24.5606; .00130. **6.** 1096.65; 33.288; 6.800. **7.** 144.894; 5.22406;

.00012. **8.** 272; 4.72; 310. **9.** .775; 9.25; 3.96. **10.** .28; 2.64; 416.13. **11.** .9, .28, .75, $.66\frac{2}{3}$, .8, .375, $.16\frac{2}{3}$, .5, .32; 90%, 28%, 75%, $66\frac{2}{3}\%$, 80%, $37\frac{1}{2}\%$, $16\frac{2}{3}\%$, 50%, 32%. **12.** \$6.40; \$95; \$24.50. **13.** \$4.05; \$47.17; \$25.55. **14.** \$.18; \$.61; \$23.11. In ex. 15–17, the estimates are given in parentheses. **15.** (80) 76; (200) 192; (1500) 1520. **16.** (150) 153; (70) 67.20; (1000) 996. **17.** (90) 91.20; (222) 223.11; (500) 495.

Page 176

Aim: To present Problem Test 5

Suggestion: This test covers problems with whole numbers, fractions, decimals, and percentage and can be used as a mid-year problem test since the work through Chapter 5 represents about one half of the text.

Key: **1.** 1 mi. = 5280 ft.; 40 × 5280 ft. = 211,200 ft.; 3 sec. = $\frac{1}{20}$ min. = $\frac{1}{1200}$ hr.; $\frac{1}{1200}$ × 211,200 ft. = 176 ft. **2.** 38 = 15 + 23; 15 kw-hr cost \$.85; 23 × \$.05 = \$1.15; \$.85 + \$1.15 = \$2.00. **3.** \$780 ÷ 12 = \$65; \$78 − \$65 = \$13; 10% of \$65 = \$6.50; 20% of \$65 = \$13; so the increase is 20%. **4.** From 8:15 A.M. to 11:30 A.M. is 3 hr. 15 min., or $3\frac{1}{4}$ hr.; 117 mi. ÷ $3\frac{1}{4}$ = 36 mi. **5.** 12 in. = 1 ft. = $\frac{1}{3}$ yd.; 33 × $\frac{1}{3}$ yd. = 11 yd.; 11 × \$.40 = \$4.40. **6.** \$1.50 − \$1.34 = \$.16; $3\frac{1}{2}$ × \$.16 = \$.56. **7.** 81 + 85 + 92 + 94 = 352; 352 ÷ 4 = 88. **8.** 4.5 × 20 = 90 (Camp Fire Girls). **9.** 100 bu. + 215 bu. = 315 bu.; 340 bu. − 315 bu. = 25 bu.; 100 × \$4.25 = \$425; 215 × \$3.00 = \$645; 25 × \$1.25 = \$31.25; \$425 + \$645 + \$31.25 = \$1101.25. **10.** 4 × $\frac{3}{4}$ hr. = 3 hr.; 3 hr. + 1 hr. + $4\frac{1}{2}$ hr. = $8\frac{1}{2}$ hr.; $8\frac{1}{2}$ × \$.60 = \$5.10.

Pages 177–178

Aim: To present diagnostic tests with references for additional practice exercises

Suggestion: These pages may be used as a mid-year test of arithmetic skills, or you may choose 20 to 25 different examples from these pages for a mid-year test.

Key: Page 177 **1.** $5070\frac{1}{2}$; $3275\frac{1}{2}$. **2.** 605; 513. **3.** 92; 324. **4.** $14\frac{1}{16}$; $9\frac{3}{5}$; 16; $10\frac{5}{12}$; $5\frac{1}{30}$. **5.** $2\frac{3}{8}$; $6\frac{5}{8}$; 3; $3\frac{1}{5}$; $2\frac{1}{6}$. **6.** $3\frac{7}{16}$; $1\frac{1}{2}$; $5\frac{1}{2}$; $5\frac{5}{12}$; $3\frac{13}{16}$. **7.** $52\frac{1}{2}$; 27; $8\frac{1}{4}$. **8.** $\frac{3}{16}$; $\frac{1}{2}$; $\frac{3}{8}$. **9.** 3; $9\frac{1}{6}$; $\frac{15}{16}$. **10.** 24; $1\frac{1}{2}$; $1\frac{3}{5}$. **11.** 25; 4; $6\frac{2}{3}$. **12.** $\frac{1}{6}$; $\frac{1}{2}$; $\frac{3}{5}$.

Page 178 **1.** 6.8; 4.7; 15.3; 36.1. **2.** .25; .875; .12; .6875. **3.** .37; .04; .105; 1.78. **4.** 25%; 50%; $87\frac{1}{2}\%$; $33\frac{1}{3}\%$. **5.** 290%; 300%; $4\frac{1}{2}\%$; $7\frac{3}{4}\%$. **6.** 6.048; .3073; .00010; 17.4584. **7.** .004; 11.375; 6.84; 1.21. **8.** 5.3; 1360; .83; 2550. **9.** \$4.62; \$27.30; \$51.30. **10.** \$39; \$26; \$45. **11.** \$.51; \$7.75; \$6.08. **12.** \$18; \$10.24; \$45. **13.** \$.38; \$5.63; \$.80.

Chapter 6

Aims of Chapter 6. The major aims of Chapter 6 are to:

1. Teach the meaning of the geometric terms *line, ray,* and *segment.*
2. Teach the drawing of circles with compasses.
3. Teach the names of parts of a circle.
4. Teach the meaning of an angle, the kinds of angles, and the names of the parts of an angle.
5. Teach how to read angles.
6. Teach the use of the circle in drawing regular inscribed hexagons and equilateral triangles.
7. Teach the use of the hexagon and the equilateral triangle in making designs.
8. Teach the measurement of angles, and the use of a protractor.
9. Teach a way to draw an inscribed square, octagon, and pentagon.
10. Teach the making of designs based on inscribed octagons and pentagons.
11. Teach how angles are used in air navigation.
12. Teach the drawing of perpendicular and parallel lines.
13. Teach the drawing of designs with perpendicular and parallel lines.
14. Extend the topic of scale drawing.
15. Review the understandings and skills taught in previous chapters.

Page 179

Aim: To introduce the topic of informal geometry

Teaching Informal Geometry. Informal geometry is concerned with a study of the more common geometric figures and with the making of simple geometric constructions. In order to succeed in this work, each pupil should be supplied with a well-sharpened pencil, a ruler, a right triangle, a pair of compasses, and a protractor. If it is not easy to obtain a right triangle, one may be made from heavy cardboard. If the pupil has a drawing board with a T-square, such as is used in simple technical drawing, he should be encouraged to use it. The making of geometric constructions can be taught only on the principle of "learn by doing." The pupils must be required actually to make the various constructions and designs indicated in the exercises, so that they will acquire a reasonable skill in this work. Precise drawing, however, is not expected of all pupils. Most pupils are interested in drawing of this kind; if supplied with proper instruments, they should have little difficulty with it. Many pupils will be interested enough to make more designs than are required.

Suggestions: The day you begin this chapter have all the required instruments in class so that the pupils can see and handle them. Ask whether any pupils have compasses, a protractor, a draftsman's triangle, a drawing board with a T-square, and, of course, a good ruler. All pupils should be required to have compasses, protractor, and ruler in class the next day. You could also have some pieces of cardboard in class, such as the back of a writing tablet, and show the pupils how to make a good right triangle.

Page 180

Aim: To teach the meanings of the geometric terms *line*, *ray*, *endpoint*, and *line segment*

Suggestions: In order for the pupils to gain experience with the terms and their representations, provide several examples and have the pupils identify which are lines, line segments, rays, and endpoints. After considerable practice in identification, the pupils should illustrate each of the terms.

Sometimes a double arrowhead is placed above the letters that identify a line, a single arrowhead above the letters that identify a ray, and a line segment above the letters that identify a line segment, thus: $\overleftrightarrow{AB}$, $\overrightarrow{AB}$, $\overline{AB}$. The arrowhead for the ray should be above the letter that is not the endpoint. In $\overrightarrow{CD}$ the C is the endpoint of the ray. After identification of examples with the proper symbols, have the pupils draw the proper examples for several given symbols.

Key: **1.** The arrowheads suggest that the line does not stop at a point. **4.** Lines: extensions of line segments without apparent ends, such as a cross-country highway; rays: beams of light from a flashlight or headlight, one's line of sight, the side of an angle; segment: the edge of a box, the segment where the wall and the floor meet, the side of any polygon, the edge of a book.

Page 181

Aims: To explain how to draw a circle with compasses and to give practice in it

Suggestions: In the work on circles it is essential at the beginning to make clear the terms *radius* (and its plural *radii*), *diameter*, and *arc*, since these terms are constantly used in giving directions concerning the drawing of geometric figures. The pupils should be given considerable practice in drawing circles and arcs with given centers and given radii. They should also become familiar with the technical instructions for doing such work, such as "With A as center, draw a circle having a radius of 2 in." In drawing circles the sharp point of the compasses should be pushed firmly into the paper; otherwise, it will slip during the process of drawing the circle. The circle is constantly used in drawing the designs taught in this chapter, so sufficient practice in drawing circles is necessary.

Key: **3.** 4 in. **4.** 2 in.; 3 in.; 5 in.; 6 in.

Pages 182–183

Aim: To teach the meaning of an angle, certain definitions relating to angles, and the method of reading angles

Suggestions: The meaning of an angle is explained in ex. 1 and 2 on page 182. In this work you should make clear that the size of an angle depends on the amount of rotation and not on the lengths of the sides of the angle. Illustrate this fact by pointing out some right angles in the schoolroom, such as the corners of a door, a window, a book, a small sheet of paper, and so on. An angle and the amount of rotation may be illustrated before the class by spreading the legs of a pair of compasses, by opening a book, or by moving the hands of a toy clock. The terms *right angle, straight angle, acute angle,* and *obtuse angle* should be explained to the pupils by blackboard diagrams. The methods of reading angles are described on page 183. Attention is called to the fact that an angle may be lettered and read in three different ways, as shown in ex. 1 on page 183. Give practice on labeling angles and reading them.

Workbook Reference: Arithmetic Workshop, Book 7, page 65

Key: Page 183 **2.** Angles c and d; yes; same amount of rotation. **3.** Angle BOD; angle AOD; obtuse angle. **5.** ABC; BCD; EFG; KLM; LMN; MNK; NKL.

Page 184

Aim: To review problem solving

Key: **1.** $16 \times \$.45 = \$7.20$; $32 \times \$.43 = \$13.76$; $\$7.20 + \$13.76 = \$20.96$; $\$20.96 - \$9.82 = \$11.14$; $\$11.14 \div 2 = \$5.57$. **2.** (1) $\frac{3}{4} \times 1000 = 750$ (watt-hours), $2 \times 500 = 1000$ (watt-hours), $1 \times 250 = 250$ (watt-hours), $750 + 1000 + 250 = 2000$ (watt-hours); (2) 2000 watt-hours = 2 kw-hr, 2 × 3¢ = 6¢. **3.** From 10:35 A.M. to 11:41 A.M. is 1 hr. 6 min. **4.** (1) 1 hr. 6 min. = 1.1 hr.; (2) 544 mi. ÷ 1.1 = 494.54 mi., which is called 494.5 mi. **5.** 11,110 gal. − 300 gal. = 10,810 gal.; 10,810 gal. ÷ 8198 = 1.31 gal., which is called 1.3 gal. **6.** $\$107.50 - \$15.00 = \$92.50$; $\$92.50 - \$25.00 = \$67.50$; $\$67.50 \div \$4.50 = 15$ (mo.). **7.** 7 hr. 50 min. = $7\frac{5}{6}$ hr.; 240 mi. − 48 mi. = 192 mi.; $7\frac{5}{6} \times 192$ mi. = 1504 mi. **8.** (1) $33 \times \frac{1}{4}$ lb. = $8\frac{1}{4}$ lb., $8\frac{1}{4} \times \$.56 = \$4.62$; (2) $8\frac{1}{4} \times \$.68 = \$5.61$.

Page 185

Aims: To give another set of improvement tests in addition and to give some oral review of percentage

Suggestion: This set of improvement tests on addition should be administered in the same manner as previous sets.

Key: **1.** 27,053; 38,834; 46,629; 30,656; 17,627. **2.** 39,008; 27,106; 33,374; 28,239; 43,628. **3.** 31,008; 21,870; 33,762; 34,074; 34,449. **4.** 12; 21; 45. **5.** 39; 50; 24. **6.** $\frac{5}{8}$ of 40; $\frac{2}{3}$ of 90. **7.** \$28.00; \$8.01. **8.** \$55; \$36; \$120. **9.** .0225; .185; .0575; .095; 1.25.

Pages 186–187

Aims: To use the circle for drawing a hexagon and to make designs based on the hexagon

Suggestions: On page 186 two methods of drawing a hexagon are given. Each of these methods produces a hexagon inscribed in a circle. The first method of drawing a hexagon, which is described in ex. 2 on page 186, is the one most frequently taught; but a satisfactory hexagon is less easily drawn by this method than by the second method, which is described in ex. 3. It is always desirable for you to have as much skill in drawing geometric figures as you expect of the pupils. If you will personally experiment with these two methods of drawing a hexagon, you will soon find that you can do much more satisfactory work by using the second method. The reason for the superiority of the second method is that it requires fewer arcs than the first method. Every time an arc is drawn there is the possibility that the sharp point of the compasses may slip on the paper. In the first method 6 arcs must be drawn, so there are 6 possibilities for error; in the second method only 2 arcs must be drawn. Successful work in drawing hexagons by either method depends upon having a sharp point on the pencil end of the compasses. Since the designs on page 187 are based on hexagons, a hexagon must first be inscribed in a circle in drawing each design. It should be pointed out at this time that the process of inscribing a hexagon in a circle is really a process of dividing the circle into 6 equal parts. In designs 2, 3, 7, and 9 on page 187, the hexagon must actually be drawn; but in designs 4, 5, 6, and 8, it is sufficient to mark the 6 points that divide the circle into 6 equal parts, without actually connecting these points to make a hexagon. In drawing each of the designs on page 187, decide on the order of the steps.

Workbook Reference: Arithmetic Workshop, Book 7, pages 60 and 61

Page 188

Aim: To explain how to draw an equilateral triangle not inscribed in a circle and how to use it in making designs

Suggestions: The method of drawing an equilateral triangle given in ex. 1 on this page is, of course, a much easier method to use than the one given at the top of page 187. In making the designs in ex. 2 and 3, follow the directions carefully; in some cases you will have to give supplementary directions in case the procedure is not self-evident to the pupil. For example, in drawing the first design in ex. 2, the pupil should start by drawing the large equilateral triangle, preferably making each side 2 in. long. After finishing the large equilateral triangle, he should mark the midpoint of each side of that triangle, which is easily found since each side is 2 in. long. These midpoints should then be connected by straight lines to form the four small triangles. In drawing the second design in ex. 2, the midpoint of each side of the large triangle should be found before drawing the arcs. The radius of the arcs, of course, is half the length of each side of the large triangle. In ex. 3 the first design requires the drawing of the large equilateral triangle, after which each side is divided into three equal parts. If each side of the large triangle is made 3 in. long, the matter of dividing each side into three equal parts will be simplified. If a smaller design is preferred, the large triangle should have sides of $1\frac{1}{2}$ in. so that each part of each side will be $\frac{1}{2}$ in. long. In all this work it is necessary to plan the procedure for each drawing before actually attempting to do the work.

Pages 189–191

Aim: To teach the measurement of angles and the use of the protractor

Suggestions: The method of measuring angles by means of degrees is explained on page 189. In this work notice that the word *degree* refers to each of the 90 small angles formed at the vertex of a right angle and to each of the 90 small arcs into which the arc of a right angle is divided. In other words, the degree is a unit for measuring arcs as well as angles.

The method of using the protractor to measure an angle, which is described in ex. 1 and 2 on page 190, should have very careful study. In this work caution the pupils to put the center of the protractor exactly over the vertex of the angle that is being measured and to see that side AB of the angle passes through the 180 mark of the protractor. The picture at the top of page 190 shows how the measurement is made. Provide ample practice in measuring angles while the pupils are in class. You may want to assign the exercises on page 190 for work outside of class.

The method of drawing an angle of a given size by means of the protractor is explained in ex. 1 on page 191. The pupil should have considerable practice in doing this work, as suggested in ex. 2. Be sure to give some practice in class before assigning homework. Two different methods of bisecting an angle are given in ex. 3 and 4 on this page. The method given in ex. 3 makes use of the protractor, while that in ex. 4 shows how to bisect an angle using only compasses. The pupil should be given practice in bisecting angles by each method.

Workbook Reference: Arithmetic Workshop, Book 7, pages 66 and 67

Key: Page 189 **3.** $\frac{1}{4}$ circle; 360°. **5.** 60°. **6.** 60°; 180°; 90°.

Page 192

Aim: To explain how to use the protractor to inscribe squares, octagons, and pentagons in circles

Suggestions: Have the pupils draw all their circles with a radius of at least 2 in. because small figures are harder to draw than large ones. In ex. 1 on this page the square is drawn in a circle by using a protractor. There is another way to make this construction which avoids the use of a protractor, but it requires knowing how to draw perpendicular lines, which is not explained until page 198. After the pupils have studied page 198, you might repeat ex. 1 on page 192, having them draw diameter CD perpendicular to AB at O instead of using the protractor to make angles of 90°.

In ex. 2 on page 192, the first step is to divide the circle into 4 equal parts as was done in ex. 1. This division will give 4 right angles at the center of the circle. Each of these right angles should then be bisected by means of compasses. The protractor must be used again in drawing a pentagon in order to make 5 angles of 72° each. Ex. 3 gives instructions for drawing a pentagon.

Page 193

Aim: To give a new set of improvement tests in subtraction

Suggestions: This set of improvement tests on subtraction should be administered in the same manner as previous sets. Have pupils check on their progress in subtraction by comparing scores on previous sets with their scores on this set.

Key: **1.** 237,992; 175,978; 295,748; 418,135. **2.** 237,358; 759,869; 326,761; 574,468. **3.** 96,145; 214,742; 467,986; 680,198. **4.** 256,829; 353,684; 200,513; 488,898. **5.** 794,756; 240,876; 435,143; 516,247. **6.** 492,921; 68,230; 338,805; 524,780. **7.** 659,032; 159,726; 51,782; 283,559. **8.** 395,371; 574,693; 177,660; 257,068. **9.** 93,641; 290,371; 270,321; 81,095.

Page 194

Aim: To draw designs based on inscribed octagons and pentagons

Suggestions: Besides having the pupils copy the 8 designs illustrated, have them try to make up three or four designs of their own that use either octagons or pentagons, or both. Require careful and neat work and let pupils use crayons of different colors or several kinds of shading.

The work in ex. 2 and 3 needs careful explanation. Give considerable practice in drawing angles larger than 180°, both in class and for homework. An understanding of the terms *clockwise* and *counterclockwise* is very necessary for a proper understanding of angles in air navigation, given on pages 196 and 197.

Workbook Reference: Arithmetic Workshop, Book 7, page 63

Page 195

Aim: To review in both problem and abstract form the work in estimating answers

Suggestion: Be sure to require both the estimated and the exact answer. Have pupils tell why they think a particular exact answer should be more or less than the estimated answer.

Key: **1.** (1) Think of \$14.95 as about \$15.00, \$10.83 as about \$11.00, and \$3.77 as about \$4.00; then \$15.00 + \$11.00 + \$4.00 = \$30.00, and \$30.00 ÷ 3 = \$10.00; (2) \$14.95 + \$10.83 + \$3.77 = \$29.55, \$29.55 ÷ 3 = \$9.85. **2.** (1) Think of $3\frac{7}{8}$ as about 4, \$1.20 as about \$1.25, $2\frac{1}{8}$ as about 2, and \$3.92 as about \$4.00; then 4 × \$1.25 = \$5.00, 2 × \$4.00 = \$8.00, \$5.00 + \$8.00 = \$13.00, and \$20.00 − \$13.00 = \$7.00; (2) $3\frac{7}{8}$ × \$1.20 = \$4.65, $2\frac{1}{8}$ × \$3.92 = \$8.33, \$4.65 + \$8.33 = \$12.98, \$20.00 − \$12.98 = \$7.02. **3.** (1) Think of 5940 as about 6000; then 6000 ÷ 15 = 400 (mi.); (2) 5940 mi. ÷ 15 = 396 mi. **4.** (1) Think of 19 as about 20 and of 52¢ as about $.50; then $20 \times \$\frac{1}{2} = \$10$; (2) 19 × $.52 = $9.88. **5.** (1) Think of 5495.8 as about 5500; then 6000 − 5500 = 500 (mi.); (2) 6000 − 5495.8 = 504.2 (mi.). **6.** (1) Think of $2.12 as about $2, $.89 as about $1, and $1.96 as about $2; then $2 + $1 + $2 = $5; (2) $2.12 + $.89 + $1.96 = $4.97. In ex. 7–13, the estimated answers are given in parentheses. **7.** (400) 392.98; (2) 2.1; ($7) $6.60. **8.** (810) 799$\frac{7}{8}$; (3600) 3549; ($30) $30.03. **9.** (1100) 1093; (55) 54$\frac{5}{8}$; ($25) $24.48. **10.** (8) 8.1; (5600) 5609; ($15) $15.34. **11.** (70) 72; ($33) $32.45; ($9) $8.74. **12.** (27) 26.68; ($60) $61.69; ($15) $15.12.

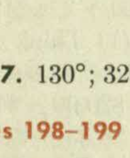
Pages 196–197

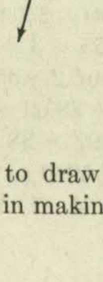
Aim: To make the work on angles more interesting by showing the use of angles in air navigation

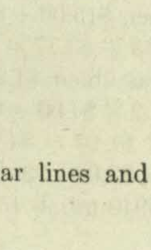
Suggestions: This work on angles in air navigation is of interest to most pupils. Circular protractors, such as the one shown in the diagram at the top of page 196, are usually sold by dealers in drawing materials. They are printed on heavy paper or Bristol board and come with diameters of 8 in. and 14 in. You may want to have such a protractor for classroom use, or some pupil might like to own one. You can make one as follows: On paper draw a circle with a radius of 4 in. or 5 in. and mark its center *O*. Draw the horizontal diameter of the circle. Place a small protractor on the paper so that its center is at point *O* and its diameter lies along the diameter of the circle. Then place dots around the circular edge of the protractor at each 5-degree mark. Draw lines through *O* and each of these dots and extend the lines in both directions until they cut the circle at two points. The circle is now marked in 5-degree intervals. By eye, divide each 5-degree space into 5 equal parts, and you will have a large protractor divided into 360°.

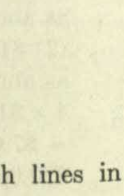
Key: **2.** 40°; 105°; 205°; 246°; 320°. **3.** 135°; 225°; 315°. **4.** North; 0°. **5.** Walk 40 paces on a course of 340°.

6.









7. 130°; 325°; 245°.

Pages 198–199

Aims: To explain how to draw perpendicular lines and to use such lines in drawing rectangles and in making designs

Suggestions: For this work each pupil should be supplied with a draftsman's triangle, a ruler, and a pencil. The most rapid and accurate way to draw a line perpendicular to another line is to use a draftsman's triangle, as explained in ex. 2 on page 198. To use a draftsman's triangle, the pupil slides the triangle along the edge of a ruler. Most rulers have a thin edge and a thicker edge. Usually the results will be more satisfactory if the pupil slides the triangle along the thicker edge of the ruler. In classes in which mechanical drawing is taught, a T-square instead of a ruler is used, together with a draftsman's triangle, to draw perpendicular lines. A picture of a T-square is shown at the bottom of page 201. If any pupils in your class have T-squares, let them use the T-squares in place of the ruler when doing the exercises on page 198. Besides the designs given on page 199, have pupils bring at least 8 designs of their own, as suggested in ex. 3.

Pages 200–201

Aims: To explain how to draw parallel lines and to use such lines in making new designs

Suggestions: Point out in ex. 1 that parallel lines may be vertical lines, horizontal lines, or *oblique* lines. If the term *oblique* is not understood by the pupils, take time to explain it and have a pupil look up the definition in the dictionary. Stress the point that parallel lines never meet in either direction no matter how long they are drawn. Parallel lines are also the same distance apart at all points. Have the pupils look around the room or out the windows to find examples of parallel lines. One method of drawing parallel lines is given in ex. 3 on page 200, but the quickest way to draw parallel lines is to use a draftsman's triangle as described in ex. 1 on page 201 or to use a T-square as explained in ex. 4 on the same page. If no pupil brings to class a drawing board with a T-square, try to borrow the equipment from the art or industrial arts teacher.

Parallel and perpendicular lines are often used in drawing designs such as those in ex. 3 on page 201. The pupils should be encouraged to search for other designs or to make up designs of their own. Designs of this kind are to be found in linoleum patterns, certain fabrics, wallpaper, rugs, tiles, and so on. Geometric design plays a prominent part in decorating the home.

Workbook Reference: Arithmetic Workshop, Book 7, pages 70 and 71

Page 202

Aim: To review problem solving and problems in which facts are missing

Suggestions: On this page pupils need to supply a missing fact for each problem. It should be understood that any missing fact that makes a sensible problem is acceptable. For example, in ex. 1 the bill must be larger than the amount Mary spends. Pupils should estimate the amount she spends and then make the answer a bill that is larger than $2.70. Remember that only one bill is asked for.

Page 203

Aims: To give the fourth set of improvement tests in multiplication and to review arithmetical terms

Suggestions: This new set of improvement tests in multiplication is to be given in the same manner as the previous sets of such tests.

Success in arithmetic depends upon understanding the language of arithmetic. Hence, the pupils should show that they understand the meaning of such words and phrases as those given in ex. 4–27.

Key: **1.** 3,716,064; 5,047,765; 625,625; 550,472. **2.** 2,902,824; 1,221,129; 8,593,128; 736,281. **3.** 1,862,788; 2,660,968; 6,252,880; 3,736,868.

Pages 204–206

Aims: To extend the topic of scale drawing to more difficult types than those studied earlier in the pupils' school experience and to include reading the distances on a map

Suggestions: The work in scale drawing on pages 204 and 206 should have very careful study; and all pupils should make the actual scale drawings, using a ruler on which the inch is divided into sixteenths. An important element necessary for success in scale drawing is to have ample time for this topic.

Scale drawing is a very practical topic. House plans, maps, and most mechanical drawings are drawn to scale. Without scale drawing, industrial activities and the building trades would be handicapped. In connection with this work, perhaps a pupil can bring to class a scale drawing of a floor plan.

Maps also are drawn to scale, as explained on page 205. It is important to know how to use the scale on a map to find the airline distance between any two towns shown on the map. The method of finding distances is explained on page 205. Ask pupils to bring to class highway maps that have printed scales. Then they can show how the scale can be used to find distances. They should also look in their social studies or geography books to find maps with scales.

Workbook Reference: Arithmetic Workshop, Book 7, pages 72 and 73

Key: Page 204 **1.** 6 squares; $1\frac{1}{2}$ in.; 4 in.; 6 in. **3.** 3 in. by 4 in.; $3\frac{3}{4}$ in. by 5 in.; $7\frac{1}{2}$ in. by 10 in. **4.** 25 ft.; 75 ft.; 50 ft.; $6\frac{1}{4}$ in.; 625 ft. **5.** $2 \times \frac{3}{4}$ in. = $1\frac{1}{2}$ in., $8\frac{1}{2}$ in. − $1\frac{1}{2}$ in. = 7 in., width of drawing; $2 \times \frac{3}{4}$ in. = $1\frac{1}{2}$ in., $10\frac{1}{2}$ in. − $1\frac{1}{2}$ in. = 9 in., length of drawing; cottage and porch measure 28 ft. × 36 ft.; $28 \div 7 = 4$, and $36 \div 9 = 4$; so the scale is 1 in. = 4 ft.

Page 205 **2.** (1) 300 mi.; (2) 160 mi.; (3) 260 mi.; (4) 350 mi.; (5) 60 mi.; (6) 200 mi.; (7) 200 mi.; (8) 390 mi.; (9) 120 mi.; (10) 320 mi. **3.** 100 mi.; 200 mi.; 300 mi. **4.** Cincinnati and Columbus; Springfield and Columbus.

Page 206 **1.** (d) 50 mi. **2.** (1) 20 ft. + 2 ft. + 2 ft. = 24 ft., and 35 ft. + 2 ft. + 2 ft. = 39 ft.; $24 \times \frac{1}{4}$ in. = 6 in., and $39 \times \frac{1}{4}$ in. = $9\frac{3}{4}$ in.; so the longest part would be $9\frac{3}{4}$ in. long; (2) $24 \times \frac{1}{8}$ in. = 3 in., and $39 \times \frac{1}{8}$ in. = $4\frac{7}{8}$ in., so the longest part would be $4\frac{7}{8}$ in. long; (3) $24 \times \frac{1}{2}$ in. = 12 in., and $39 \times \frac{1}{2}$ in. = $19\frac{1}{2}$ in., so the longest part would be $19\frac{1}{2}$ in. long. **4.** 6; 6; 1 in. = 4 ft.; 1 in. = 6 ft. **5.** 1 in. = 4 ft.; 1 in. = 3 ft.; 1 in. = 2 ft.

Page 207

Aim: To review problem solving

Key: **1.** $\frac{3}{4}$ hr. + $1\frac{1}{2}$ hr. + $1\frac{1}{4}$ hr. + $1\frac{1}{2}$ hr. + 1 hr. = 6 hr.; 6 hr. ÷ 5 = $1\frac{1}{5}$ hr. **2.** $8 \times 38\frac{1}{2}$ mi. = 308 mi. **3.** $\frac{3}{4} \times \frac{2}{3}$ cup = $\frac{1}{2}$ cup. **4.** 20 min. = $\frac{1}{3}$ hr.; $6\frac{1}{2} \times \frac{1}{3}$ hr. = $2\frac{1}{6}$ hr., or 2 hr. 10 min.; 2 hr. 10 min. before 6 P.M. is 3:50 P.M.

5.

Money Received				Money Paid Out			
Feb. 3	Cash on hand		65	Feb. 5	Movies		55
Feb. 4	Errand		35	Feb. 9	School supplies		20
Feb. 7	Shoveling snow	1	00	Feb. 13	Book	1	49
Feb. 8	Working		90	Feb. 15	Ticket to play		50
Feb. 11	Helping Mrs. Wood	1	50				
Feb. 16	Selling old papers		75		Cash on hand	2	41
	Total	5	15			5	15
Feb. 17	Cash on hand	2	41				

6. (1) $9 \times \frac{2}{3}$ cup = 6 cups; (2) $15 \times \frac{2}{3}$ cup = 10 cups. **7.** 2 × \$.55 = \$1.10; $1\frac{1}{2}$ × \$.69 = \$1.03½, which is called \$1.04; \$1.10 + \$1.04 + \$.49 + \$.45 + \$.39 = \$3.47.

Page 208

Aim: To sum up the work on informal geometry taught in Chapter 6

Suggestions: If you wish to test the pupils on this work, page 208 can be used for such a test. If you want only 10 exercises in such a test, omit ex. 10. However, be sure to take up ex. 10 with the class.

Key: **1.** 68°. **3.** 36°. **5.** 90°; less than 90°; more than 90°; 180°. **6.** An acute angle; 75°. **7.** The size of an angle depends upon how much one side of the angle is rotated from the other side. **8.** 180°. **9.** Yes; yes. **10.** 45°; 315°; 135°. **11.** Southwest.

Page 209

Aim: To present Problem Test 6

Suggestion: Be sure to remind the pupils to graph their scores on their individual bar graphs.

Workbook Reference: Arithmetic Workshop, Book 7, page 75

Key: 1. 3 × $9.75 = $29.25; 7 × $11.25 = $78.75; $29.25 + $78.75 = $108.00; 3 wk. + 7 wk. = 10 wk.; $108.00 ÷ 10 = $10.80. **2.** .15 × $68.00 = $10.20; 12 × $10.20 = $122.40. **3.** $\frac{1}{4}$ lb. = 4 oz.; 4 oz. ÷ $\frac{1}{2}$ oz. = 8 (balls); 8 × $.59 = $4.72. **4.** $10.86 + $9.58 + $10.52 + $12.72 + $9.80 + $11.44 = $64.92; $64.92 ÷ 12 = $5.41. **5.** $15\frac{1}{2} \times \$.89 = \$13.79\frac{1}{2}$, which is called $13.80. **6.** 13 lb. 11 oz. − 6 lb. 6 oz. = 7 lb. 5 oz.; 7 lb. 5 oz. = $7\frac{5}{16}$ lb.; $7\frac{5}{16} \times \$.64 = \$4.68$. **7.** 588 mi. ÷ 35 = 16.8 mi. **8.** 500 ÷ 50 = 10 (fifties); 10 × $.23 = $2.30; $2.30 − $1.78 = $.52. **9.** $1\frac{3}{4}$ mi. in $\frac{1}{2}$ min. is at the rate of $3\frac{1}{2}$ mi. in 1 min.; 60 × $3\frac{1}{2}$ mi. = 210 mph. **10.** 5 lb. = $\frac{1}{2}$ of 10 lb.; $\frac{1}{2} \times 39¢ = 19\frac{1}{2}¢$, which is called 20¢; 3 × 13¢ = 39¢; $1\frac{1}{2} \times 15¢ = 22\frac{1}{2}¢$, which is called 23¢; $\frac{1}{2} \times 67¢ = 33\frac{1}{2}¢$, which is called 34¢; $.20 + $.39 + $.23 + $.34 = $1.16.

Page 210

Aim: To provide a diagnostic test with page references for remedial work

Suggestion: This page is a review of all the abstract computation reviewed or taught up to this point.

Key: 1. 9603 R11; 8635. **2.** $12\frac{3}{5}$; $9\frac{5}{12}$. **3.** $1\frac{3}{4}$; $1\frac{1}{2}$; $2\frac{1}{24}$. **4.** $27\frac{1}{2}$; $\frac{3}{5}$; $6\frac{1}{4}$. **5.** $\frac{3}{4}$; $7\frac{1}{2}$; 4. **6.** .625; .65; .8125; .84. **7.** .06; .125; .0225; 1.37. **8.** 169.02; 537.99; .01392; .09776. **9.** 16.75; .008; 26.8; 1.3. **10.** 9.6; 2470; .7; 1260. **11.** $1.95; $22.50; $26. **12.** $1.56; $51; $37.82. **13.** $.15; $.51; $1.00.

Chapter 7

Aims of Chapter 7. The major aims of Chapter 7 are to:

1. Teach the comparison of two numbers by division by finding what part the smaller number is of the larger number.
2. Teach the comparison of two numbers by a per cent when the per cent is a whole number.
3. Teach the comparison of two numbers by a per cent when the per cent ends in a fraction.
4. Apply the work of Aims 2 and 3 to the rate of discount.
5. Teach the comparison of two numbers by a per cent to the nearest whole per cent.
6. Teach the comparison of two numbers by division by finding how many times as large as the smaller number is the greater number.
7. Teach the comparison of two numbers by a per cent when the per cent is 100% or larger.
8. Teach per cent of increase and decrease.
9. Teach a brief introduction to profit and loss.
10. Review the understandings and skills taught in previous chapters.

Page 211

Aim: To introduce the topic of comparison of numbers by division by means of a project dealing with a subject of pupil interest

Suggestion: This page should be a class exercise to introduce the idea of comparing a smaller number with a larger one. You might add two or three more topics of this kind, such as buying a 45-dollar bicycle by paying \$5 a week; buying a used radio for \$15 and paying \$3 a week.

Key: **1.** \$36 ÷ \$3 = 12 (weekly payments); 12 − 2 = 10 (weekly payments). **3.** Yes; \$9; $\frac{1}{4}$. **4.** \$12; \$15; \$18; \$21; \$24; $\frac{1}{3}$; $\frac{5}{12}$; $\frac{1}{2}$; $\frac{7}{12}$; $\frac{2}{3}$.

Page 212

Aim: To show how to compare a number with a larger number by using division

Suggestions: In all the exercises on this page a given number is compared with a larger number by finding what fractional part the smaller number is of the larger one. To make this comparison the smaller number is written over the larger one to form a fraction, and this fraction is changed to lowest terms. For example, 24 is compared with 36 by finding what part 24 is of 36. The work is done by writing 24 over 36, which gives the fraction $\frac{24}{36}$, or $\frac{2}{3}$. The result shows that 24 is $\frac{2}{3}$ of 36. Since $\frac{24}{36}$ means 24 ÷ 36, you see that this comparison is made by division. Ex. 2 shows that it is very important for amounts to be expressed in the same unit of measure before they are compared by division. Make this point very clear to the pupils.

Workbook Reference: Arithmetic Workshop, Book 7, page 77

Key: **4.** (1) 12 ÷ 36 = $\frac{12}{36}$ = $\frac{1}{3}$ (yd.); (2) 24 ÷ 36 = $\frac{24}{36}$ = $\frac{2}{3}$ (yd.); (3) 27 ÷ 36 = $\frac{27}{36}$ = $\frac{3}{4}$ (yd.); (4) 30 ÷ 36 = $\frac{30}{36}$ = $\frac{5}{6}$ (yd.); (5) 2 ft. 3 in. = 27 in., 27 ÷ 36 = $\frac{27}{36}$ = $\frac{3}{4}$ (yd.); (6) 1 ft. 3 in. = 15 in., 15 ÷ 36 = $\frac{15}{36}$ = $\frac{5}{12}$ (yd.). **5.** (1) 15 ÷ 60 = $\frac{15}{60}$ = $\frac{1}{4}$ (hr.); (2) 40 ÷ 60 = $\frac{40}{60}$ = $\frac{2}{3}$ (hr.); (3) 48 ÷ 60 = $\frac{48}{60}$ = $\frac{4}{5}$ (hr.); (4) 50 ÷ 60 = $\frac{50}{60}$ = $\frac{5}{6}$ (hr.). **6.** (1) 440 ÷ 5280 = $\frac{440}{5280}$ = $\frac{1}{12}$ (mi.); (2) 528 ÷ 5280 = $\frac{528}{5280}$ = $\frac{1}{10}$ (mi.); (3) 660 ÷ 5280 = $\frac{660}{5280}$ = $\frac{1}{8}$ (mi.); (4) 880 ÷ 5280 = $\frac{880}{5280}$ = $\frac{1}{6}$ (mi.); (5) 1320 ÷ 5280 = $\frac{1320}{5280}$ = $\frac{1}{4}$ (mi.); (6) 3960 ÷ 5280 = $\frac{3960}{5280}$ = $\frac{3}{4}$ (mi.). **7.** (1) 100 ÷ 2000 = $\frac{100}{2000}$ = $\frac{1}{20}$ (T.); (2) 200 ÷ 2000 = $\frac{200}{2000}$ = $\frac{1}{10}$ (T.); (3) 500 ÷ 2000 = $\frac{500}{2000}$ = $\frac{1}{4}$ (T.); (4) 1200 ÷ 2000 = $\frac{1200}{2000}$ = $\frac{3}{5}$ (T.). **8.** (1) 1 ÷ 12 = $\frac{1}{12}$ (ft.); (2) 5 ÷ 12 = $\frac{5}{12}$ (ft.); (3) 4 ÷ 12 = $\frac{1}{3}$ (ft.); (4) 6 ÷ 12 = $\frac{6}{12}$ = $\frac{1}{2}$ (ft.); (5) 8 ÷ 12 = $\frac{8}{12}$ = $\frac{2}{3}$ (ft.). **9.** (1) 20 ÷ 60 = $\frac{20}{60}$ = $\frac{1}{3}$ (min.); (2) 25 ÷ 60 = $\frac{25}{60}$ = $\frac{5}{12}$ (min.); (3) 45 ÷ 60 = $\frac{45}{60}$ = $\frac{3}{4}$ (min.). **10.** (1) 10¢ ÷ 100¢ = $\frac{10}{100}$ = $\frac{1}{10}$ (dollar); (2) 20¢ ÷ 100¢ = $\frac{20}{100}$ = $\frac{1}{5}$ (dollar); (3) 25¢ ÷ 100¢ = $\frac{25}{100}$ = $\frac{1}{4}$ (dollar); (4) 35¢ ÷ 100¢ = $\frac{35}{100}$ = $\frac{7}{20}$ (dollar); (5) 40¢ ÷ 100¢ = $\frac{40}{100}$ = $\frac{2}{5}$ (dollar); (6) 50¢ ÷ 100¢ = $\frac{50}{100}$ = $\frac{1}{2}$ (dollar); (7) 60¢ ÷ 100¢ = $\frac{60}{100}$ = $\frac{3}{5}$ (dollar); (8) 75¢ ÷ 100¢ = $\frac{75}{100}$ = $\frac{3}{4}$ (dollar); (9) 85¢ ÷ 100¢ = $\frac{85}{100}$ = $\frac{17}{20}$ (dollar); (10) 95¢ ÷ 100¢ = $\frac{95}{100}$ = $\frac{19}{20}$ (dollar). **11.** 135 ÷ 324 = $\frac{135}{324}$ = $\frac{5}{12}$. **12.** $\frac{2}{5}$; $\frac{2}{3}$; $\frac{7}{8}$; $\frac{2}{5}$; $\frac{1}{2}$. **13.** $\frac{2}{9}$; $\frac{5}{6}$; $\frac{2}{3}$; $\frac{3}{8}$; $\frac{2}{3}$.

Page 213

Aims: To give another set of improvement tests in division and to review problems in estimating answers

Suggestion: This set of improvement tests should be administered in the same way as preceding sets.

Key: **1.** 91.5; 68.0; 58.2. **2.** 71.8; 75.5; 62.3. **3.** 53.9; 40.3; 72.8. **4.** 69.1; 50.9; 96.5. **5.** 63.4; 43.9; 93.5. **6.** 81.5; 47.3; 85.5. **7.** (1) Think of \$.98 as about \$1.00; 4 × \$1.00 = \$4.00; (2) 4 × \$.98 = \$3.92. **8.** (1) Think of \$7.48 as about \$7.50, \$1.94 as about \$2.00, and \$4.99 as about \$5.00; \$7.50 + \$2.00 + \$5.00 = \$14.50, \$20.00 − \$14.50 = \$5.50; (2) \$7.48 + \$1.94 + \$4.99 = \$14.41, \$20.00 − \$14.41 = \$5.59. **9.** (1) Think of 389 mi. as about 400 mi., 400 mi. ÷ 8 = 50 mi.; (2) 389 mi. ÷ 8 = 48.625 mi., which is called 48.6 mi. **10.** (1) Think of \$3.95 as about \$4.00 and \$.49 as about \$.50, \$4.00 + \$.50 = \$4.50; (2) \$3.95 + \$.49 = \$4.44.

Page 214

Aim: To show how to compare a number with a larger number by using a per cent (On this page the per cent is always a whole number.)

Suggestions: In ex. 1 on page 214 you compare 18 with 20 by showing that 18 is $\frac{18}{20}$, or $\frac{9}{10}$, of 20. Since the answer is required in the form of a per cent instead of a fraction, change $\frac{9}{10}$ to a fraction having a denominator of 100. The change gives $\frac{90}{100}$, or 90%. On this page the work of comparing numbers by a per cent is easy because the number in the denominator of the simplified fraction is a factor of 100, and finding a number to use as a multiplier to make the denominator 100 is very simple. Notice that all the numbers used in the exercises on page 214 are easily changed to fractions having denominators of 2, 4, 5, 10, 20, 25, and 50. These are whole numbers that can be multiplied by a whole number to make 100. This work can probably be done mentally, but most pupils should be required to write the work. For example, to find what per cent 13 is of 50, think of 50 as becoming 100 and that makes 13 become 26. So the answer is 26%. Also, to find what per cent 6 is of 25, think of 25 as becoming 100 and that makes 6 become 24. So the answer is 24%. This method is not the usual one of comparing two numbers by means of a per cent, and it will work easily only in certain cases. The usual method is explained on page 215.

Key: **3.** 75%; 75%; 26%; 50%; 75%. **4.** 50%; 25%; 75%; 24%; 80%. **6.** 85; 12; 32. **7.** 28; 24; 68.

Page 215

Aim: To teach the comparison of two numbers by a per cent when the per cent ends in a fraction

Suggestions: The usual method of comparing two numbers by means of a per cent is explained in ex. 1 on page 215. There are three steps to this comparison:

(1) Compare the two numbers in fractional form, as 37 is $\frac{37}{40}$ of 40.
(2) Change the fraction to a two-place decimal which represents *hundredths.* The result is $.92\frac{1}{2}$.
(3) Change the two-place decimal to a per cent, which is $92\frac{1}{2}$%.

By following these three steps, you can compare any two numbers by means of a per cent. Sometimes the per cent is a whole number, and often it ends in a fraction. The work of comparing two numbers by means of a per cent is sometimes called Case 2 of percentage. In such problems you are finding what per cent one number is of another number. Finding a given per cent of a number, such as finding 12% of 150, is sometimes called Case 1 of percentage.

Workbook Reference: Arithmetic Workshop, Book 7, pages 78 and 79

Key: **2.** $97\frac{2}{9}$%. **3.** No. **4.** $\frac{37}{40} = 92\frac{1}{2}$%; yes.

Page 216

Aim: To give problems and practice on comparing two numbers by a per cent

Workbook Reference: Arithmetic Workshop, Book 7, page 81

Key: **1.** $18 \div 32 = 56\frac{1}{4}$%. **2.** (1) 247 ÷ 475 = 52%; (2) smaller. **3.** 100% − 52% = 48%. **4.** (1) 840 ÷ 875 = 96%; (2) 100% − 96% = 4%. **5.** \$360 ÷ \$9600 = $3\frac{3}{4}$%. **6.** \$45 − \$39 = \$6; \$6 ÷ \$45 = $13\frac{1}{3}$%. **7.** 18 ÷ 300 = 6%. **8.** 75 ÷ 300 = 25%. **9.** $37\frac{1}{2}$; $16\frac{1}{4}$; 28. **10.** 75; $22\frac{1}{2}$; 60. **11.** 60; 84; 18. **12.** $33\frac{1}{3}$; 12; 44. **13.** $42\frac{1}{2}$; $12\frac{1}{2}$; 48. **14.** $53\frac{1}{3}$; 26; 52.

Page 217

Aim: To give a review of problem solving of all kinds

Key: **1.** 40% = $\frac{2}{5}$; $\frac{2}{5}$ × \$35 = \$14; \$35 − \$14 = \$21; \$21 ÷ 6 = \$3.50. **2.** $\frac{1}{4}$ hr. = 15 min.; 15 × 1400 ft. = 21,000 ft.; 21,000 ÷ 5280 = 3.97 (mi.), which is called 4.0 mi. **3.** 356 × \$.12 = \$42.72; .05 × \$20.10 = \$1.01, \$20.10 + \$1.01 = \$21.11, $\frac{2}{3}$ × \$20.10 = \$13.40, .05 × \$13.40 = \$.67, \$13.40 + \$.67 = \$14.07, 3 × \$14.07 = \$42.21, \$21.11 + \$42.21 = \$63.32; \$63.32 − \$42.72 = \$20.60, so the trip is \$20.60 cheaper by automobile. **4.** 1200 ÷ 240 = 5, so 1200 mi. is 5 times as far as 240 mi.; 5 × 49 min. = 245 min., or 4 hr. 5 min.; 4 hr. 5 min. after 8:30 A.M. is 12:35 P.M. **5.** \$950 − \$100 = \$850; .05 × \$850 = \$42.50; \$60 + \$42.50 = \$102.50. **6.** 1 gal. = 16 half pints; 40 × 16 = 640 (half pints); 640 × \$.60 = \$384.00. **7.** $2\frac{3}{4}$ mi. ÷ $\frac{1}{4}$ mi. = 11 (quarter miles); 11 − 1 = 10; 10 × 5¢ = 50¢; \$.35 + \$.50 + \$.50 = \$1.35. **8.** 50¢ ÷ 12 = $4\frac{1}{6}$¢; 33¢ ÷ 8 = $4\frac{1}{8}$¢; since $4\frac{1}{8}$¢ is the lowest cost, the oranges selling at 8 for 33¢ are cheapest.

Page 218

Aim: To find the rate of discount when a purchase is made at a reduced price

Suggestions: The *rate* of discount means the *per cent* of discount. This work is an excellent practical application of comparing two numbers by means of a per cent. The two numbers to be compared are the amount of the discount and the original price before the article was marked down. The first exercise on page 218 can be solved by dividing the discount, \$2.25 (found by subtracting \$12.75 from \$15.00), by the original price, \$15.00, or by first changing $\frac{\$2.25}{\$15.00}$ to lowest terms, to get $\frac{3}{20}$. Then change $\frac{3}{20}$ to $\frac{15}{100}$, which equals 15%.

Key: **1.** \$15.00 − \$12.75 = \$2.25; \$2.25 ÷ \$15.00 = 15%. **2.** \$10.00 − \$8.50 = \$1.50; \$1.50 ÷ \$10.00 = 15%. **3.** \$4.00 − \$3.00 = \$1.00; \$1.00 ÷ \$4.00 = 25%. **4.** (1) \$18.00 − \$12.00 = \$6.00, \$6.00 ÷ \$18.00 = $33\frac{1}{3}$%; (2) \$10.00 − \$8.40 = \$1.60, \$1.60 ÷ \$10.00 = 16%; (3) \$9.50 − \$7.60 = \$1.90, \$1.90 ÷ \$9.50 = 20%; (4) \$8.75 − \$7.00 = \$1.75, \$1.75 ÷ \$8.75 = 20%; (5) \$1.25 − \$1.00 = \$.25, \$.25 ÷ \$1.25 = 20%; (6) \$16.00 − \$10.00 = \$6.00, \$6.00 ÷ \$16.00 = $37\frac{1}{2}$%.

Pages 219–220

Aim: To teach the comparison of two numbers by a per cent when the nearest whole per cent is required

Suggestions: In comparing any two numbers by means of a per cent, the result is usually not a whole per cent. In such cases, the result is often satisfactory in applications when it is rounded off to the nearest whole per cent. If this is to be done, there must be three places in the decimal quotient so that it can be rounded off to two places representing per cent. Ex. 2 on page 219 gives practice in rounding off three-place decimals. If this is not sufficient practice, you will find much more on page 340, as noted at the bottom of page 219. Real problems of interest to the pupils are found on pages 219 and 220.

Workbook Reference: Arithmetic Workshop, Book 7, page 82

Key: Page 219 **2.** 28%; 48%; 22%; 10%; 1%; 13%. **3.** 11 ÷ 12 = .916, or 92%. **4.** (1) 31 ÷ 36 = .861, or 86%; (2) 27 ÷ 43 = .627, or 63%; (3) 9 ÷ 17 = .529, or 53%; (4) 18 ÷ 29 = .620, or 62%. **5.** 6 ÷ 11 = .545, which is called 55%; 5 ÷ 9 = .555, which is called 56%; so Jean had the better per cent.

Page 220 **1.** 91%. **2.** 44%. **3.** 64%. **4.** 89%. **5.** 81%. **6.** 57%. **7.** 36%; 47%; 17%. **8.** 185 da. − 6 da. = 179 da.; 179 ÷ 185 = .967, which is called 97%. **9.** 130 − 19 = 111 (boys); 111 ÷ 130 = .853, which is called 85%. **10.** 69%. **11.** 36%; 74%. **12.** 31%; 78%.

Page 221

Aim: To review all types of previous work

Key: **1.** 13,662. **2.** $9.38. **3.** 15%; 40%. **4.** $\frac{3}{4}$ hr.; $\frac{1}{3}$ hr.; $\frac{3}{5}$ hr. **5.** 104.2. **6.** $60\frac{1}{2}$; 70; $4\frac{3}{8}$. **7.** $1\frac{3}{4}$; $\frac{2}{3}$; $\frac{1}{20}$. **8.** 75. **9.** .85. **10.** 40. **11.** .6. **12.** 75 oz. **13.** $2\frac{4}{6} + 7\frac{1}{6} + 9\frac{3}{6} + 11\frac{2}{6} = 29\frac{10}{6} = 30\frac{4}{6} = 30\frac{2}{3}$; $30\frac{2}{3} \div 4 = 7\frac{2}{3}$. **14.** .05 × 240 = 12; .08 × 175 = 14; so 8% of 175 is greater. **15.** 9339 − 3993 = 5346; 5346 ÷ 54 = 99. **16.** 21 hr. 53 min. **17.** (1) 1.10 × $250 = $275; (2) .90 × $250 = $225; (3) .10 × $250 = $25. **18.** (1) 98 ÷ 245 = $\frac{98}{245}$ = 40%; (2) 81 ÷ 93 = $\frac{81}{93}$ = .870, which is called 87%. **19.** (1) Since the time is 3 hr. earlier in California, it is 5 A.M. in California when it is 8 A.M. in New York; (2) since the time is 2 hr. earlier in California, it is 10 A.M. in California when it is noon in Illinois. **20.** $2.50 + $2.25 + $.30 + $.55 + $.31 = $5.91; $1.00 + $3.75 + $1.20 + $.30 + $.17 = $6.42; $6.42 − $5.91 = $.51; so the second group of coins amounts to $.51 more.

Page 222

Aim: To compare two numbers by division by finding how many times as large as the smaller number is the greater number

Suggestions: The work on page 222 is in preparation for the work on page 224. In ex. 1 you compare 950 lb. with 95 lb. by dividing 950 by 95 to get 10. So the horse is 10 times as heavy as the colt. To compare a larger number with a smaller one, you divide the larger number by the smaller one. The result shows how many times as great as one number is the other. The answer is always a whole number or a mixed number. On pages 211 and 212 division was also used in comparing a smaller number with a larger one, but in that case you found what part the smaller number was of the larger one. The answer for such cases is always a fraction less than one.

Key: **1.** 950 lb. ÷ 95 lb. = 10 (times). **2.** $3000 ÷ $6 = 500 (times). **3.** 1680 bu. ÷ 320 bu. = $5\frac{1}{4}$ (times). **4.** 480 mi. ÷ 45 mi. = $10\frac{2}{3}$ (times). **5.** 200. **6.** 30. **7.** $6\frac{2}{5}$. **8.** $8\frac{1}{2}$. **9.** 40.

Page 223

Aim: To give another set of improvement tests in addition

Key: **1.** 454; 366; 578; 445; 542; 402; 481; 602. **2.** 531; 377; 566; 378; 678; 555; 534; 451. **3.** 580; 375; 393; 406; 634; 396; 529; 417.

Pages 224–225

Aim: To compare a larger number with a smaller one by using per cents larger than 100%

Suggestions: Since the work on page 224 is difficult for pupils, proceed slowly so that they will become adjusted to thinking in terms of large per cents. Call attention to the fact that while 200% is a large per cent, actually it represents only 2 times a number. Likewise, 750%, which is a very large per cent, represents only 7½ times a number. Go over ex. 1–3 on page 224 very carefully, making certain that the pupils can easily change a large per cent to a whole number or a decimal, and vice versa. The work on page 224 is a good illustration of the fact that the *language* of percentage is the main new element in the subject and time is needed to become familiar with it. Percentage is a new way of talking about numbers and comparing them. It is an easy matter to understand a statement such as "Bob sold twice as many tickets as Tom," but much practice is needed to understand this fact when expressed in percentage language, such as "Bob sold 200% as many tickets as Tom." The project on page 225 gives practice on finding large per cents by using familiar material of interest to pupils.

Key: Page 224 **2.** 3; 2.35; 5; 14; 3.75; 2.8; 1.45; 8.25; 2.05. **4.** 600%; 345%; 1200%; 467%; 750%; 1150%. **6.** 2½ times; 250%. **7.** 225%. **8.** 250%; 100%; 125%; 112½%; 325%. **9.** 133⅓%; 150%; 110%; 100%; 260%. **10.** 200%; 120%; 137½%; 400%; 225%.

Page 225 **1.** 160%. **2.** 275%. **3.** 375%. **4.** 4½ times; 450%. **5.** 466%. **6.** 240%.

Page 226

Aim: To review the language of arithmetic by completing statements

Suggestions: This exercise can be an oral one; but if the words are written by each pupil, you will know whether the spelling, as well as the meaning, has been learned. Pupils should write only the number of an exercise and the missing word or words for that exercise. It is not necessary to write the whole statement.

Key: **1.** Quotient; dividend; divisor. **2.** Thousandths. **3.** Estimating. **4.** Least common denominator. **5.** Discount. **6.** Decimal. **7.** Rounding off; nearest tenth. **8.** Inverted; multiplication. **9.** Fraction; per cent. **10.** Billions. **11.** Millionths. **12.** Remainder; difference; minuend; subtrahend. **13.** Less; more.

Pages 227–229

Aim: To teach how to find the per cent of increase or decrease

Suggestions: These pages teach a new idea in percentage and introduce two new language expressions peculiar to percentage; namely, *per cent of increase* and *per cent of decrease.* The per cent of increase is explained in ex. 1 on page 227.

After finding the *amount* of increase, pupils have difficulty in remembering by which number they should divide the amount of increase: the original number or the increased number. Emphasize the fact that they always divide by the original number since that is the number which has been increased. Ex. 2 gives the explanation of per cent of decrease. The same reasoning holds in this case as in the per cent of increase. Find the amount of decrease and divide it by the original number since the original number has been decreased. The increase or decrease must always be compared with the number before the increase or decrease took place. Pages 228 and 229 give situations in which per cents of increase and decrease occur.

Workbook Reference: Arithmetic Workshop, Book 7, pages 83 and 84

Key: Page 227 **3.** $70 − $60 = $10; $10 ÷ $60 = .16⅔ = 16⅔%. **4.** $40.94 − $35.60 = $5.34; $5.34 ÷ $35.60 = .15 = 15%. **5.** $9.00 − $6.75 = $2.25; $2.25 ÷ $9.00 = .25 = 25%. **6.** $3.00 − $2.25 = $.75; $.75 ÷ $3.00 = .25 = 25%. **7.** 179 − 132 = 47; 47 ÷ 132 = .356, which is called 36%. **8.** 88 bu. − 40 bu. = 48 bu.; 48 ÷ 40 = 1.2 = 120%. **9.** 12 ÷ 43 = .279, which is called 28%.

Page 228 **1.** 6 × $.17 = $1.02; $1.02 − $.93 = $.09; $.09 ÷ $1.02 = .088, which is called 9%. **2.** 63¢ − 59¢ = 4¢; 4 ÷ 63 = .063, which is called 6%. **3.** 79¢ − 69¢ = 10¢; 3½ × 10¢ = 35¢. **4.** 79¢ − 69¢ = 10¢; 10 ÷ 79 = .126, which is called 13%.

Page 229 **1.** Sugar: 10¢ − 8¢ = 2¢, 2 ÷ 8 = .25 = 25% (increase); milk: 25¢ − 23¢ = 2¢, 2 ÷ 23 = .086, which is called 9% (increase); eggs: 57¢ − 53¢ = 4¢, 4 ÷ 53 = .075, which is called 8% (increase); beef: 62¢ − 55¢ = 7¢; 7 ÷ 55 = .127, which is called 13% (increase); oranges: 49¢ − 38¢ = 11¢, 11 ÷ 49 = .224, which is called 22% (decrease); potatoes: 5¢ − 4¢ = 1¢, 1 ÷ 4 = .25 = 25% (increase); white bread: 23¢ − 19¢ = 4¢, 4 ÷ 19 = .210, which is called 21% (increase); chicken: 49¢ − 42¢ = 7¢, 7 ÷ 42 = .16⅔ = 16⅔% (increase). **2.** 70¢ − 42¢ = 28¢; 28 ÷ 70 = .4 = 40%. **3.** 29¢ − 10¢ = 19¢; 19 ÷ 10 = 1.9 = 190%. **4.** (1) 52¢ − 40¢ = 12¢, 12 ÷ 40 = .3 = 30%; (2) 56¢ − 49¢ = 7¢, 7 ÷ 56 = .125 = 12½%. **5.** .10 × $45 = $4.50.

Pages 230–231

Aim: To give an introductory treatment of the subject of profit and loss where only simple cases in finding the profit, or the per cent of profit, are given

Suggestions: The topic of profit and loss has its technical language which must be fully learned if this topic is to be understood. The pupils should become familiar with the terms *expenses, overhead, profit, loss, cost,* and *selling price.* As used in this book, the terms *expenses* and *overhead* mean the same thing; hence, the words are interchangeable. The word *profit* as used here means the *actual profit,* or what is sometimes called the *net profit.* The word *profit* differs from the term *gross profit;* since gross profit is not used in any of the exercises, the pupils need not become familiar with it. The *cost* is the price actually paid for goods that are bought, while the *selling price* is the price received for them when they are sold.

The pupils' success in the work on profit and loss depends upon a full understanding of the following formula:

Selling Price = Cost + Expenses + Profit

Pupils must remember that *overhead* means *expenses*, and *profit* means *actual profit*.

The exercises on page 231 are devoted to finding the *per cent of profit*, and in each case the profit is found as a per cent of the *selling price*. It is the general practice in business to compute profit as a per cent of the *selling price*. Years ago profit was frequently computed as a per cent of the *cost* of the goods, but that practice has gradually been replaced by the substitution of the selling price as the base for computing per cent of profit.

In modern business the selling price is used also as a basis for computing other items. For example, *expenses* are computed as a per cent of the selling price. When clerks are paid a commission, this commission is computed as a per cent of the amount of the clerks' sales, which, of course, are selling prices. When discounts are allowed to customers, they also are computed on the selling price. Taxes are likewise computed on the sales, or the selling price. In view of all these situations, people in modern business have found it convenient to use the selling price as a common base for all computations. In this book, each exercise states clearly that the profit, or loss, is to be computed as a per cent of the selling price. In all the exercises on page 231 pupils must first find the amount of the profit in each case and then find what per cent this profit is of the selling price.

Workbook Reference: Arithmetic Workshop, Book 7, pages 88 and 89

Key: Page 230 **2.** $9.50 + $1.50 + $.10 = $11.10; $15.00 − $11.10 = $3.90. **3.** (1) $15 + $4 + $.50 = $19.50, $25.00 − $19.50 = $5.50 (profit); (2) $4 + $.50 = $4.50 (expenses). **4.** (1) .6 × $25.00 = $15.00; (2) .3 × $25.00 = $7.50; (3) .1 × $25.00 = $2.50. **5.** $15.00 + $7.50 = $22.50; $22.50 − $19.50 = $3.00 (loss).

Page 231 **1.** (1) $10.50 + $35 = $45.50, $65.00 − $45.50 = $19.50; (2) $19.50 ÷ $65.00 = .30, or 30%. **2.** (1) $13.53 + $1.78 + $.19 + $2.50 = $18.00, $24.00 − $18.00 = $6.00; (2) $6 ÷ $24 = .25 = 25%. **3.** (1) 6 bu. = 24 pk., 6 × $2.25 = $13.50, $13.50 + $.35 + $1.00 = $14.85, 24 × $.75 = $18.00, $18.00 − $14.85 = $3.15; (2) $3.15 ÷ $18.00 = .17½ = 17½%. **4.** (1) $2.75 + $1.75 = $4.50, $5.00 − $4.50 = $.50; (2) $.50 ÷ $5.00 = .10 = 10%. **5.** (1) $6.00 + $2.75 = $8.75, $10.00 − $8.75 = $1.25; (2) $1.25 ÷ $10.00 = .12½ = 12½%. **6.** (1) $9.75 + $4.25 = $14.00, $17.50 − $14.00 = $3.50; (2) $3.50 ÷ $17.50 = .20 = 20%. **7.** (1) $7.75 + $3.50 = $11.25, $12.50 − $11.25 = $1.25; (2) $1.25 ÷ $12.50 = .10 = 10%. **8.** (1) $18.90 + $9.45 = $28.35, $31.50 − $28.35 = $3.15; (2) $3.15 ÷ $31.50 = .10 = 10%.

Page 232

Aim: To present a mental review of percentage

Key: **1.** 10%; $16\frac{2}{3}\%$; 75%; $87\frac{1}{2}\%$; $83\frac{1}{3}\%$; $12\frac{1}{2}\%$; 60%; $33\frac{1}{3}\%$; $62\frac{1}{2}\%$; 20%. **2.** .125, $\frac{1}{8}$; .5, $\frac{1}{2}$; 1.25, $1\frac{1}{4}$; .375, $\frac{3}{8}$; $.66\frac{2}{3}$, $\frac{2}{3}$; .15, $\frac{3}{20}$; .06, $\frac{3}{50}$; .045, $\frac{9}{200}$. **3.** 12 pupils; 36 pupils. **4.** 6 pupils; 72 pupils. **5.** \$50; \$36; \$60. **6.** \$3. **7.** \$4. **8.** 75%. **9.** \$15,000. **10.** \$18,000. **11.** \$48. **12.** 25%.

Page 233

Aim: To give another set of improvement tests in subtraction

Key: **1.** \$337.83; \$417.65; \$759.99; \$364.49. **2.** \$188.59; \$579.89; \$457.47; \$79.46. **3.** \$282.99; \$406.66; \$582.36; \$24.79. **4.** \$147.55; \$377.89; \$61.49; \$654.16. **5.** \$152.38; \$750.85; \$298.03; \$123.56. **6.** \$30.18; \$347.67; \$935.87; \$132.83. **7.** \$101.04; \$535.50; \$322.46; \$257.02. **8.** \$526.61; \$429.72; \$420.36; \$281.69. **9.** \$767.11; \$96.11; \$436.29; \$340.88.

Page 234

Aim: To review problem solving

Key: **1.** From 3:15 to 5:45 is $2\frac{1}{2}$ hr.; from 8:30 A.M. to 1:00 P.M. is $4\frac{1}{2}$ hr.; $5 \times 2\frac{1}{2}$ hr. $= 12\frac{1}{2}$ hr.; $12\frac{1}{2}$ hr. + $4\frac{1}{2}$ hr. = 17 hr.; 17 × \$.60 = \$10.20. **2.** 8 × \$10.20 = \$81.60. **3.** 3 + 5 = 8 (packages); 12 − 8 = 4 (packages); 3 × 20¢ = 60¢; 5 × 10¢ = 50¢; 4 × 15¢ = 60¢; \$.60 + \$.50 + \$.60 = \$1.70. **4.** \$16,500 − \$10,000 = \$6500; \$6500 ÷ \$10,000 = .65 = 65%. **5.** 2 × 100 = 200 (stamps); 2 × 75 = 150 (stamps); 200 + 150 = 350 (stamps); 500 − 350 = 150 (stamps); 150 stamps ÷ 25 stamps = 6 (packages); 2 × \$.50 = \$1.00; 2 × \$.65 = \$1.30; 6 × \$.35 = \$2.10; \$1.00 + \$1.30 + \$2.10 = \$4.40. **6.** 93 + 86 + 88 + 92 = 359; 5 × 90 = 450; 450 − 359 = 91. **7.** $\frac{1}{3}$ × \$58.50 = \$19.50, \$58.50 − \$19.50 = \$39.00; $\frac{1}{8}$ × \$50.00 = \$6.25, \$50.00 − \$6.25 = \$43.75; \$43.75 − \$39.00 = \$4.75; so the first chair costs \$4.75 less. **8.** 40 mi. ÷ 1.1 = 36.36 mi., or 36.4 mi.

Page 235

Aim: To present a written review of percentage

Key: **1.** 75%; $66\frac{2}{3}\%$; $87\frac{1}{2}\%$; 80%; 15%; 60%; $9\frac{1}{2}\%$; 273%. **2.** .17; .06; .125; 2.45; 6; 1.3. **3.** \$5.70; \$3.00; \$2450. **4.** \$23.25; \$19.50; \$3750. **5.** \$2.07; \$59; \$10,238.40. **6.** 50%; $12\frac{1}{2}\%$; $33\frac{1}{3}\%$; 75%; 400%. **7.** $66\frac{2}{3}\%$; 20%; $37\frac{1}{2}\%$; $166\frac{2}{3}\%$; $16\frac{2}{3}\%$. **8.** \$.49; \$.42; 29%. **9.** \$6.56; \$3.34; 82%. **10.** \$2.18; \$4.47; 34%. **11.** (1) $\frac{1}{3}$ × \$10.95 = \$3.65; (2) \$10.95 − \$3.65 = \$7.30. **12.** (1) \$5.95 + \$2.80 = \$8.75, \$10.00 − \$8.75 = \$1.25; (2) \$1.25 ÷ \$10.00 = $.12\frac{1}{2} = 12\frac{1}{2}\%$. **13.** .3 × \$120 = \$36.

Page 236

Aim: To summarize and review the work of Chapter 7

Suggestion: This page prepares the pupils for the test on page 238. Carefully check pupils' mistakes on this review so that corrections can be made and the topic retaught if necessary.

Key: **1.** $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{2}$; $\frac{5}{6}$; $\frac{3}{4}$. **2.** $\frac{2}{3}$; $\frac{4}{5}$; $\frac{1}{3}$; $\frac{2}{5}$; $\frac{2}{3}$. **3.** 75%; 200%; 68%; 100%; 175%. **4.** $11\frac{1}{4}$%; $37\frac{1}{2}$%; $33\frac{1}{3}$%; 125%; $14\frac{3}{4}$%. **5.** 23%; 38%; 24%; 88%; 321%. **6.** 25%; 90%; $62\frac{1}{2}$%; 50%; $43\frac{3}{4}$%; $66\frac{2}{3}$%; $83\frac{1}{3}$%; 35%; $12\frac{1}{2}$%. **7.** 1 ft. 6 in. = 18 in.; 18 in. ÷ 36 in. = $\frac{1}{2}$ (yd.). **8.** 1 lb. 4 oz. = $1\frac{1}{4}$ lb.; $1\frac{1}{4}$ lb. ÷ 5 lb. = $\frac{1}{4}$. **9.** 84¢ − 56¢ = 28¢; 28 ÷ 56 = .5 = 50%. **10.** \$21.75 ÷ \$72.50 = .3 = 30%. **11.** \$125 ÷ \$875 = .142, which is called 14%. **12.** 50 ÷ 10 = 5 = 500%. **13.** 24 times; 8 times. **14.** .12 × \$900 = \$108; \$485 + \$108 = \$593; \$900 − \$593 = \$307.

Page 237

Aim: To present Problem Test 7

Workbook Reference: Arithmetic Workshop, Book 7, page 91

Key: **1.** 6 × 15¢ = 90¢; 90¢ − 75¢ = 15¢. **2.** In $\frac{1}{4}$ hr. the train travels $\frac{1}{3}$ of 42.9 mi., or 14.3 mi.; in $\frac{4}{4}$ hr. (1 hr.) the train travels 4 × 14.3 mi., or 57.2 mi. **3.** 19 − 15 = 4 (words); 4 × $5\frac{1}{2}$¢ = 22¢; \$1.20 + \$.22 = \$1.42; 1.10 × \$1.42 = \$1.562, which is called \$1.56. **4.** 12 × \$6.75 = \$81.00; \$25 + \$81 = \$106. **5.** $\frac{18}{20}$ = 90%; $\frac{9}{12}$ = $\frac{3}{4}$ = 75%; so today her mark was 15% better. **6.** 100 × \$.05 = \$5.00; \$5.00 − \$3.75 = \$1.25. **7.** (1) \$1.75 + \$.80 = \$2.55; \$3.00 − \$2.55 = \$.45; (2) \$.45 ÷ \$3.00 = .15 = 15%. **8.** From 9 A.M. to 6 P.M. is 9 hr.; $2\frac{1}{2}$ hr. + $1\frac{1}{2}$ hr. = 4 hr.; 9 hr. − 4 hr. = 5 hr.; $2\frac{1}{2}$ × 36 mi. = 90 mi.; 305 mi. − 90 mi. = 215 mi.; 215 mi. ÷ 5 = 43 mi. **9.** \$7.95 − \$7.39 = \$.56; 9 × \$.56 = \$5.04. **10.** 16 oz. ÷ 4 oz. = 4 (times); 4 × \$.15 = \$.60.

Page 238

Aim: To give a diagnostic test with page references for remedial work

Key: **1.** 138%; 425%; 700%; 1173%; 175%. **2.** 3; 4.12; 1; 15.5. **3.** $\frac{1}{3}$; $\frac{7}{8}$; $\frac{7}{10}$. **4.** 75; $48\frac{3}{4}$. **5.** 125; 250. **6.** 73; 51. **7.** (1) \$20 − \$15 = \$5, \$5 ÷ \$15 = $.33\frac{1}{3}$ = $33\frac{1}{3}$% (increase); (2) \$400 − \$200 = \$200, \$200 ÷ \$200 = 1 = 100% (increase); (3) \$396 − \$297 = \$99, \$99 ÷ \$396 = .25 = 25% (decrease). **8.** (1) \$30 − \$18 = \$12, \$12 ÷ \$30 = .4 = 40% (decrease); (2) \$1.75 − \$1.50 = \$.25, \$.25 ÷ \$1.50 = $.16\frac{2}{3}$ = $16\frac{2}{3}$% (increase); (3) \$6.60 − \$4.80 = \$1.80, \$1.80 ÷ \$4.80 = $.37\frac{1}{2}$ = $37\frac{1}{2}$% (increase). **9.** 14%; 10%; $12\frac{1}{2}$%. **10.** 15%; 20%; 11%. **11.** (1) \$9.36 − \$7.02 = \$2.34, \$2.34 ÷ \$9.36 = .25 = 25%; (2) \$6.50 ÷ \$4.55 = \$1.95, \$1.95 ÷ \$6.50 = .3 = 30%; (3) \$82 − \$41 = \$41; \$41 ÷ \$82 = .5 = 50%.

Chapter 8

Aims of Chapter 8. The major aims of Chapter 8 are to:

1. Reteach and extend the reading of pictographs.
2. Reteach and extend the construction of pictographs.
3. Review and extend the reading of bar graphs.
4. Review and extend the construction of bar graphs.
5. Review and extend the reading of line graphs.
6. Review and extend the construction of line graphs.
7. Teach the construction and reading of circle graphs when the data are given in per cents.
8. Teach the construction and reading of circle graphs when the data are given in numbers.
9. Review the understandings and skills taught in previous chapters.

Page 239

Aim: To reteach the reading of a simple pictograph

Suggestions: Easy pictographs (picture graphs) were introduced in Grade 6 with numbers going only into the thousands. In this chapter the work on pictographs is retaught and extended to include very large numbers. The pictograph on page 239 represents the facts about apple production in 6 states. A pictograph makes use of pictures to represent number facts. Pupils should learn to interpret them since they are frequently found in newspapers and magazines as well as in school texts on different subjects. Remind the pupils that a whole basket represents 2 million bushels; $\frac{1}{2}$ basket, therefore, represents 1 million bushels. The primary purpose of the pictograph on page 239 is to enable the reader to compare the apple production in the 6 states, one with the other, and not to give exactly the number of bushels produced in each state. In reading the pictograph the general comparisons are much more important than the numbers obtained by figuring the apple production of each state.

Key: **2.** 8,000,000 bu.; 20,000,000 bu.; 1,000,000 bu.; 7,000,000 bu. **3.** 23,000,000 bu.; 15,000,000 bu.; 11,000,000 bu.; 8,000,000 bu.; 6,000,000 bu.; 6,000,000 bu. **4.** 8,000,000 bu.; 4,000,000 bu.; 2,000,000 bu. **5.** About 4 times.

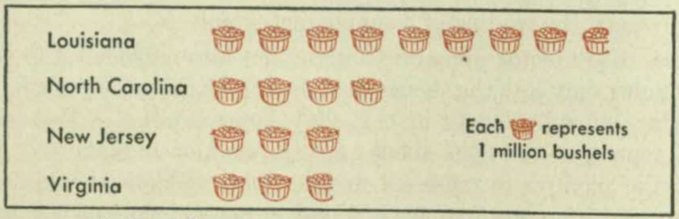
Pages 240–241

Aim: To extend the topic of pictographs to a more difficult case

Suggestions: The symbol in the pictograph on page 240 represents 500,000 people, or $\frac{1}{2}$ million people. Since there are $22\frac{1}{2}$ symbols representing Pennsylvania, the population of Pennsylvania can be found by finding $22\frac{1}{2} \times 500{,}000$ people or by finding $22\frac{1}{2} \times \frac{1}{2}$ million people. Either way the result is 11,250,000 people. The colored background at the left end of this row of men represents the city of Philadelphia, which has 4 men standing in it. The population of Philadelphia, therefore, is $4 \times \frac{1}{2}$ million, or 2,000,000, people. From this pictograph it is seen that $4 \div 22\frac{1}{2} = \frac{8}{45}$, or about $\frac{1}{6}$, of the total population of the state lives in Philadelphia. Next have the pupils study the pictograph for New York State and compare the population of New York City with the population of the state. They should compare also the states of New York and Pennsylvania and the cities of New York and Philadelphia. Similar comparisons can be made with each of the other states shown in the pictograph on page 240. In the answers for the exercises on page 241, the fractions showing what part of the population of each state lives in its largest city are to be given only approximately since pupils will probably be able to recognize the symbols only as whole figures or half figures of men.

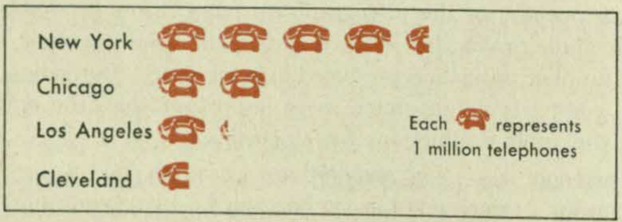
Key: **1.** $3\frac{1}{2}$ million. **2.** (1) $16\frac{3}{4}$ million; (2) $7\frac{3}{4} \div 16\frac{3}{4} = \frac{31}{67}$, or a little less than $\frac{1}{2}$. **3.** (1) $11\frac{1}{4}$ million; (2) $2 \div 11\frac{1}{4} = \frac{8}{45}$, or about $\frac{1}{6}$. **4.** $10\frac{1}{4}$ million; less than half. **5.** $15\frac{3}{4}$ million; $2\frac{1}{2} \div 15\frac{3}{4} = \frac{20}{126}$, or about $\frac{1}{6}$. **6.** (1) $1\frac{1}{2}$ million; (2) $7\frac{3}{4}$ million; (3) $1\frac{1}{2} \div 7\frac{3}{4} = \frac{6}{31}$; so the population of Detroit is about $\frac{1}{5}$ that of Michigan. **7.** $16\frac{3}{4} \div 3\frac{1}{4} = 5\frac{2}{13}$, or about $5\frac{1}{6}$ (times). **8.** $7\frac{3}{4} \div 1\frac{1}{2} = 5\frac{1}{6}$ (times).

9.



Bushels of Sweet Potatoes Grown in Four States

10.



Number of Telephones in Four Cities

Page 242

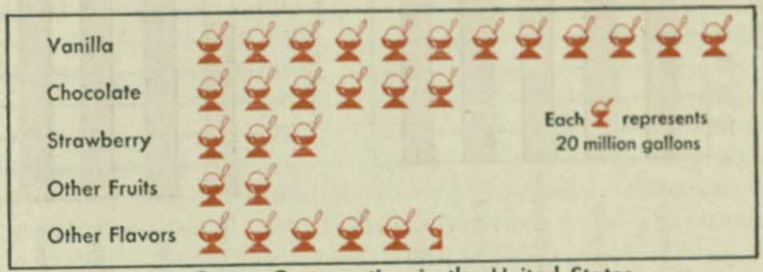
Aim: To give more practice in reading and constructing a pictograph

Suggestions: The primary purpose of this pictograph is to show the relative consumption of the different flavors of ice cream. This topic will be of interest to all pupils. From this pictograph the pupils can easily see that about twice as much vanilla ice cream is consumed as chocolate ice cream. The pictograph shows also that about 7 times as much vanilla ice cream is consumed as strawberry ice cream. Call the pupils' attention to the fact that chocolate ice cream is represented by $6\frac{1}{2}$ dishes; since each whole dish represents 20 million gallons, $\frac{1}{2}$ dish represents 10 million gallons.

Workbook Reference: Arithmetic Workshop, Book 7, page 92

Key: **1.** Vanilla; chocolate. **2.** 290 million gal.; 130 million gal.; 40 million gal.; 30 million gal.; 100 million gal.; 590 million gal. **3.** 2 times; 7 times; $\frac{29}{59}$, or about $\frac{1}{2}$.

4.



Ice Cream Consumption in the United States

Page 243

Aim: To review and extend the making of easy bar graphs

Suggestions: The topic of bar graphs is not a new one since it was first presented in *American Arithmetic, Grade 5* and repeated in *American Arithmetic, Grade 6*. In the graph in ex. 1 the new feature is to learn how to use a part of a square. In all earlier work pupils were taught to round off numbers to the nearest 10, nearest 100, and so on, before drawing the bars. The method on page 243 should be carefully studied. On this graph the bar representing 73% covers 7 whole squares and $\frac{3}{10}$ of the next square. In drawing this bar it is necessary to estimate $\frac{3}{10}$ of a square as closely as possible by the eye. Similarly, the bar representing 87% covers 8 squares and $\frac{7}{10}$ of the next square. In work of this kind the purpose is to get the bars approximately the right length. Precise drawing in such cases is not possible. If a person were reading this graph and did not know in advance that the third bar represented 87%, the closest that he could read it would probably be anywhere from 86% to 88%. The main purpose of this graph is to provide an easy way of comparing Jack's marks for the 4 weeks. In this comparison it is evident that each week's mark was considerably better than that of the week before. It is evident also that Jack's marks jumped from 60% the first week to well over 90% the fourth week.

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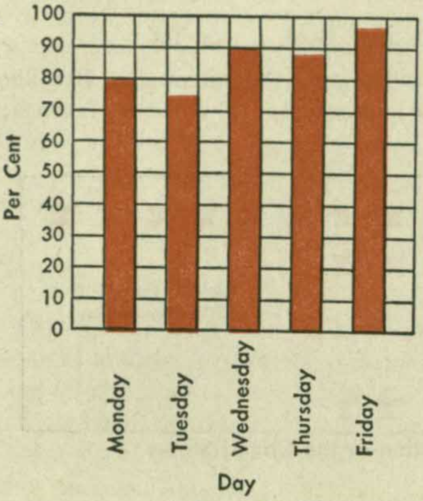
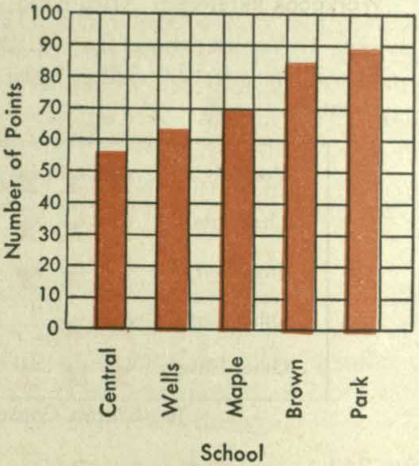
Key: **2.** $7\frac{2}{5}$ squares; $8\frac{3}{5}$ squares; $7\frac{9}{10}$ squares; $9\frac{4}{5}$ squares; $8\frac{1}{5}$ squares.

3.



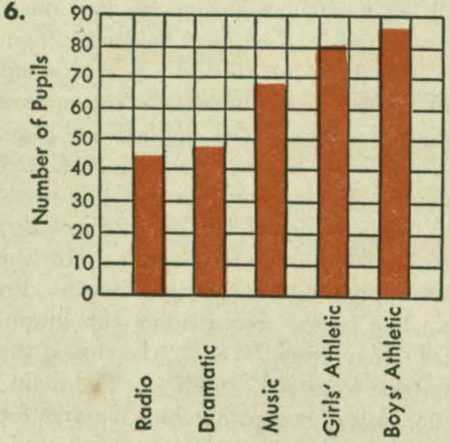
Marks in English

5.



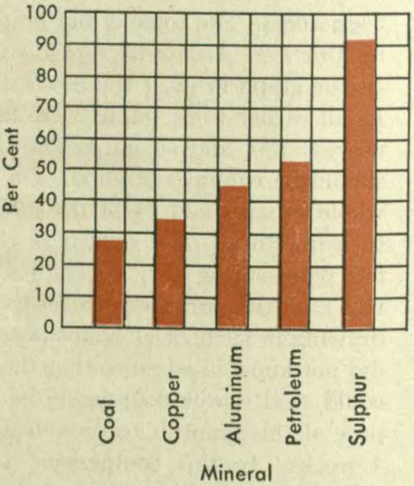
Points Made by Competing Schools

6.



Membership of Clubs in Seventh Grades

7.



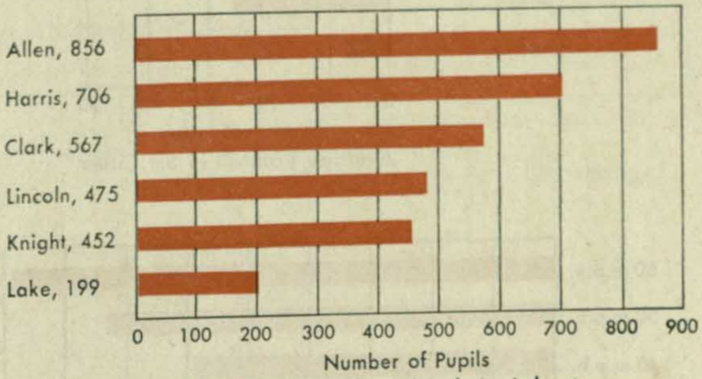
Per Cents of Minerals Produced in the United States

Page 244

Aim: To review and extend the making of bar graphs with large numbers

Suggestions: In the graph in ex. 1 each space on the scale represents 100; hence, such a graph is more difficult to draw than one in which each space represents 10. The directions for drawing this graph, which are given in ex. 1, require careful study. According to these directions, each bar is so drawn that its length is *approximately* correct. For example, to represent 981 mi., which is the length of the Ohio River, you divide 981 by 100, which gives 9.81; you then round off 9.81 to the nearest tenth, which gives 9.8. Hence, the bar representing 981 is drawn 9.8 spaces long. This length of 9.8 spaces is, of course, an *approximate* representation. These 9.8 spaces not only represent 981, but they could equally well represent any of the following numbers: 975, 976, 977, 978, 979, 980, 982, 983, or 984. To give the reader more information in a situation of this kind, the exact length of each river is printed at the left end of the bar. For example, the length of the Ohio River is given as 981 mi., the length of the Yellowstone is given as 669 mi., and so on. In a case like this the question naturally arises, "Why do you draw the bars and take the trouble to make them approximately correct in length when the numbers are already there?" The main reason for drawing this bar graph is to make easy a *comparison* of the lengths of the rivers. The first bar shows quickly that the Ohio River is much longer than the Yellowstone River. The Ohio River is also about twice as long as the Wabash River and about three times as long as the Hudson River. The bars also show that the Hudson and Potomac rivers are almost the same length. All these facts are seen at a glance. Bar graphs are drawn primarily to make comparisons easy. When the pupils are drawing the graph for ex. 2 on page 244, make sure that they use at least $\frac{1}{2}$ inch for each space on the scale.

Key: 2.



Enrollment in Six Schools in Johnstown

Page 245

Aim: To give some general directions for constructing bar graphs

Suggestions: In addition to the suggestions for making graphs given on page 245, the following will be helpful to the pupils in determining the number of spaces on the scale that will be needed to represent a given number approximately: *To find the number of spaces needed to represent a given number, divide the given number by the value of each space and round off the result correct to the nearest tenth.* For example, if each space represents 1000, the number of spaces necessary to represent 9482 approximately is obtained by dividing 9482 by 1000, which gives

9.482, and then rounding off 9.482 correct to the nearest tenth. This gives 9.5 as the required number of spaces. Likewise, if each space represents 1,000,000, the number of spaces necessary to represent 7,439,100 approximately is obtained by dividing 7,439,100 by 1,000,000. This operation is done, of course, by moving the decimal point 6 places to the left, and rounding off to 7.4.

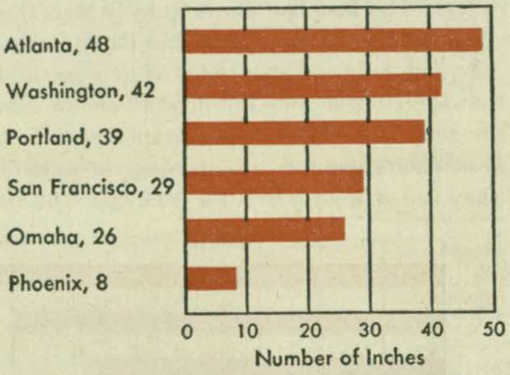
Page 246

Aim: To give practice on making bar graphs

Suggestion: These exercises give practice on all the new difficulties encountered on pages 243–245.

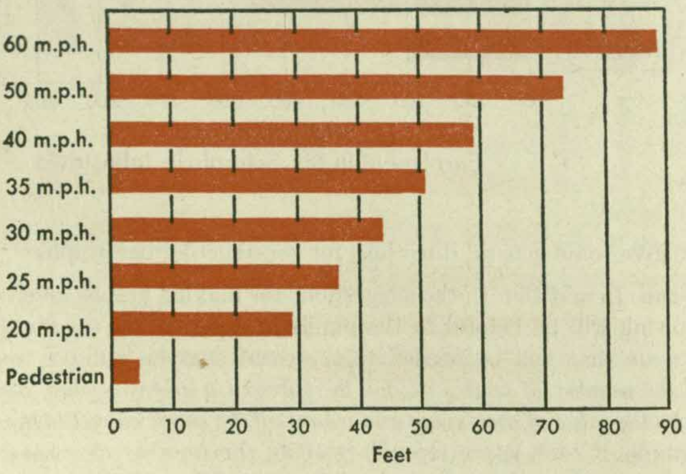
Workbook Reference: Arithmetic Workshop, Book 7, page 94

Key: 1.



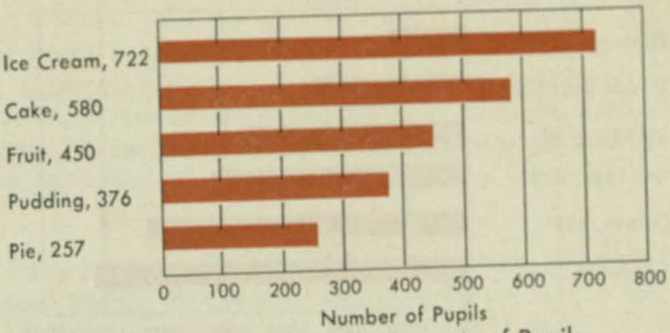
Average Rainfall in Six Cities

2.



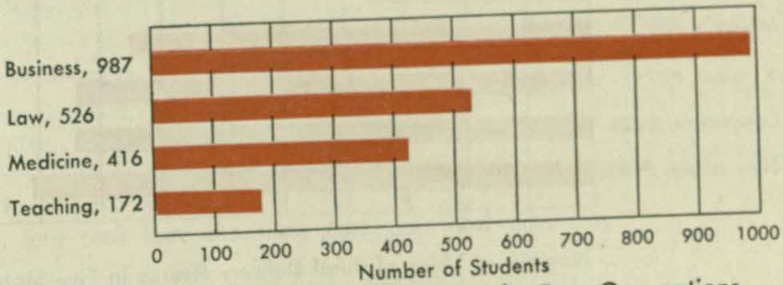
Distances Covered in One Second

3.



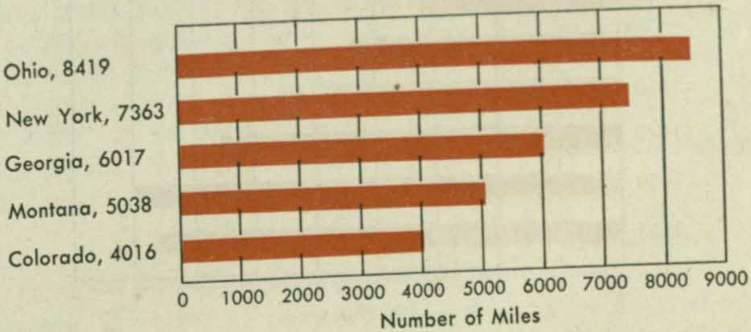
The Favorite Desserts of Pupils

4.



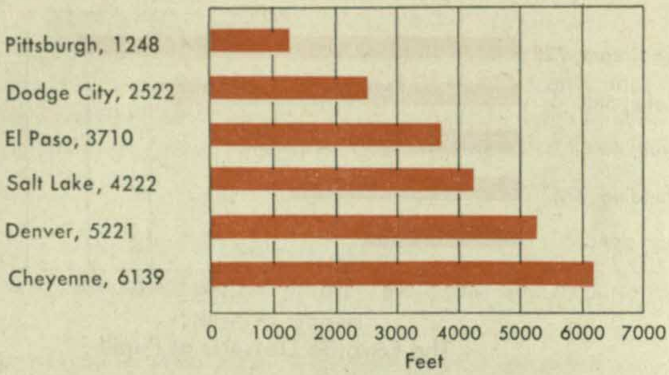
College Students Preparing for Four Occupations

5.



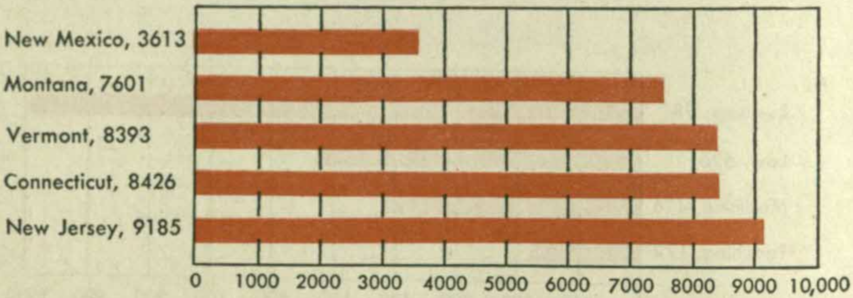
Railroad Mileage in Five States

6.



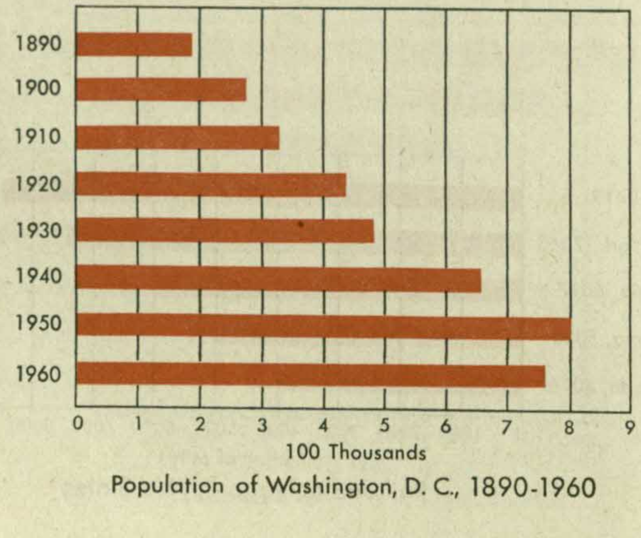
Altitudes of Six Cities

7.



Number of Miles of Rural Delivery Routes in Five States

8.



Population of Washington, D. C., 1890-1960

Page 247

Aim: To present new sets of improvement tests on multiplication and division

Key: **1.** 5,256,279; 6,971,547; 1,206,446; 5,444,352; 2,825,550. **2.** 3,367,296; 1,010,242; 6,333,600; 516,076; 8,052,050. **3.** 1,793,885; 1,291,176; 2,063,875; 2,804,772; 4,502,880. **4.** 43.1; 58.5. **5.** 88.6; 65.8. **6.** 73.2; 53.5. **7.** 27.1; 39.2. **8.** 54.8; 36.3. **9.** 43.4; 26.5.

Page 248

Aim: To give pupils practice in reading two bar graphs with very large numbers

Suggestions: Caution the pupils to study carefully the scale on each graph before giving any answer. The last two questions in ex. 1 show that the bar for the Golden Gate Bridge could represent any length from 4150 to 4249 since these two numbers and all between them could be rounded off to 4200. Therefore, 4200 is an approximate number, and any answer that could be rounded off to 4200 should be accepted. However, pupils should give answers to the nearest 100 or nearest 50. While the answer book and the key give only one answer for each bar, you should accept any other answer that is approximately correct. For ex. 2 suggest that all answers be given in millions, as 76 million for 1900.

Workbook Reference: Arithmetic Workshop, Book 7, page 93

Key: **1.** 500 ft.; Golden Gate, 4200 ft.; George Washington, 3500 ft.; Whitestone, 2300 ft.; Delaware Memorial, 2150 ft.; Brooklyn, 1600 ft.; $\frac{2}{5}$; yes; yes. **2.** 10 million; 10; 1900, 76 million; 1910, 92 million; 1920, 106 million; 1930, 122 million; 1940, 132 million; 1950, 150 million; 1960, 179 million.

Pages 249–250

Aim: To review fractions and general problem solving

Key: Page 249 **1.** $\frac{3}{4}$ mi. + $\frac{3}{8}$ mi. = $1\frac{1}{8}$ mi.; $3\frac{1}{4}$ mi. − $1\frac{1}{8}$ mi. = $2\frac{1}{8}$ mi. **2.** $5\frac{1}{2}$ mi. **3.** $1\frac{3}{8}$ in. ÷ $\frac{1}{4}$ in. = $5\frac{1}{2}$ (times); $5\frac{1}{2}$ × 10 mi. = 55 mi. **4.** $14\frac{1}{8}$; $6\frac{3}{5}$; $19\frac{1}{8}$; $10\frac{1}{6}$; $11\frac{1}{4}$; $10\frac{3}{4}$; 18. **5.** $5\frac{2}{5}$; $2\frac{13}{24}$; $6\frac{5}{6}$; $3\frac{1}{8}$; $4\frac{13}{16}$; $2\frac{1}{2}$; $2\frac{3}{10}$. **6.** $1\frac{1}{6}$; $\frac{13}{16}$; $5\frac{1}{3}$; $3\frac{13}{15}$; $1\frac{7}{10}$; $1\frac{5}{12}$; $6\frac{2}{3}$. **7.** $7\frac{1}{2}$; $7\frac{1}{2}$; $2\frac{1}{2}$; $49\frac{1}{2}$. **8.** $1\frac{1}{4}$; $\frac{3}{4}$; $3\frac{3}{4}$; $19\frac{4}{5}$. **9.** $\frac{5}{16}$; 1; $6\frac{1}{4}$; $5\frac{1}{3}$. **10.** $\frac{7}{10}$; 12; $\frac{3}{10}$; 10.

Page 250 **1.** \$2.75 + \$3.00 + \$2.25 + \$2.10 + \$2.95 = \$13.05; 6 × \$2.75 = \$16.50; \$16.50 − \$13.05 = \$3.45. **2.** At the rate of 29.4 mi. in $\frac{3}{4}$ hr., in $\frac{1}{4}$ hr. she would go $\frac{1}{3}$ × 29.4 mi., or 9.8 mi.; in 1 hr. ($\frac{4}{4}$) she would go 4 × 9.8 mi., or 39.2 mi. **3.** 1 lb. ÷ $\frac{1}{8}$ lb. = 8 (times); 8 × 5¢ = 40¢. **4.** 16 × 17.5 mi. = 280 mi. **5.** (1) 195 bu. ÷ 30 bu. = $6\frac{1}{2}$ (times); (2) $6\frac{1}{2}$ = 650%. **6.** $\frac{3}{4}$ × 50¢ = $37\frac{1}{2}$¢, which is called 38¢. **7.** (1) 185 ÷ 365 = $\frac{185}{365}$ = $\frac{37}{73}$; (2) $\frac{37}{73}$ = .506, or 51%. **8.** 70 min. ÷ $3\frac{1}{2}$ = 20 min. **9.** 378 mi. ÷ 72 mi. = $5\frac{1}{4}$ (times). **10.** 500 ÷ 5 = 100 (fives); 100 × 2 min. = 200 min.; 200 min. = 3 hr. 20 min.

Page 251

Aim: To review the reading of simple line graphs

Suggestions: The topic of line graphs was introduced in *American Arithmetic, Grade 6*, so the work on page 251 is in the nature of a review. Put emphasis on the fact that each point on the graph represents *two different values.* On this particular graph, each point represents a certain hour and a certain temperature. Line graphs, like bar graphs, are used largely as a means of comparing numerical facts. For example, the graph on page 251 shows how the temperature changes from hour to hour. It is easily seen from the graph that during certain hours the changes occur somewhat slowly, while at other hours the changes are more rapid. The great advantage of the graph is that it gives you at a glance a picture of the various changes that have taken place during the entire day. These facts could have been given in paragraph form or by making a table (see ex. 1 and 2 on page 252), but the graph shows the information at a glance.

Workbook Reference: Arithmetic Workshop, Book 7, page 95

Key: **2.** 70° at 9 A.M.; 70° at 10 A.M.; 71° at 11 A.M.; 74° at 12 noon; 76° at 1 P.M.; 77° at 2 P.M.; 77° at 3 P.M.; 76° at 4 P.M.; 75° at 5 P.M.; 72° at 6 P.M.; 68° at 7 P.M.; 66° at 8 P.M. **3.** From 2 to 3; 8 P.M. **4.** From 11 to 12; from 6 to 7. **5.** 70°; 75°; $75\frac{1}{2}$°; 67°.

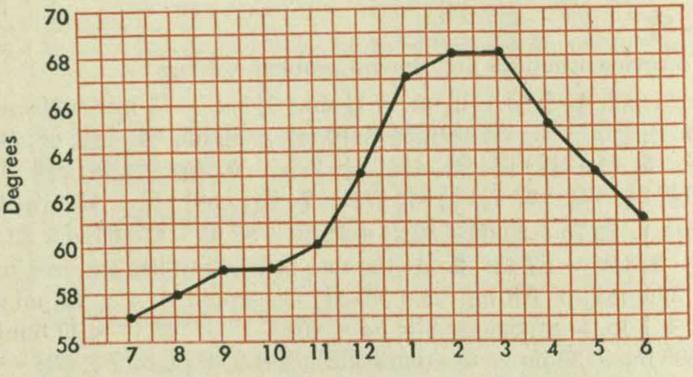
Page 252

Aim: To give practice in drawing line graphs

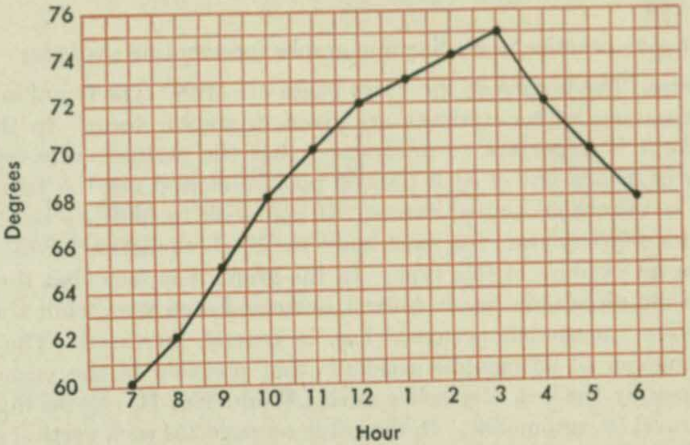
Suggestion: When the pupils bring in their finished graphs for page 252, you might ask about graphs 1 and 2 some questions of the type given in ex. 2–5 on page 251.

Workbook Reference: Arithmetic Workshop, Book 7, page 96

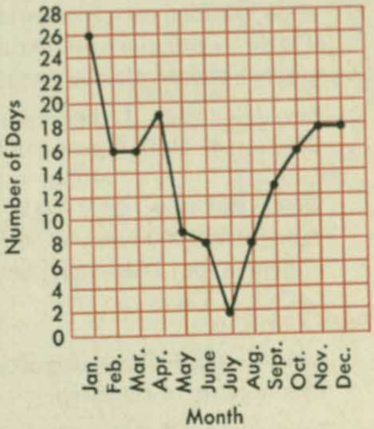
Key: 1.



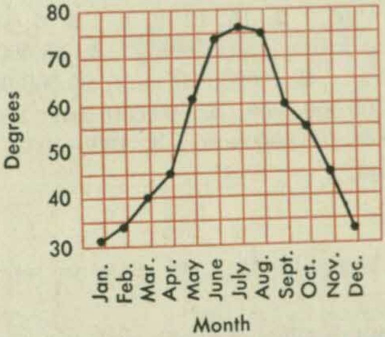
2.


Degrees
76
74
72
70
68
66
64
62
60
7
8
9
10
11
12
1
2
3
4
5
6
Hour

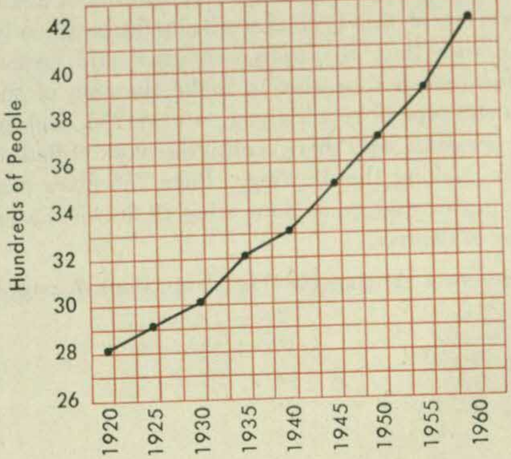
5.


Number of Days
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.
Month

3.


Degrees
80
70
60
50
40
30
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.
Month

6.


Hundreds of People
42
40
38
36
34
32
30
28
26
1920
1925
1930
1935
1940
1945
1950
1955
1960
Year

Pages 253–254

Aim: To give practice in reading line graphs representing statistics

Suggestions: The graphs on these two pages are of the type found in newspapers and magazines, where statistics are given in graphic form. In the graph on page 253 it is important to understand that the vertical scale represents the number of deaths out of each 100,000 population and not the total number of deaths in the whole United States. If you wish to find the total number of deaths for any one year, you must know the total population for that year. See ex. 5 for an exercise of this type. In the graph it is seen that the number of deaths from diphtheria has, in general, decreased each year, while the number of deaths from automobile accidents has, in general, increased. The sharp drop in the number of automobile accidents that occurred in the years 1941–1943 was caused by the lack of gasoline during World War II. Rationing greatly reduced travel by automobile. In the graph on page 254 each vertical space represents 1 million cars, so answers can be given to the nearest million cars. However, if answers are given to tenths of a million, such as 7.2 million or 7,200,000, these answers should also be accepted if they appear to be correct.

Key: Page 253 **1.** Decreasing; 1921; 1955. **2.** 15; 12; 2; 1. **3.** 6; about 29; 23; 23. **4.** 1937; because of less driving during gas rationing. **5.** 38,400 deaths. Page 254 **1.** 1 million cars; 1 year. **2.** 8,000,000 cars; 20,000,000 cars; 23,000,000 cars; 22,500,000 cars; 27,500,000 cars; 27,500,000 cars; 25,500,000 cars; 40,000,000 cars; 52,000,000 cars; 61,700,000 cars. **3.** 1929–1933, because of depression; 1941–1944, because of war.

Pages 255–256

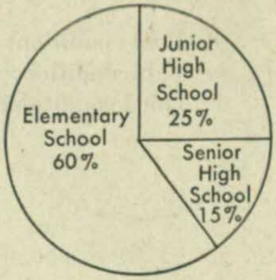
Aim: To teach the construction of circle graphs when per cents are represented by the parts of the circle

Suggestions: Circle graphs require careful drawing of angles with the protractor; therefore, a review of this type of work should be given before circle graphs are studied. Be sure that all pupils can draw and measure angles accurately; otherwise, they will not be able to make the sum of the angles 360° in their drawing. In the type of graph given on page 255, each part of the whole is expressed by a per cent; and there is only the work of finding the correct per cents of 360° before making the drawing. Page 256 gives practice on drawing this type of circle graph, and ex. 5 shows what to do when a per cent does not give a whole number of degrees.

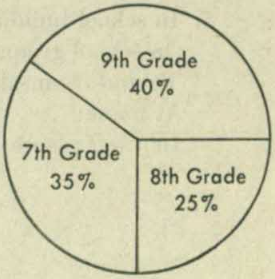
Workbook Reference: Arithmetic Workshop, Book 7, pages 99 to 101

Key: Page 255 **2.** $60\% + 25\% = 85\%$; $100\% - 85\% = 15\%$.

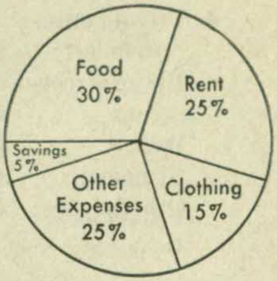
Elementary School: $.60 \times 360^\circ = 216^\circ$
Junior High School: $.25 \times 360^\circ = 90^\circ$
Senior High School: $.15 \times 360^\circ = 54^\circ$



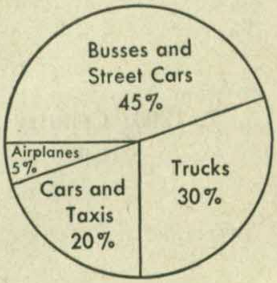
Page 256 **1.** 7th grade: $.35 \times 360^\circ = 126^\circ$
8th grade: $.25 \times 360^\circ = 90^\circ$
9th grade: $.40 \times 360^\circ = 144^\circ$



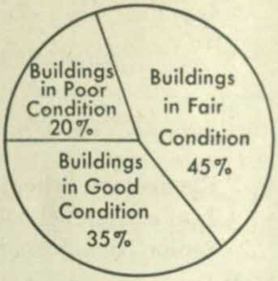
2. Food: $.30 \times 360^\circ = 108^\circ$
Rent: $.25 \times 360^\circ = 90^\circ$
Clothing: $.15 \times 360^\circ = 54^\circ$
Other expenses: $.25 \times 360^\circ = 90^\circ$
Savings: $.05 \times 360^\circ = 18^\circ$



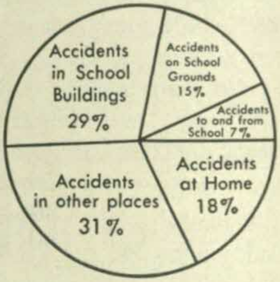
3. Busses and street cars: $.45 \times 360^\circ = 162^\circ$
Trucks: $.30 \times 360^\circ = 108^\circ$
Cars and taxis: $.20 \times 360^\circ = 72^\circ$
Airplanes: $.05 \times 360^\circ = 18^\circ$



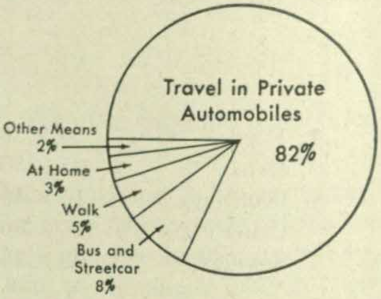
4. Poor condition: $.20 \times 360^\circ = 72^\circ$
Fair condition: $.45 \times 360^\circ = 162^\circ$
Good condition: $.35 \times 360^\circ = 126^\circ$



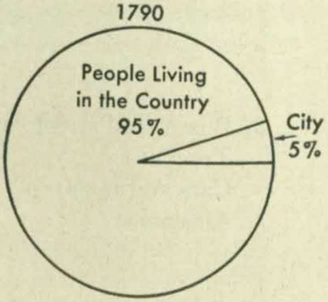
5. In school buildings: $.29 \times 360^\circ = 104.4^\circ$, or 104°
On school grounds: $.15 \times 360^\circ = 54^\circ$
To and from school: $.07 \times 360^\circ = 25.2^\circ$, or 25°
At home: $.18 \times 360^\circ = 64.8^\circ$, or 65°
In other places: $.31 \times 360^\circ = 111.6^\circ$, or 112°



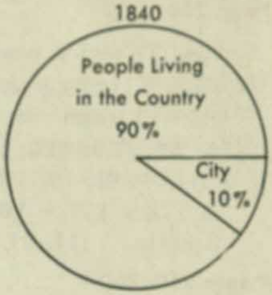
6. Private automobiles: $.82 \times 360^\circ = 295.2^\circ$, or 295°
Bus or streetcar: $.08 \times 360^\circ = 28.8^\circ$, or 29°
Walk: $.05 \times 360^\circ = 18^\circ$
At home: $.03 \times 360^\circ = 10.8^\circ$, or 11°
Other means: $.02 \times 360^\circ = 7.2^\circ$, or 7°



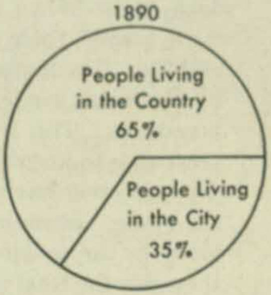
7. 1790: Country: $.95 \times 360^\circ = 342^\circ$
City: $.05 \times 360^\circ = 18^\circ$



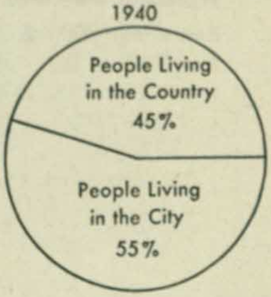
1840: Country: $.90 \times 360° = 324°$
City: $.10 \times 360° = 36°$



1890: Country: $.65 \times 360° = 234°$
City: $.35 \times 360° = 126°$



1940: Country: $.45 \times 360° = 162°$
City: $.55 \times 360° = 198°$



Page 257

Aim: To give the last set of improvement tests in addition

Suggestion: At this stage nearly all pupils should be able to secure a high score, if not a perfect one, on addition with whole numbers. It may be that individual pupils will need to take again the test on the 100 addition facts on page 4. If remedial work on the facts is needed, be sure to begin such work.

Key: **1.** 4500; 5054; 3935; 4167; 6020; 5364. **2.** 5465; 4424; 3941; 6441; 5403; 5623. **3.** 4157; 4924; 6431; 5244; 4896; 4977.

Page 258

Aim: To review work on decimals

Key: **1.** 12.799; 1648.4. **2.** \$7.82; \$6951.23. **3.** 12.673; .45; 59.032; .00009; 1.592; 2.16500. **4.** .028; 17.9; .053; 114; 10.3. **5.** 79; 97.25; 5600; 4.75; 24.625. **6.** 2.6; 27.9; 22.5; .9; 8.0. **7.** 5.8; .5; 2.4; 13.8; 130.1. **8.** 4.10; 2.15; .99; .48; .31; .07; .01. **9.** 170; 17; .17; .017. **10.** $2.27 + 2.14 + 2.94 + 3.50 + 1.83 + 4.24 + 5.73 + 4.77 + 5.08 + 4.31 + 2.70 + 2.06 = 41.57$ (in.); $41.57 \div 12 = 3.464$, or 3.46 (in.). **11.** 41.57 in. − 14.09 in. = 27.48 in. **12.** 12 × 2.53 in. = 30.36 in.

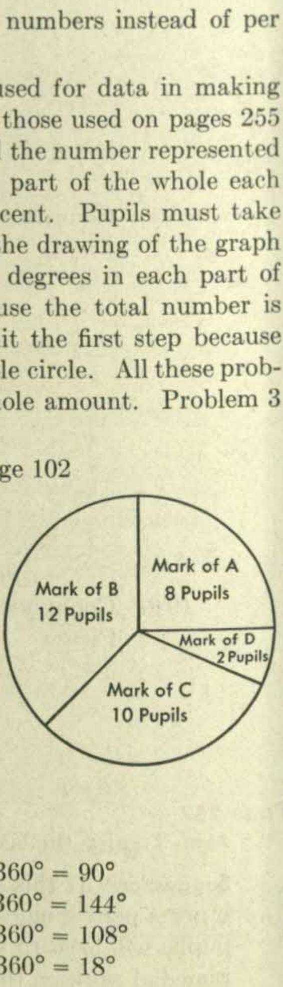
Pages 259–260

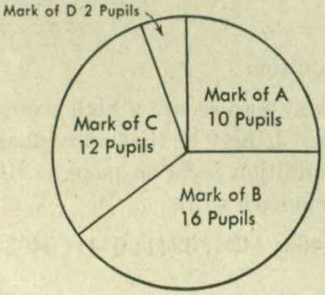
Aim: To teach the construction of circle graphs when numbers instead of per cents are represented by the parts of the circle

Suggestions: When numbers instead of per cents are used for data in making circle graphs, there are usually two extra steps beyond those used on pages 255 and 256. The first step is to add all the numbers to find the number represented by the whole circle. The second step is to find what part of the whole each number is. This fraction may be expressed as a per cent. Pupils must take great care to keep this work free from mistakes, since the drawing of the graph depends upon having correct computation to find the degrees in each part of the circle. Sometimes the first step is omitted because the total number is given in the problem. The problems on page 260 omit the first step because they give the total number to be represented by the whole circle. All these problems, however, could be reworded to leave out the whole amount. Problem 3 at the bottom of page 259 omits the whole amount.

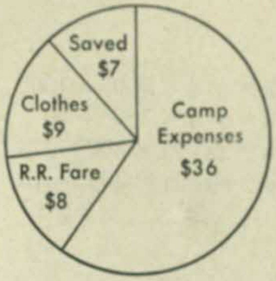
Workbook Reference: Arithmetic Workshop, Book 7, page 102

Key: Page 259 **2.** Mark of A: $\frac{8}{32} \times 360^\circ = 90^\circ$
Mark of B: $\frac{12}{32} \times 360^\circ = 135^\circ$
Mark of C: $\frac{10}{32} \times 360^\circ = 112\frac{1}{2}^\circ$
Mark of D: $\frac{2}{32} \times 360^\circ = 22\frac{1}{2}^\circ$



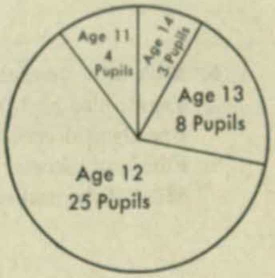


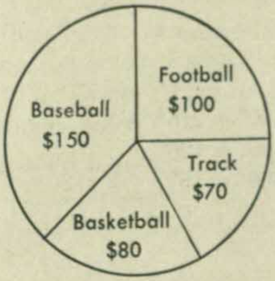
3. Mark of A: $\frac{10}{40} \times 360^\circ = 90^\circ$
Mark of B: $\frac{16}{40} \times 360^\circ = 144^\circ$
Mark of C: $\frac{12}{40} \times 360^\circ = 108^\circ$
Mark of D: $\frac{2}{40} \times 360^\circ = 18^\circ$



Page 260 **1.** Camp expenses: $\frac{36}{60} \times 360^\circ = 216^\circ$
Railroad fare: $\frac{8}{60} \times 360^\circ = 48^\circ$
Clothes: $\frac{9}{60} \times 360^\circ = 54^\circ$
Savings: $\frac{7}{60} \times 360^\circ = 42^\circ$

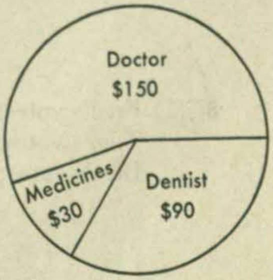
2. Pupils age 11: $\frac{4}{40} \times 360^\circ = 36^\circ$
Pupils age 12: $\frac{25}{40} \times 360^\circ = 225^\circ$
Pupils age 13: $\frac{8}{40} \times 360^\circ = 72^\circ$
Pupils age 14: $\frac{3}{40} \times 360^\circ = 27^\circ$

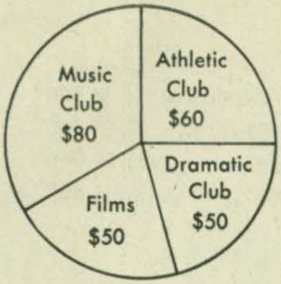




3. Football: $\frac{100}{400} = \frac{1}{4};\ \frac{1}{4} \times 360^\circ = 90^\circ$
Baseball: $\frac{150}{400} = \frac{3}{8};\ \frac{3}{8} \times 360^\circ = 135^\circ$
Basketball: $\frac{80}{400} = \frac{1}{5};\ \frac{1}{5} \times 360^\circ = 72^\circ$
Track: $\frac{70}{400} = \frac{7}{40};\ \frac{7}{40} \times 360^\circ = 63^\circ$

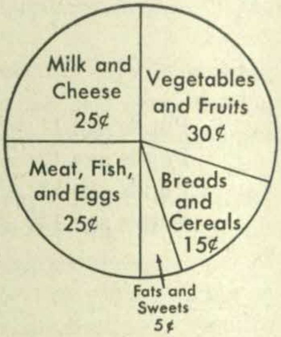
4. Doctor: $\frac{150}{270} = \frac{5}{9};\ \frac{5}{9} \times 360^\circ = 200^\circ$
Dentist: $\frac{90}{270} = \frac{1}{3};\ \frac{1}{3} \times 360^\circ = 120^\circ$
Medicines: $\frac{30}{270} = \frac{1}{9};\ \frac{1}{9} \times 360^\circ = 40^\circ$

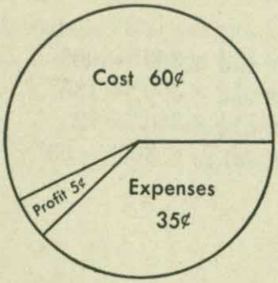




5. Music club: $\frac{80}{240} = \frac{1}{3};\ \frac{1}{3} \times 360^\circ = 120^\circ$
Athletic club: $\frac{60}{240} = \frac{1}{4};\ \frac{1}{4} \times 360^\circ = 90^\circ$
Dramatic club: $\frac{50}{240} = \frac{5}{24};\ \frac{5}{24} \times 360^\circ = 75^\circ$
Films: $\frac{50}{240} = \frac{5}{24};\ \frac{5}{24} \times 360^\circ = 75^\circ$

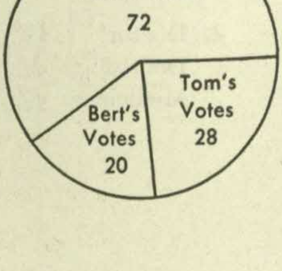
6. Milk and cheese: $\frac{25}{100} \times 360^\circ = 90^\circ$
Vegetables and fruits: $\frac{30}{100} \times 360^\circ = 108^\circ$
Breads and cereals: $\frac{15}{100} \times 360^\circ = 54^\circ$
Fats and sweets: $\frac{5}{100} \times 360^\circ = 18^\circ$
Meat, fish, and eggs: $\frac{25}{100} \times 360^\circ = 90^\circ$

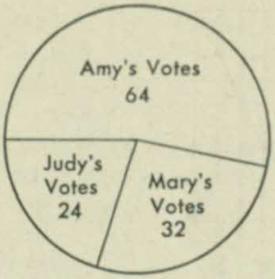




7. Cost: $\frac{60}{100} \times 360^\circ = 216^\circ$
Expenses: $\frac{35}{100} \times 360^\circ = 126^\circ$
Profit: $\frac{5}{100} \times 360^\circ = 18^\circ$

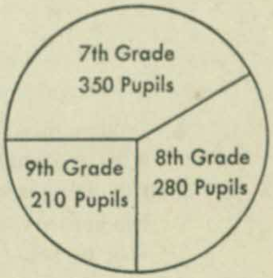
8. (1) Fred's votes: $\frac{72}{120} \times 360^\circ = 216^\circ$
Tom's votes: $\frac{28}{120} \times 360^\circ = 84^\circ$
Bert's votes: $\frac{20}{120} \times 360^\circ = 60^\circ$





8. (2) Amy's votes: $\frac{64}{120} \times 360° = 192°$
Mary's votes: $\frac{32}{120} \times 360° = 96°$
Judy's votes: $\frac{24}{120} \times 360° = 72°$

9. 7th grade: $\frac{350}{840} \times 360° = 150°$
8th grade: $\frac{280}{840} \times 360° = 120°$
9th grade: $\frac{210}{840} \times 360° = 90°$



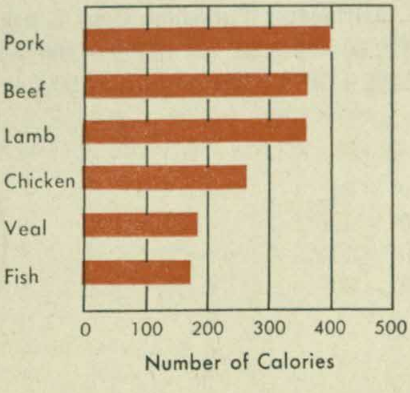
Page 261

Aim: To review bar, line, and circle graphs

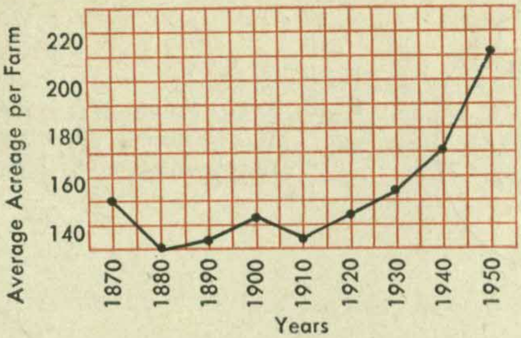
Suggestions: In ex. 6 on page 261 the pupils are asked to draw a circle graph. The number of degrees in each part of this graph is shown in the key below. Notice that the number of degrees for each angle is given to tenths of a degree rather than to the nearest whole degree. If each angle had been rounded off to the nearest whole degree, the sum of the four angles would have been 361°, which is 1° too large. When each angle is given to tenths of a degree the sum of all the angles is 360°.

Key: **2.** Crossing in middle of block, 3712; crossing at street corners, 2944; walking on highway, 1792; playing in streets, 1408; coming from behind parked cars, 1152; miscellaneous, 1792.

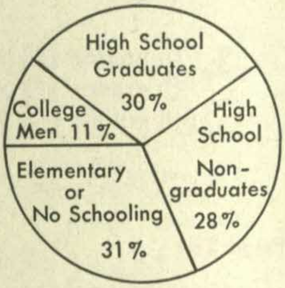
4.

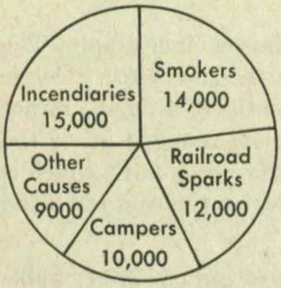


5.



6. College men: $.11 \times 360° = 39.6°$
H.S. graduates: $.30 \times 360° = 108°$
H.S. nongraduates: $.28 \times 360° = 100.8°$
Elementary school
or no schooling: $.31 \times 360° = 111.6°$





7. Incendiaries: $\frac{15,000}{60,000} = \frac{1}{4}; \frac{1}{4} \times 360° = 90°$
Smokers: $\frac{14,000}{60,000} = \frac{7}{30}; \frac{7}{30} \times 360° = 84°$
Railroad sparks: $\frac{12,000}{60,000} = \frac{1}{5}; \frac{1}{5} \times 360° = 72°$
Campers: $\frac{10,000}{60,000} = \frac{1}{6}; \frac{1}{6} \times 360° = 60°$
Other causes: $\frac{9,000}{60,000} = \frac{3}{20}; \frac{3}{20} \times 360° = 54°$

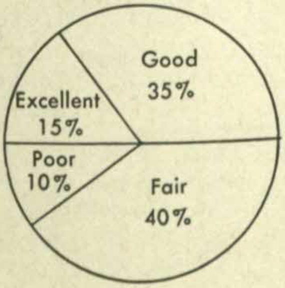
Page 262

Aim: To review all types of percentage problems

Workbook Reference: Arithmetic Workshop, Book 7, page 107

Key: **1.** (1) $1.80 \times \$22.50 = \$40.50$; (2) $180\% - 100\% = 80\%$. **2.** $.10 \times \$750 = \$75$. **3.** $15\% + 35\% + 40\% = 90\%; 100\% - 90\% = 10\%$.

Excellent: $.15 \times 360° = 54°$
Good: $.35 \times 360° = 126°$
Fair: $.40 \times 360° = 144°$
Poor: $.10 \times 360° = 36°$



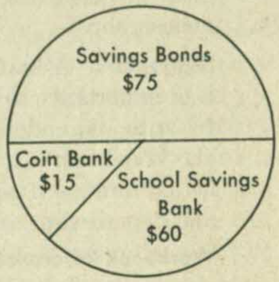
4. (1) .15 × 40 pupils = 6 pupils (excellent); (2) .35 × 40 pupils = 14 pupils (good); (3) .40 × 40 pupils = 16 pupils (fair); (4) .10 × 40 pupils = 4 pupils (poor). **5.** (1) 1.40 × $\$100 = \$140$; (2) .60 × $\$100 = \$60$; (3) 2.40 × $\$100 = \$240$. **6.** $\$962.50 \div \$17{,}500 = .05\frac{1}{2} = 5\frac{1}{2}\%$. **7.** $\$18.60 \div \$49.60 = .37\frac{1}{2} = 37\frac{1}{2}\%$. **8.** 10,649 ÷ 9260 = 1.15 = 115%. **9.** 10,649 − 9260 = 1389; 1389 ÷ 9260 = .15 = 15%. **10.** (1) $\$15.50 + \$6.50 = \$22.00$, $\$25.00 - \$22.00 = \$3.00$; (2) $\$3.00 \div \$25.00 = .12 = 12\%$.

Page 263

Aim: To present Problem Test 8

Key: **1.** 7¢ − 5¢ = 2¢; 2¢ ÷ 5¢ = $\frac{2}{5}$ = 40%. **2.** From 5:30 to 10:30 is 5 hr.; 7 × 5 hr. = 35 hr.; 2 × 75 watts = 150 watts; 100 watts + 150 watts = 250 watts; 35 × 250 = 8750 (watt-hr.); 8750 ÷ 1000 = 8.75 (kw-hr); 8.75 × 4¢ = 35¢. **3.** (1) 50 mi. ÷ 60 = $\frac{5}{6}$ mi. per minute; 1 mi. = 5280 ft.; $\frac{5}{6}$ × 5280 ft. = 4400 ft.; (2) 4400 ft. ÷ 60 = $73\frac{1}{3}$ ft. **4.** 1442 kw-hr − 1396 kw-hr = 46 kw-hr; 46 × $4\frac{1}{2}$¢ = 207¢, or $\$2.07$.

5. $\$75 + \$60 + \$15 = \$150$.
Savings bonds: $\frac{75}{150} = \frac{1}{2}$; $\frac{1}{2} \times 360° = 180°$
School savings bank: $\frac{60}{150} = \frac{2}{5}$; $\frac{2}{5} \times 360° = 144°$
Coin bank: $\frac{15}{150} = \frac{1}{10}$; $\frac{1}{10} \times 360° = 36°$



6. $\$.25 + \$.40 + \$.10 + \$.25 + \$.35 = \$1.35$; $\$2.50 - \$1.35 = \$1.15$; $\$1.15 \div 5 = \$.23$. **7.** 8 buttons + 2 buttons + 2 buttons = 12 buttons; 12 ÷ 3 = 4 (times); 4 × $\$.25 = \$1.00$. **8.** 4171.2 mi. − 3892.8 mi. = 278.4 mi.; 278.4 mi. ÷ 16 = 17.4 mi. **9.** .04 × 22,825 pupils = 913 pupils. **10.** $176 \div 528 = .33\frac{1}{3} = 33\frac{1}{3}\%$.

Page 264

Aim: To give a diagnostic test with page references for remedial work

Key: **1.** $14\frac{5}{8}$; $9\frac{3}{5}$; $12\frac{1}{12}$; $16\frac{23}{30}$. **2.** $3\frac{1}{3}$; $4\frac{3}{8}$; $2\frac{7}{10}$; $\frac{13}{16}$. **3.** .50; .27; .008; 1.199. **4.** $67\frac{1}{2}$; $7\frac{1}{2}$; $\frac{3}{8}$. **5.** 6; $3\frac{1}{2}$; $1\frac{5}{16}$. **6.** .36; .420; .0096. **7.** 2.37; 12.25; 2.94; 1.17. **8.** .92; .03; .74; 15.05. **9.** $\$5.40$; $\$8.75$; $\$18$. **10.** $\$6.11$; $\$81$; $\$220$. **11.** $\$.16$; $\$1.06$; $\$1.88$.

Chapter 9

Aims of Chapter 9. The major aims of Chapter 9 are to:

1. Teach how to open a checking account and to write deposit slips and checks.
2. Teach how to endorse a check and fill in check stubs.
3. Teach the computation of simple interest for 1 yr. and $\frac{1}{2}$ yr.
4. Teach finding interest by the interest formula.
5. Teach how to write a promissory note and find the interest.
6. Explain the procedure for opening a savings account.
7. Discuss safe ways of sending money with problems in finding the cost.

Page 265

Aim: To describe the services offered by commercial banks

Page 266

Aim: To teach the procedure for opening a checking account and making out deposit slips

Suggestions: A knowledge of certain of the more common procedures in banking is of importance to everyone. Pupils should be taught to make out deposit slips, to write and endorse checks correctly, and to keep a record of bank balances on checkbook stubs. These things can be learned properly only by filling out the actual forms. If possible, the school should provide the pupils with blank checks and deposit slips.

Workbook Reference: Arithmetic Workshop, Book 7, page 108

Key: **2.** \$449.33. **3.** \$638.62. **4.** \$743.59. **5.** \$149.40.

Page 267

Aim: To teach how to write a check

Suggestions: The illustrated check should be carefully studied. Point out that the check is the part to the right of the vertical dotted line, while the stub is the part to the left of that line. After the check is written, it is torn out of the checkbook, but the stub remains in the book and serves as a record. In writing a check the amount is written in figures immediately after the dollar sign and as close to the sign as possible. If a space is left between the dollar sign and the amount, a dishonest person might raise the amount by inserting another number. The amount is written in words also; in this case the writing should begin at the extreme left-hand end of the line of writing so that the amount cannot be raised. If the amount is less than one dollar, such as \$0.75, it may be written in words, as "Seventy-five cents" or "Only seventy-five cents," in which case the word *Dollars* is crossed out. The amount "$\frac{75}{100}$" may be written at the extreme left-hand end of the line used for writing the amount in words, in which case the word *Dollars* is not crossed out. If the amount written in figures differs from the amount written in words, the bank usually returns the check to the depositor, who informs the writer of the check of the discrepancy.

Workbook Reference: Arithmetic Workshop, Book 7, pages 109 and 110

Key: **1.** The check serves as a receipt since the bank returns the check marked *Paid* to Mr. Baker at the end of the month. **2.** When the amount of a check is written both in words and in figures, the check cannot be altered easily. **3.** No space is left in order that alterations and the insertion of a figure following the $ sign can be prevented. **4.** A decimal point is easily erased to alter the check. **5.** No, because the check might not be good. **6.** Suppose you buy something from Mr. Jones who lives in another city and who does not know anything about you. If you send him a certified check, he will know at once that the check is good; but if you send him a check that is not certified, he may have no way of knowing whether the check is good.

Page 268

Aim: To teach how to endorse checks

Workbook Reference: Arithmetic Workshop, Book 7, page 111

Key: **4.** When a check that you have written is paid by the bank, it is canceled and returned to you at the end of the month. This check shows that you have paid the money to the person to whom you sent it. The endorsement on the check shows that that person received the money. **5.** (1) A check should not be endorsed until it is deposited in the bank. If you endorse a check and lose it before it is deposited, the person who finds it may be able to cash it. (2) When a check is endorsed in full, it can be cashed only by the person named in the endorsement; but when it is endorsed in blank, it can be cashed by any person who receives it. **6.** When a check is written to "Cash" and presented at the bank for payment, it may or may not require endorsement. Find out what the practice is at a bank in your town.

Page 269

Aim: To teach how to fill out a check stub and to balance the account

Workbook Reference: Arithmetic Workshop, Book 7, pages 112 and 113

Key: **2.** The stub of a check should be filled out when the check is written; otherwise you may forget that you wrote the check. **3.** It is important for Mr. Baker to keep a record of his bank balance on the stubs of his checks so that he will know how much money he has left in the bank. **4.** Checks should be numbered so that you can refer to them quickly when they are returned to you in canceled form by the bank at the end of the month. When the checks are returned to you, you should check them off on the stubs of your checkbook to see whether all the checks that you have drawn have been paid. **5.** \$370.34 − \$69.58 = \$300.76; \$300.76 + \$37.81 = \$338.57. **6.** \$241.82.

Page 270

Aim: To review problem solving

Workbook Reference: Arithmetic Workshop, Book 7, page 115

Key: **1.** (1) 40 mi. an hour = $\frac{2}{3}$ mi. a minute; 1 mi. = 5280 ft., $\frac{2}{3} \times 5280$ ft. = 3520 ft., 3 sec. = $\frac{1}{20}$ min., $\frac{1}{20} \times 3520$ ft. = 176 ft.; (2) 60 mi. an hour = 1 mi. a minute, $\frac{1}{20} \times 5280$ ft. = 264 ft. **2.** 14 gal. + 35 gal. = 49 gal.; 49 gal. − 4 gal. = 45 gal.; 756 mi. ÷ 45 = 16.8 mi. **3.** $\$92 - \$80 = \$12; \$12 \div \$80 = \frac{3}{20} = 15\%$. **4.** 210 mi. ÷ 35 mi. = 6 (hr.); 6 hr. + 45 min. = 6 hr. 45 min.; 6 hr. 45 min. before 4 P.M. is 9:15 A.M. **5.** $5\frac{1}{4} \times \$.48 = \$2.52$; \$5.00 − \$2.52 = \$2.48; \$2.00 + \$.25 + \$.10 + \$.03 = \$2.38; the change was short \$.10. **6.** 10 × 25.9¢ = 259¢, or \$2.59; 5 × 26.6¢ = 133¢, or \$1.33; 10 × 27.8¢ = 278¢, or \$2.78; \$2.59 + \$1.33 + \$2.78 = \$6.70; \$6.70 ÷ 25 = \$.268, or 26.8¢. **7.** 310 ÷ 1155 = .268, which is called 27%. **8.** $\frac{1}{2} \times \$12.50 = \$6.25$.

Page 271

Aim: To present a set of improvement tests in subtraction

Suggestion: This set of improvement tests should be given in the same manner as the preceding improvement tests in subtraction.

Key: **1.** 5,890,939; 6,921,547; 7,917,979. **2.** 3,876,749; 3,054,789; 4,787,614. **3.** 4,538,358; 840,671; 2,661,982. **4.** 3,883,135; 4,564,446; 2,415,283. **5.** 2,398,142; 977,081; 6,873,958. **6.** 2,035,565; 5,373,790; 7,669,799. **7.** 2,868,897; 2,442,867; 7,692,586. **8.** 3,941,438; 4,360,247; 6,154,642. **9.** 6,513,845; 3,234,355; 648,201.

Page 272

Aim: To teach how to find interest for 1 yr. and $\frac{1}{2}$ yr.

Suggestions: The interest on ordinary loans seldom runs beyond a year without being paid. Problems involving time beyond one year usually serve to let one know in advance what the total amount of interest would be for such a long period, rather than to indicate that this interest is all paid at one time. On loans running for a year or more, *the interest is usually paid quarterly, semiannually, or annually.* Emphasize the fact that the rate of interest means the rate *for 1 year* unless there is some statement to the contrary. For example, if money is borrowed at 5%, the rate indicates 5% for 1 year. If money is borrowed for 6 months at 5%, you find the interest for 1 year at 5% and then take $\frac{1}{2}$ of it to find the interest for $\frac{1}{2}$ year.

Workbook Reference: Arithmetic Workshop, Book 7, page 116

Key: **2.** (1) .06 × \$500 = \$30; (2) \$500 + \$30 = \$530. **3.** \$8; \$48; \$175. **4.** \$300; \$14.20; \$140. **5.** \$136.75; \$117; \$185.52. **6.** \$5.72; \$612.50; \$188.27. **7.** (1) .035 × \$1200 = \$42; (2) $\frac{1}{2} \times \$42 = \$21$; (3) \$1200 + \$21 = \$1221. **8.** \$10; \$72; \$29.25. **9.** \$43.75; \$116.25; \$117.75. **10.** (1) $.04\frac{1}{2} \times \$1237 = \$55.665$; $\$55.665 \div 2 = \$27.832\frac{1}{2}$, or \$27.83; (2) $.03\frac{1}{4} \times \$4740 = \$154.050$; $\$154.05 \div 2 = \$77.02\frac{1}{2}$, or \$77.03; (3) $.06\frac{1}{2} \times \$1900 = \$123.50$; \$123.50 ÷ 2 = \$61.75. **11.** (1) $.02\frac{3}{4} \times \$1400 = \$38.50$; \$38.50 ÷ 2 = \$19.25; (2) .07 × \$2000 = \$140; \$140 ÷ 2 = \$70; (3) $.02\frac{1}{4} \times \$1600 = \$36$; \$36 ÷ 2 = \$18.

Page 273

Aim: To teach how to use the formula for finding interest

Suggestions: The method of finding interest by using the formula is the best one to use since it applies to all situations, is easy to remember, and has computation that can be simplified by the use of cancellation. The substitution in the formula is very plainly discussed in ex: 3. The pupils must understand that in the formula the rate of interest is expressed as a fraction or as a decimal, and the time is expressed in years. Notice that 5 mo. is $\frac{5}{12}$ yr. and that 8 mo. is $\frac{2}{3}$ yr. Simplification of the fraction before the work is begun lessens the amount of computation.

Workbook Reference: Arithmetic Workshop, Book 7, page 117

Key: **4.** (1) $\$300 \times \frac{6}{100} \times \frac{1}{6} = \$3$; (2) $\$600 \times \frac{6}{100} \times \frac{7}{12} = \$21$; (3) $\$900 \times \frac{4}{100} \times \frac{1}{3} = \$12$. **5.** (1) $\$600 \times \frac{4}{100} \times \frac{11}{12} = \$22$; (2) $\$800 \times \frac{7}{100} \times \frac{3}{4} = \$42$; (3) $\$300 \times \frac{4}{100} \times \frac{2}{3} = \$8$. **6.** (1) $\$240 \times \frac{3}{100} \times \frac{7}{12} = \$4.20$; (2) $\$480 \times \frac{6.5}{100} \times \frac{5}{6} = \$26$; (3) $\$420 \times \frac{5}{100} \times \frac{1}{2} = \$10.50$.

Page 274

Aim: To teach how to find interest for a given number of days

Suggestions: In finding interest for a given number of days, as in ex. 1 on this page, it is preferable first to express the given number of days as a fractional part of a year and to change this fraction to lowest terms before computing the interest. For example, in finding the interest for 24 days, first express 24 days as $\frac{24}{360}$ of a year and then change $\frac{24}{360}$ to $\frac{1}{15}$ before computing the interest. This makes the cancellation less complicated. All written work should follow the model solution given in ex. 1.

Workbook Reference: Arithmetic Workshop, Book 7, page 118

Key: **2.** $\frac{24}{360} = \frac{1}{15}$; $\$1000 \times \frac{6}{100} \times \frac{1}{15} = \$4$. **3.** (1) $\frac{30}{360} = \frac{1}{12}$, $\$900 \times \frac{6}{100} \times \frac{1}{12} = \$4.50$; (2) $.05 \times \$900 = \$45$, $\$45 - \$4.50 = \$40.50$. **4.** $\frac{75}{360} = \frac{5}{24}$; $\$600 \times \frac{6}{100} \times \frac{5}{24} = \$7.50$; $\$100 - \$7.50 = \$92.50$. **5.** (1) $\frac{60}{360} = \frac{1}{6}$, $\$600 \times \frac{5}{100} \times \frac{1}{6} = \$5$; (2) $\frac{24}{360} = \frac{1}{15}$, $\$1500 \times \frac{3}{100} \times \frac{1}{15} = \$3$. **6.** (1) $\frac{45}{360} = \frac{1}{8}$, $\$800 \times \frac{6}{100} \times \frac{1}{8} = \$6$; (2) $\frac{27}{360} = \frac{3}{40}$, $\$1200 \times \frac{4}{100} \times \frac{3}{40} = \$3.60$. **7.** (1) $\frac{90}{360} = \frac{1}{4}$, $\$700 \times \frac{7}{100} \times \frac{1}{4} = \$12.25$; (2) $\frac{66}{360} = \frac{11}{60}$, $\$8000 \times \frac{3}{100} \times \frac{11}{60} = \$44$.

Page 275

Aim: To present a variety of real problems involving interest

Key: **1.** $.04 \times \$85 = \$3.40$. **2.** (1) $.05 \times \$1500 = \$75$; (2) $3 \times \$75 = \$225$. **3.** (1) $.04 \times \$400 = \$16$, $5 \times \$16 = \$80$; (2) $\$400 + \$80 = \$480$. **4.** (1) $.03 \times \$400 = \$12$, $1\frac{1}{2} \times \$12 = \$18$; (2) $\$400 + \$18 = \$418$. **5.** $.05 \times \$2000 = \$100$. **6.** $.04 \times \$15{,}000 = \$600$. **7.** $12 \times 2\% = 24\%$; yes. **8.** $.03 \times \$425 = \$12.75$.

Page 276

Aim: To explain promissory notes and how they are used

Suggestions: The promissory note shown in ex. 1 is the type used when one person borrows money from another person. The teaching of borrowing of money from a bank or a loan company is postponed until Grade 8. In connection with this work, the pupils should be familiar with the terms *promissory note, face,* and *date of maturity.*

Attention is called to ex. 7, in which the interest is paid every 3 months. In this case, the rate of interest, which is 6%, means the rate for 1 year. To find the interest for 3 months, first find the interest for a year at 6% and then take $\frac{1}{4}$ of it.

Key: **2.** \$1000; Sept. 1; Thomas White; $\$1000 \times \frac{5}{100} \times \frac{1}{2} = \$25$. **4.** (1) $\$700 \times \frac{5}{100} \times \frac{1}{2} = \$17.50$; (2) Oct. 1, 1963. **5.** (1) $.045 \times \$1200 = \$54$; (2) Sept. 10, 1965. **6.** (1) $\$2350 \times \frac{6}{100} \times \frac{1}{4} = \$35.25$; (2) Oct. 1, 1964. **7.** (1) $\$1000 \times \frac{6}{100} \times \frac{1}{4} = \$15$; (2) July 1, Oct. 1, Jan. 1, Apr. 1; (3) $4 \times \$15 = \$60$.

Page 277

Aim: To give information on opening a savings account

Suggestions: In connection with this work have the pupils inquire at the savings department of a commercial bank, or at savings banks, about the rate of interest paid by the banks in your locality. Have them bring in any printed folders supplied by the banks. Attention is called to ex. 4, which includes the statement that savings banks do not pay interest on a fractional part of a dollar. In other words, they pay interest only on the whole number of dollars on deposit. For example, if you have \$104.35 in the savings bank for a full interest period, the bank will pay you interest on only \$104; the \$.35 will not receive interest.

Key: **2.** $\$15 \times \frac{4}{100} \times \frac{1}{2} = \$.30$. **3.** $\$15 + \$.30 + \$17 = \$32.30$. **4.** $\$32 \times \frac{4}{100} \times \frac{1}{2} = \$.64$; $\$32.30 + \$.64 = \$32.94$. **5.** $\$137 \times \frac{4}{100} \times \frac{1}{2} = \$2.74$; $\$137.83 + \$2.74 = \$140.57$. **7.** $.035 \times \$100 = \$3.50$; $.04 \times \$100 = \$4.00$; $\$4.00 - \$3.50 = \$.50$.

Pages 278–279

Aim: To give information about postal money orders and how to buy them

Suggestions: Further information concerning postal money orders, including the current fees charged for such money orders, may be obtained from your local post office. The rates for money orders, registered mail, and other postal services vary from time to time according to an Act of Congress. Have the pupils secure the latest rates at the post office. The method of finding the cost of any postal service, as given on these pages, applies to any new rates that may be adopted. In other words, if the pupils know how to do the exercises on these pages, they can do similar exercises based on any new rates. If pupils find that there are some new rates, have them work the exercises based on the new rates. Then compare the former cost with the new cost.

Key: **2.** \$4.00; \$8.95; \$25.15; \$62.85; \$88.33. **3.** \$18. **4.** (1) $\$4{,}958{,}000{,}000 \div 264{,}000{,}000 = \$18.78$; (2) $\$35{,}300{,}000 \div 1{,}630{,}000 = \$21.656$, which is called

\$21.66. **6.** (1) 35¢ and 20¢; \$110.55. **7.** Cost of order for \$50 is 30¢; cost of order for \$60 is 35¢; cost of both is 65¢, which is 10¢ more than the plan in ex. 6 costs. **8.** (1) \$7.70; (2) \$2.09; (3) \$86.85; (4) \$22.28; (5) \$103.50 + \$.35 + \$.20 = \$104.05; (6) \$150 + \$.35 + \$.30 = \$150.65; (7) \$185 + \$.35 + \$.35 = \$185.70; (8) \$215.25 + \$.35 + \$.35 + \$.30 = \$216.25; (9) \$360 + \$.35 + \$.35 + \$.35 + \$.35 = \$361.40. **9.** A postal money order for \$3 costs 20¢; the registration fee for \$3 is 60¢; in addition, a 5-cent stamp must be put on the letter in each case; therefore, the postal money order is cheaper. **10.** (1) 30¢; (2) \$37.50 + \$.80 + \$.30 = \$38.60. **11.** \$6.50 + \$.50 + \$.20 = \$7.20.

Page 280

Aim: To give a miscellaneous review of preceding work

Key: **1.** 3.0. **2.** $18\frac{2}{3}$; $69\frac{1}{2}$. **3.** $3\frac{5}{16} + 9\frac{1}{2} + 7\frac{3}{4} + 5\frac{3}{16} = 25\frac{3}{4}$; $25\frac{3}{4} \div 4 = 6\frac{7}{16}$. **4.** 12,307. **5.** .627. **6.** 200%. **7.** $22\frac{1}{2}$; 9. **8.** $\frac{1}{4}$ bu.; $\frac{3}{8}$ bu.; $\frac{3}{4}$ bu. **9.** \$1.77. **10.** 5190. **11.** $16 \times 12\frac{1}{3} = 197\frac{1}{3}$; $197\frac{1}{3} \times 10\frac{7}{8} = 2146$. **12.** .035 × \$12,555 = \$439.425, which is called \$439.43. **13.** \$82.50. **14.** .002 × 31.8 = .0636; .0636 ÷ .03 = 2.12. **15.** 9.68 + 4.42 + 6.12 + 8.07 + 1.51 = 29.80; 29.80 ÷ 5 = 5.96. **16.** 62 + 312 + 76 = 450; 12 × 450 = 5400. **17.** \$2400 − \$1350 = \$1050; \$1050 ÷ \$2400 = .4375, which is called 44%. **18.** \$28,247.58 − \$17,959.50 = \$10,288.08; \$10,288.08 ÷ 12 = \$857.34. **19.** \$1674 − \$1550 = \$124, \$124 ÷ \$1550 = .08 = 8%. **20.** 4 × 660 ft. = 2640 ft.; 2640 ft. = $\frac{2640}{5280}$ mi. = $\frac{1}{2}$ mi. **21.** \$7.95 − \$6.36 = \$1.59; \$1.59 ÷ \$7.95 = .20 = 20%. **22.** $\frac{241}{716}$ = .336, which is called 34%.

Page 281

Aim: To explain how to send money by telegraph

Suggestions: If you are away from home in a strange city and need money quickly, because of an emergency, one way to get it is to telegraph your family or friends at home, asking them to send you money by telegraph. This method is much quicker than having your friends send you money by air mail. The details of sending money by telegraph are given on page 281.

Key: **2.** (1) \$37.50 + \$.70 + \$1.20 + \$.12 = \$39.52; (2) \$37.50 + \$.70 + \$1.30 + \$.13 = \$39.63; (3) \$37.50 + \$.70 + \$1.45 + \$.15 = \$39.80; (4) \$37.50 + \$.70 + \$1.60 + \$.16 = \$39.96. **3.** (1) \$51.75 + \$.90 + \$1.75 + \$.18 = \$54.58; (2) \$51.75 + \$.90 + \$1.90 + \$.19 = \$54.74. **4.** (1) \$63 + \$1.00 + \$1.10 + \$.11 = \$65.21; (2) \$63 + \$1.00 + \$1.30 + \$.13 = \$65.43; (3) \$63 + \$1.00 + \$1.45 + \$.15 = \$65.60; (4) \$63 + \$1.00 + \$1.60 + \$.16 = \$65.76. **5.** (1) A telegraphic money order costs \$77.53, and a postal money order costs \$75.00 + \$.35, or \$75.35; \$77.53 − \$75.35 = \$2.18. If 5¢ postage for regular mail or 8¢ postage for air mail is included in the cost, the difference becomes \$2.13 or \$2.10. (2) If the money is needed immediately, the time saved by sending it by telegraph might be very important. In this case the extra expense would be justified.

Page 282

Aim: To give information about travelers checks and how to buy them

Suggestions: When a person is traveling, he should not carry a large amount of money in cash, which might easily be lost or stolen. For this reason many travelers carry money in the form of travelers checks, which can be cashed at most hotels, banks, and stores. If the travelers checks are lost, the company that issues them will under normal circumstances repay the loss. Travelers checks are sold by banks and some travel agencies.

Key: **3.** .01 × \$200 = \$2. **4.** (1) 8 × \$10 = \$80; 3 × \$20 = \$60; 2 × \$50 = \$100; \$80 + \$60 + \$100 = \$240; (2) .01 × \$240 = \$2.40. **5.** (1) (a) .01 × \$150 = \$1.50, (b) \$150 + \$1.50 = \$151.50; (2) (a) .01 × \$340 = \$3.40, (b) \$340 + \$3.40 ≐ \$343.40; (3) (a) 01. × \$500 = \$5, (b) \$500 + \$5 = \$505; (4) (a) .01 × \$875 = \$8.75, (b) \$875 + \$8.75 = \$883.75; (5) (a) .01 × \$1000 = \$10, (b) \$1000 + \$10 = \$1010. **6.** If you are traveling and hold travelers checks, these checks can be cashed without personal identification because the matching of the signatures on the travelers check is regarded as sufficient identification. If you are in a strange place, however, and offer one of your personal checks in payment of a bill, you may be required to furnish some means of identification before the check will be accepted. A stranger has no way of knowing whether your personal check is good.

Page 283

Aims: To give a quick oral review and to review the language of arithmetic

Key: **2.** 46 oz. **3.** $\frac{2}{5}$. **4.** 49 min. **5.** $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2; $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $1\frac{1}{8}$, $1\frac{3}{8}$, $1\frac{5}{8}$, $1\frac{7}{8}$. **6.** 14.5; 2.36; .0387. **7.** 1310; 511; 601; 44; 909; 1969. **8.** \$400; \$200. **9.** $\frac{3}{4}$; 16; $\frac{5}{8}$. **10.** 96; $22\frac{1}{2}$; 20. **11.** 20,000; 12; 120.

Page 284

Aim: To sum up the different topics taught in Chapter 9

Suggestions: Make a careful check on each pupil's answers to the questions on this page. If any individual has difficulty, be sure to see that he has a re-explanation of the difficult topic and a chance to do some remedial work.

Key: **1.** Banks earn money by lending money to reliable people. The interest paid to the bank on such loans is greater than the interest that the bank pays on savings accounts. **2.** To open a checking account a person must give references and also leave a copy of his signature at the bank. See page 266 of the text. **4.** If you do not fill out the stub each time, you may forget that you wrote the check. **5.** See page 267 of the text for certified checks. **8.** The canceled checks serve as receipts for the payments that were made. Such checks serve also as records of past business transactions. **9.** If you draw a check for an amount that is larger than your balance in the bank, you are said to *overdraw* your account. In such cases the bank usually refuses to cash the check until you have deposited enough money in the bank to cover the amount of the check. **10.** (1) .04 × \$275 = \$11; (2) .05 × \$275 = \$13.75. **11.** (1) \$400 × $\frac{4}{100}$ × $\frac{1}{2}$ = \$8; (2) \$400 × $\frac{6}{100}$ × $\frac{1}{2}$ = \$12. **12.** (1) \$500 × $\frac{4}{100}$ × $\frac{1}{4}$ = \$5; (2) \$850 × $\frac{6}{100}$ × $\frac{1}{3}$ = \$17. **13.** 18%. **14.** \$200 × $\frac{4}{100}$ × $\frac{1}{4}$ = \$2. **16.** (1) The fee for the money order is 20¢, and the fee for registry is 60¢; so the fee for the money order is cheaper; (2) 60¢ − 20¢ = 40¢.

Page 285

Aim: To present Problem Test 9

Key: **1.** 21 − 15 = 6 (words); 6 × $5\frac{1}{2}$¢ = 33¢; \$1.20 + \$.33 = \$1.53; .10 × \$1.53 = \$.153, or \$.15; \$1.53 + \$.15 = \$1.68. **2.** $\frac{1}{4}$ × 16 gal. = 4 gal.; 16 gal. − 4 gal. = 12 gal.; 12 gal. ÷ 1.2 = 10 gal. **3.** 21 min. 48 sec. = 21.8 min.; 175 mi. ÷ 21.8 = 8.02 mi.; 60 × 8.02 mi. = 481.20 mi., or about 481 mi. **4.** Rent: $\frac{700}{3500}$ = $\frac{1}{5}$ = .20, or 20%; food: $\frac{800}{3500}$ = .228, or 23%; clothing: $\frac{490}{3500}$ = $\frac{7}{50}$ = .14, or 14%. **5.** 560 bu. − 420 bu. = 140 bu.; $\frac{140}{560}$ = .25, or 25%. **6.** \$1500 × $\frac{4}{100}$ × $\frac{3}{2}$ = \$90. **7.** 14 lb. = 2 lb. + 12 lb.; 12 × \$.16 = \$1.92; \$.58 + \$1.92 = \$2.50. **8.** 12 × \$98.50 = \$1182; \$982 + \$1182 = \$2164. **9.** \$3.50 + \$1.25 = \$4.75; $\frac{4}{5}$ × \$4.75 = \$3.80; 10 × \$3.80 = \$38; \$40 − \$38 = \$2. **10.** 8 × 275 mi. = 2200 mi.

Page 286

Aim: To provide a diagnostic test with page references for remedial work

Suggestions: Since the work of Chapter 9 is largely informational, this diagnostic test reviews three processes with fractions, multiplication of decimals, and finding a per cent of a number. At this stage of the work there should be very few mistakes in computation.

Key: **1.** $9\frac{3}{4}$; $9\frac{1}{20}$; $15\frac{1}{30}$; $5\frac{5}{24}$. **2.** $2\frac{7}{8}$; $1\frac{13}{16}$; $\frac{4}{5}$; $2\frac{1}{2}$. **3.** $2\frac{5}{12}$; $3\frac{19}{24}$; $3\frac{11}{20}$; $\frac{13}{15}$. **4.** .056; 224.88; 56.5536; 16.200. **5.** 24; $4\frac{1}{6}$; $1\frac{1}{24}$. **6.** $6\frac{2}{3}$; $1\frac{11}{16}$; 7. **7.** $\frac{3}{8}$; 8; $\frac{15}{16}$. **8.** \$1.40; \$39; \$4. **9.** \$.30; \$2.52; \$9.40. **10.** \$13; \$230; \$86.50. **11.** \$.19; \$5.41; \$9.38.

Chapter 10

Aims of Chapter 10. The major aims of Chapter 10 are to:

1. Review some units of measure.
2. Reteach addition and subtraction of measurements expressed in two units.
3. Teach the multiplication of measurements expressed in two units.
4. Discuss approximate nature of measurement.
5. Discuss precision of measurement and greatest possible error in a measurement.
6. Show how to measure distances approximately by pacing.
7. Explain how to estimate short lengths by eye.
8. Explain how to estimate a height by comparison with a known height.
9. Review area of a rectangle and the formula for finding it.
10. Review area of a square and develop the formula for finding it.
11. Review the table of square measure.
12. Teach how to find the area of a parallelogram and develop the formula for finding it.
13. Teach how to find the area of a triangle and develop the formula for finding it.
14. Teach how to find the circumference of a circle and how to verify the formula for it experimentally.
15. Reteach finding the volume of a rectangular prism, and develop a formula for the volume of a cube.
16. Reteach the table of cubic measure.
17. Review the year's work by means of computation and problems.

Page 287

Aim: To discuss measurement and the meaning of standard units of measure

Suggestions: In connection with this page you might discuss the different standard units of measure that have been studied since Grade 3. Pages 342 and 343 give tables of measures; you might refer to them. Let pupils try to give the table of liquid measure without referring to the book. In this way also review measures of length, time, weight, and counting articles.

Page 288

Aim: To review measures of length and measures of weight

Suggestions: In the table of measures of weight, a new measure has been included that was not used in earlier grades. This is *hundredweight*, which is used in certain situations, such as in weighing cattle for sale or purchase and weighing large items sent by freight. Give some extra practice on finding the number of hundredweight in 225 lb., 175 lb., 450 lb., 410 lb., and other weights. You can also use problems such as finding the cost of shipping 225 lb. at $1.70 a hundredweight.

Key: **4.** 36 in.; 108 in. **5.** 5280 ft.; 1320 ft.; 2640 ft. **8.** 48 oz.; 128 oz.; 40 oz. **9.** $3\frac{1}{2}$ cwt.; 47 cwt. **10.** 5 T.; $3\frac{1}{2}$ T.

Pages 289–290

Aims: To reteach the addition and subtraction of measurements expressed in two units and to teach the multiplication of such measurements

Suggestions: In everyday life, measurements are often made by using two units, such as 3 ft. 2 in., but they are seldom made by using more than two units. Hence the work on these pages is limited to two units in a single measurement.

You should point out to the pupils that any measurement in two units can be changed to a measurement in one unit as shown in ex. 2 on page 289. Sometimes it is more convenient to change measurements to one unit before performing computation with them. If it were necessary to divide 8 ft. 6 in. by 3, you could first change 8 ft. 6 in. to 102 in., and then divide 102 in. by 3.

Key: Page 289 **4.** 66; $5\frac{1}{2}$. **5.** 116; $7\frac{1}{4}$. **6.** 26; $8\frac{2}{3}$. **7.** 185; $3\frac{1}{12}$. **8.** 1, 20; $1\frac{1}{3}$. **9.** 7, 8; $7\frac{2}{3}$. **11.** 20 ft. 10 in.; 20 ft. 6 in.; 13 hr. 49 min.; 20 lb. 12 oz.

Page 290 **2.** (1) 9 lb. 12 oz. − 4 lb. 9 oz. = 5 lb. 3 oz.; (2) 7 ft. = 6 ft. 12 in., 6 ft. 12 in. − 3 ft. 4 in. = 3 ft. 8 in.; (3) 4 hr. 15 min. = 3 hr. 75 min., 3 hr. 75 min. − 1 hr. 40 min. = 2 hr. 35 min. **3.** (1) 8 yd. 7 in. = 7 yd. 43 in., 7 yd. 43 in. − 3 yd. 14 in. = 4 yd. 29 in.; (2) 23 lb. 5 oz. = 22 lb. 21 oz., 22 lb. 21 oz. − 19 lb. 13 oz. = 3 lb. 8 oz.; (3) 12 hr. = 11 hr. 60 min., 11 hr. 60 min. − 3 hr. 37 min. = 8 hr. 23 min. **5.** 4 × 4 yd. 16 in. = 16 yd. 64 in. = 17 yd. 28 in., or 18 yd. (to the nearest whole yard). **6.** (1) 6 × 3 yd. 13 in. = 18 yd. 78 in., 18 yd. 78 in. = 20 yd. 6 in.; (2) 8 × 3 yd. 13 in. = 24 yd. 104 in., 24 yd. 104 in. = 26 yd. 32 in.; (3) 10 × 3 yd. 13 in. = 30 yd. 130 in., 30 yd. 130 in. = 33 yd. 22 in. **7.** (1) 3 × 4 ft. 7 in. = 12 ft. 21 in., 12 ft. 21 in. = 13 ft. 9 in.; (2) 8 × 8 lb. 9 oz. = 64 lb. 72 oz., 64 lb. 72 oz. = 68 lb. 8 oz.; (3) 5 × 6 lb. 14 oz. = 30 lb. 70 oz., 30 lb. 70 oz. = 34 lb. 6 oz.; (4) 4 × 4 hr. 27 min. = 16 hr. 108 min., 16 hr. 108 min. = 17 hr. 48 min.

Page 291

Aim: To teach how to express measurement with different precision and to provide experience with actual measurements requiring different precision

Suggestions: Plan to have enough measuring instruments so that all pupils can participate in numerous measurements. Have a list of the different objects that the pupils are to measure, and state the precision that you expect. Prepare a sheet of the correct results so that each pupil can check his measurements against the key and recheck those where he erred. Nothing can substitute for the actual experience of doing the measurement.

Key: **6.** 29 ft. by 15 ft. **7.** 17 ft.; 43 ft.; 90 ft.; 24 ft.

Page 292

Aim: To teach how to express a measurement correct to a fractional part of a unit of measure

Suggestions: Conduct measurements with an ordinary ruler or yardstick; express measurements to the nearest $\frac{1}{2}$ inch, then the nearest $\frac{1}{4}$ inch, and then the nearest $\frac{1}{8}$ inch. Discuss the various measuring instruments that pupils have used, and invite presentation of the different precision that the various instruments could provide reasonably. You should, if possible, have a micrometer for the pupils to handle, read, and use. Point out that the precision of a measurement refers to the size of the smallest subdivision used on the measuring instrument.

Key: **4.** 62.45 ft.

Page 293

Aim: To teach the meaning of greatest possible error and how to determine the greatest possible error from the precision of the measurement

Suggestions: Give considerable practice in the comparison of a measurement and the markings on the measurement instrument. Have the pupils give the reading of the various subdivisions of the ruler to be certain that they can identify the fractional part used. Select a particular length, such as segment A, and state its length as more than 2 in. but less than 3 in. Always start with a large enough unit to be able to make obviously true statements of inequality. Now look at the subdivision that divides the unit in half and compare the length with it. Segment A is more than 2 in. but less than $2\frac{1}{2}$ in. The subdivision halfway between 2 and $2\frac{1}{2}$ is $2\frac{1}{4}$, so compare the length with that marking. Is the line more or less than $2\frac{3}{4}$ in.? Is the line nearer in length to 2 in. or to $2\frac{1}{2}$ in.? The conclusion that the length is nearer to $2\frac{1}{2}$ in. indicates that the length was more than or equal to $2\frac{1}{4}$ in. The difference of $\frac{1}{4}$ in. is the greatest possible error when measurement is given to the nearest $\frac{1}{2}$ in. *Note that the greatest possible error is always one half of the smallest subdivision of the unit of measure that is used in expressing the measurement.* Measurement to the nearest inch has $\frac{1}{2}$ in. as the greatest possible error, to the nearest $\frac{1}{4}$ in. has $\frac{1}{8}$ in. as the greatest possible error, and to the nearest $\frac{1}{8}$ in. has $\frac{1}{16}$ in. as the greatest possible error. This study of measurement provides an excellent opportunity to develop a need for the inequality symbols presented in Grade 8.

Key: **1.** Segment B is more than $1\frac{1}{2}$ in. but less than 2 in. **3.** Segment C is more than $2\frac{3}{4}$ in. but less than $2\frac{7}{8}$ in.; therefore, the length to the nearest $\frac{1}{4}$ in. is $2\frac{3}{4}$ in. instead of 3 in. The greatest possible error is $\frac{1}{8}$ in.

Page 294

Aim: To show how easily mistakes can be made when making measurements

Suggestions: This page shows how results in measuring vary according to the instruments used for measuring and the care with which the measurements are made. If you have, or can borrow, a 100-foot steel tape, have the pupils measure the classroom, as suggested in ex. 3. Have several sets of three pupils measure with ruler, yardstick, and 100-foot steel tape and compare the results.

Key: **1.** 9 yd. 15 in. = 27 ft. 15 in.; 28 ft. 10 in. = 27 ft. 22 in.; 27 ft. 22 in. − 27 ft. 15 in. = 7 in. **2.** (1) (a) 28 ft. 10 in. − 28 ft. 2 in. = 8 in.; (b) 28 ft. 2 in. = 338 in., $\frac{8}{338}$ = .023, or 2%; (2) (a) 9 yd. 15 in. = 339 in., 28 ft. 2 in. = 338 in., 339 in. − 338 in. = 1 in.; (b) $\frac{1}{339}$ = .0029, or .29%, which is 0% to the nearest whole per cent. **4.** 73.85 + 73.86 + 73.87 + 73.90 = 295.48 (ft.); 295.48 ft. ÷ 4 = 73.87 ft.

Page 295

Aim: To present new sets of improvement tests in multiplication and division

Key: **1.** 34,765,256; 7,823,625; 18,248,832; 15,776,100. **2.** 10,077,650; 8,859,356; 49,835,968; 18,057,300. **3.** 39,662,262; 9,895,788; 45,082,436; 32,002,500. **4.** 60.4; 34.5. **5.** 78.3; 77.4. **6.** 40.0; 91.8. **7.** 80.5; 14.2. **8.** 82.2; 37.7. **9.** 23.5; 87.5.

Page 296

Aim: To show how to measure distances approximately by pacing

Suggestions: In teaching pupils to measure distances by pacing, you should remember that some pupils naturally take much longer steps than others; hence, it is necessary for each pupil to find the average length of *his step* as explained in ex. 1. In measuring the length, caution each pupil to walk at his natural pace. Sometimes a pupil tries to take long steps when pacing. In this work it is essential that each pupil walk at his usual rate. After each pupil finds the average length of his step, he should practice measuring distances approximately by pacing, as explained in ex. 2. Considerable practice will be needed in order to pace distances easily and naturally. The work should be confined to distances of reasonable length, preferably not over 250 ft. long.

Key: **2.** $177\frac{1}{2}$ ft. **3.** $2\frac{1}{4}$ ft.; 135 ft.; 180 ft.; 270 ft.; $83\frac{1}{4}$ ft. **4.** 785 ft.; 2112 steps. **5.** 6 steps; 16 steps.

Page 297

Aim: To show how to estimate short distances by the eye

Suggestions: The work on this page requires careful observation and much practice on the part of the pupil. Encourage the pupils to estimate lengths and distances in the room, such as the height and width of the window, the length of the room, and so on.

Key: **5.** (1) $\frac{1}{4} \times 16$ ft. = 4 ft., 16 ft. + 4 ft. = 20 ft., yes; (2) 20 ft. and 12 ft.

Page 298

Aim: To show how to estimate the height of a tall object by comparing its height with some known height

Suggestions: This work requires the ability to estimate by eye the number of times a *known* height is contained in a greater height. In this work the greater height is *unknown*. If the greater height is estimated to be 3 times the known height, then the greater height is easily found by multiplying the known height by 3.

Key: **1.** $5\frac{1}{2}$ times; 88 ft. **2.** 24 ft. **3.** No.

Pages 299–300

Aims: To review the work of finding the areas of rectangles and squares and to develop formulas for finding these areas

Suggestions: In teaching the formula for the area of a rectangle, point out to the pupils that the formula is one of the *most useful* mathematical devices that we have. People in all parts of the world use formulas to express briefly the rules of mathematics and the laws of science. Formulas are very useful to businessmen, engineers, electricians, navigators, and surveyors. Without formulas it would be impossible to guide a ship safely at sea or to build a bridge or a skyscraper.

The pupil should memorize the formula $A = lw$ and learn how to substitute numerical values in it. Emphasize the fact that when two letters are written *side by side*, they are to be *multiplied*. This is an important convention of algebra. We have no such convention in arithmetic, since multiplication is not indicated when two numerals are written side by side. For example, 35 does *not* mean 3×5. When, however, a letter is written beside a numeral, or several letters are written side by side, multiplication is indicated. Thus $3a$ means $3 \times a$ and abc means $a \times b \times c$.

For ex. 7 on page 299 review the way to find an unknown factor of a product: if one of two factors of a product is known, the unknown factor can be found by dividing the product by the known factor. So, in ex. 7, $60 = l \times 4$, and $l = 60 \div 4$, or 15.

In the work on these pages remind the pupils that both dimensions of a rectangle must be given in, or changed to, the same unit of measure, as stated in ex. 1 on page 300.

Workbook Reference: Arithmetic Workshop, Book 7, page 122

Key: Page 299 **5.** 45 sq. ft.; 28 sq. in.; 13.68 sq. in. **6.** 27 sq. ft.; 96 sq. yd.; 64.74 sq. ft. **7.** 15 in. **8.** 14 in.; 35 in.; 7 in.

Page 300 **2.** (1) 8 ft. × 2 ft. = 16 sq. ft., or 96 in. × 24 in. = 2304 sq. in.; (2) 8 ft. × $\frac{1}{2}$ ft. = 4 sq. ft., or 96 in. × 6 in. = 576 sq. in.; (3) 9 yd. × 2 yd. = 18 sq. yd., or 27 ft. × 6 ft. = 162 sq. ft. **3.** (1) 4 ft. × $\frac{3}{4}$ ft. = 3 sq. ft., or 48 in. × 9 in. = 432 sq. in.; (2) 3 ft. × $1\frac{1}{3}$ ft. = 4 sq. ft., or 36 in. × 16 in. = 576 sq. in.; (3) $3\frac{2}{3}$ yd. × 1 yd. = $3\frac{2}{3}$ sq. yd., or 11 ft. × 3 ft. = 33 sq. ft. **5.** 16; 25. **6.** 36; 81; 144. **8.** 36 sq. in.; 25 sq. ft.; 4 sq. mi.; $\frac{9}{16}$ sq. mi.; 841 sq. in.; 151.29 sq. in.

Pages 301–303

Aims: To review the table of square measure and to use it in practical problems in gardening and in home repair

Suggestions: The first two lines of the table of square measure, given in ex. 5 on page 301, should be memorized. The rest of the table need not be memorized, and the pupils should be permitted to refer to it as necessary. A pupil should not be expected to remember that 43,560 sq. ft. = 1 acre since he uses that fact so infrequently. If he needs this information at any time, he can easily look it up in a mathematics textbook or in a dictionary. Ex. 3 on page 301 suggests how the pupils can get a concept of the size of an acre. If a square 209 ft. on each side is measured off on the playground, it will represent approximately an acre. In measuring 209 ft. on the ground, a piece of twine 20 ft. long can be used to shorten the work if you do not have a 100-foot steel tape. Stakes can be driven into the ground to mark the four corners of the square. Naturally, a measurement of this kind made by the pupils will be somewhat crude; but it will serve to show the pupils the size of an acre.

Ex. 1–3 on page 302 explain important work in gardening and show how to find the number of tomato plants that can be set out in a garden of a given size. In this work the pupils should first draw the garden to scale, as directed in ex. 1. In making this drawing, they might use a scale of 1 ft. × $\frac{1}{4}$ in. Another drawing should be made in connection with ex. 2. After studying these drawings the pupils will be ready for ex. 3, which shows how the answers in ex. 2 can be obtained in a shorter way *without making a drawing.* Allow ample time for all the work on page 302 in order to make it meaningful to the pupils.

Ex. 7 on page 303 should have careful study since it explains a method of finding the amount of fertilizer that is needed for a garden which is *smaller* than an acre in size. In problems of this kind one first finds what decimal part of an acre the garden is, using the fact that 1 acre = 43,560 sq. ft.

Workbook Reference: Arithmetic Workshop, Book 7, page 123

Key: Page 301 **3.** 43,264 sq. ft.; 43,681 sq. ft.; 209 ft. × 209 ft. **4.** 360 × 242 = 87,120 (sq. ft.); 2 × 43,560 sq. ft. = 87,120 sq. ft.; yes. **6.** (1) 3 ft. = 1 yd.; 75 ft. = 25 yd.; 1 × 25 = 25 (sq. yd.); (2) 25 × $4.75 = $118.75. **7.** 2 × 1.5 = 3 (sq. mi.); 3 × 640 acres = 1920 acres; 1920 × $50 = $96,000. **8.** 720; 3. **9.** 1280; 15. **10.** 10,890; 2.

Pages 302–303 **1.** 128 plants. **2.** 96 rectangles; 96 plants. **3.** Yes. **4.** 4 × 3 = 12 (sq. ft.); 43,560 sq. ft. ÷ 12 sq. ft. = 3630 (plants). **5.** (1) 3 × 2 = 6 (sq. ft.), 43,560 sq. ft. ÷ 6 sq. ft. = 7260 (plants); (2) 4 × 1 = 4 (sq. ft.), 43,560 sq. ft. ÷ 4 sq. ft. = 10,890 (plants); (3) 4 × $1\frac{1}{2}$ = 6 (sq. ft.), 43,560 sq. ft. ÷ 6 sq. ft. = 7260 (plants). **6.** (1) 200 × 70 = 14,000 (sq. ft.), 14,000 ÷ 1000 = 14 (thousands), 14 × 50 lb. = 700 lb. (fertilizer); (2) 14 × 75 lb. = 1050 lb. (limestone); (3) 700 ÷ 100 = 7 (bags), 1050 ÷ 100 = $10\frac{1}{2}$. He must buy 7 bags of fertilizer and 11 bags of limestone. **7.** 100 × 88 = 8800 (sq. ft.); $\frac{8800}{43560}$ acre = $\frac{20}{99}$ acre; $\frac{20}{99}$ = .202, or .20; .20 × 900 lb. = 180 lb.; to get 180 lb. he will have to buy two 100-pound bags. **8.** (1) 30 × 40 = 1200 (sq. ft.), $\frac{1200}{43560}$ acre = $\frac{10}{363}$ acre, $\frac{10}{363}$ = .027, or .03 (acre); (2) 60 × 75 = 4500 (sq. ft.), $\frac{4500}{43560}$ acre = $\frac{25}{242}$ acre, $\frac{25}{242}$ = .103, or .10 (acre); (3) 60 × 100 = 6000 (sq. ft.), $\frac{6000}{43560}$ acre = $\frac{50}{363}$ acre, $\frac{50}{363}$

$= .137$, or $.14$ (acre); (4) $80 \times 125 = 10{,}000$ (sq. ft.), $\frac{10000}{43560}$ acre $= \frac{250}{1089}$ acre, $\frac{250}{1089} = .229$, or $.23$ (acre). **9.** (1) $30 \times 24 = 720$ (sq. ft.), $720 \div 20 = 36$ (bags); (2) $36 \times \$1.29 = \$46.44$. **10.** $13 \times 21 = 273$ (sq. ft.); $12 \times 14 = 168$ (sq. ft.); 273 sq. ft. + 168 sq. ft. = 441 sq. ft.; $441 \div 150 = 2\frac{141}{150}$ (qt.), or 3 qt.

Pages 304–305

Aims: To teach how to find the area of a parallelogram and to develop the formula for finding its area

Suggestions: See that the pupils become familiar with such terms as *parallelogram, base, height, perpendicular, opposite sides,* and *adjacent sides.* Make clear that the *opposite* sides of a parallelogram are parallel. In this connection it will help if you draw on the board a 4-sided figure, such as a trapezoid, which is *not* a parallelogram. Emphasize the fact that the height of a parallelogram is *always* perpendicular to the base, as stated in ex. 2 on page 304. Make sure that the pupils do not confuse the height with the length of the slanting side AD of the parallelogram (see the figure in ex. 1 on page 304). In order to make sure that the pupils understand how the height of a parallelogram is drawn, you should place parallelograms of different shapes on the board and ask the pupils to draw the height of each.

The experiment outlined at the top of page 305 should actually be carried out in order to show how the rule for finding the area of a parallelogram is obtained. This rule should be expressed also as a formula. After understanding has been attained, both the rule and the formula should be memorized by the pupils.

Workbook Reference: Arithmetic Workshop, Book 7, page 124

Key: Page 304 **5.** Height is $1\frac{1}{2}$ in. **6.** Height is 2 in.

Page 305 **3.** 450 sq. ft. **4.** $85 \times 200 = 17{,}000$ (sq. ft.). **5.** 56 sq. in.; 57 sq. ft.; 14,250 sq. ft. **6.** 15 sq. in.; 6 sq. ft.; 6960 sq. ft.

Pages 306–307

Aims: To show the development of the formula for finding the area of a triangle and to teach the use of this formula in solving problems

Suggestions: In the study of the triangle the pupils must be made familiar with the terms *base* and *height.* Emphasize the fact that the *height* of a triangle must be measured on a line perpendicular to the base, as shown in ex. 1 on page 306. The experiment in ex. 2 on this page shows how to develop the formula for finding the area of a triangle. Have each pupil perform this experiment in class, using triangles large enough to handle easily. The shortest side should be at least 4 inches long. In the formula $80 = \frac{1}{2} \times 16 \times h$ in ex. 9 on page 307, the pupils should first find $\frac{1}{2} \times 16$ and write the formula as $80 = 8 \times h$. Then the unknown factor can easily be found.

Workbook Reference: Arithmetic Workshop, Book 7, page 125

Key: Page 307 **1.** $A = \frac{1}{2}bh$; $A = \frac{1}{2} \times 9 \times 7 = 31\frac{1}{2}$ (sq. ft.). **2.** (1) $31\frac{1}{2} \times 2\frac{1}{2}$ lb. $= 78\frac{3}{4}$ lb.; (2) $31\frac{1}{2} \times 5$ lb. $= 157\frac{1}{2}$ lb. **3.** (a) $A = \frac{1}{2} \times 7\frac{1}{2} \times 8 = 30$ (sq. ft.), safe; (b) $A = \frac{1}{2} \times 8\frac{1}{2} \times 8 = 34$ (sq. ft.), unsafe; (c) $A = \frac{1}{2} \times 10 \times 6\frac{1}{4} = 31\frac{1}{4}$ (sq. ft.), safe; (d) $A = \frac{1}{2} \times 9 \times 7\frac{1}{2} = 33\frac{3}{4}$ (sq. ft.), unsafe; (e) $A = \frac{1}{2} \times 9\frac{1}{2} \times 8\frac{1}{2} = 40\frac{3}{8}$ (sq. ft.), unsafe; (f) $A = \frac{1}{2} \times 10 \times 8\frac{1}{2} = 42\frac{1}{2}$ (sq. ft.), unsafe. **4.** 35 sq. yd.; 1105 sq. yd. **5.** 60 sq. in.; $246\frac{1}{2}$ sq. in. **6.** 13.69 sq. ft.; 683.28 sq. ft. **7.** $16\frac{1}{4}$ sq. in; $19\frac{1}{8}$ sq. in. **8.** 88.80 sq. in., or 88.8 sq. in.; $46\frac{1}{2}$ sq. ft. **9.** 10 in. **10.** 15 ft.; 12 ft.

Pages 308–311

Aims: To teach how to develop experimentally the formula for the circumference of a circle and to use the formula in solving problems

Suggestions: Be sure that the pupils know the meaning of the terms *circle*, *radius*, *radii*, *diameter*, and *circumference* and how to spell them. Draw a circle on the board, using a piece of string for a radius. Ask a pupil to draw a radius and a diameter, and to point out the circumference. Emphasize the fact that the diameter always passes through the center of the circle. Also show by measuring with a yardstick that the diameter is the longest distance across a circle. This fact is a helpful one to know when the center of the circle is not known. If you wish to find the diameter of the circular top of a wastebasket, you make use of this fact because the center of the circular top is not known.

The experiments described in ex. 4, 5, and 6 on page 309 should be carried out. The purpose of these experiments is merely to show that in any circle the circumference is about 3.14 times as long as the diameter. Thus the pupil develops confidence in the formulas $C = \pi d$ and $C = 2\pi r$.

Workbook Reference: Arithmetic Workshop, Book 7, page 126

Key: Pages 308–309 **1.** 6 in.; 8 in. **2.** 66 ft.; 110 in.; 154 in.

Page 310 **3.** 37.68 in.; 20.41 ft.; 78.50 in.; 31.4 ft.; 314 ft. **4.** 198 in.; 176 ft.; 220 in.; 11 ft. **6.** 21 ft.; $10\frac{1}{2}$ in.; 28 ft. **7.** 20 ft.; 7 in.; 14.9 in.; 42.3 ft. **8.** (1) $3\frac{1}{7} \times 28$ in. $= 88$ in.; (2) 88 in.; (3) $88 \div 12 = 7\frac{1}{3}$ (ft.); (4) 5280 ft. $\div\ 7\frac{1}{3}$ ft. $= 720$ (revolutions). **9.** 3.14.

Page 311 **2.** 18.84 ft. **3.** 31.40 ft. or 31.4 ft.; 188.40 ft. or 188.4 ft.; 62.80 in. or 62.8 in.; 150.72 ft.; 376.80 ft. or 376.8 ft.; 251.20 yd. or 251.2 yd. **4.** 22 ft.; 66 ft.; 110 in.; 132 in. **5.** $18\frac{6}{7}$ ft.; $62\frac{6}{7}$ yd.; 264 ft.; 308 in. **6.** $C = \pi d$; $C = 3\frac{1}{7} \times 3\frac{1}{2}$, or 11 (in.); 11 in. + $\frac{3}{8}$ in. = $11\frac{3}{8}$ in. **7.** (1) $C = 3.14 \times 15 = 47.1$ (in.); $1\frac{1}{2} \times 47.1$ in. = 70.65 in., or about 71 in.; (2) since she needs 71 in., she should buy 2 yd. of lace. **8.** (1) $C = 3.14 \times 200 = 628.00$ (ft.), or 628 ft., circumference of inside circle; (2) 200 ft. + 25 ft. + 25 ft. = 250 ft., diameter of outside circle; $C = 3.14 \times 250 = 785.00$ (ft.), or 785 ft., circumference of outside circle. **9.** 785 ft. − 628 ft. = 157 ft. **10.** $22 = 2 \times \frac{22}{7} \times r$, so $22 = \frac{44}{7} \times r$. The radius can be found by dividing 22 by $\frac{44}{7}$; $22 \div \frac{44}{7} = 22 \times \frac{7}{44} = 3.5$ (ft.).

Pages 312–315

Aims: To reteach finding the volume of a rectangular prism and to develop the formula for finding these volumes

Suggestions: The work on cubic measure was first taught in *American Arithmetic, Grade 6*, so it will be reteaching for many pupils; for others, this will be new work. The pupils must be thoroughly familiar with the terms *rectangular prism, cube, height, width, length,* and *dimensions.* The concept of volume is a difficult one for pupils; hence, you should try to make this work as real as possible. Have in class as many rectangular prisms as possible, such as boxes of different sizes, blocks, and a brick. Point out that a rectangular prism has square corners and that there are six sides that are rectangles. Have different pupils point out the length, width, and height of the various models. Make sure that the pupils understand that *volume* is the amount of space enclosed by a geometric figure or the amount that it will hold if it is filled with something. You can fill an empty box to illustrate the meaning of volume. The volume is called the *capacity.* As in ex. 3 on page 312, it will be helpful to use inch cubes to fill the box.

In connection with cubic measure on page 313, have some pupils make a cubic inch, a cubic foot, and a cubic yard from stiff paper. The cubic foot and the cubic yard will have to be supported by thin pieces of wood tacked together. These three cubes will help pupils to understand the relationships in the table in ex. 6. Encourage all pupils to measure the dimensions of boxes of various sizes and to compute their volumes. The more real and practical this topic can be made, the better it will be understood by the pupils.

In finding volumes remind the pupils that all three dimensions must be given in, or changed to, the same unit of measure, as explained in ex. 1 on page 313. The problems on pages 314 and 315 represent practical applications of cubic measure.

Workbook Reference: Arithmetic Workshop, Book 7, page 127

Key: Page 312 **2.** 1 cu. ft.; 1 cu. yd. **5.** 135 cu. in.; 864 cu. in. **6.** 70 cu. in.; $262\frac{1}{2}$ cu. in. **7.** 6 cu. ft.; $11\frac{1}{4}$ cu. ft.

Page 313 **1.** 864 cu. in.; $\frac{1}{2}$ cu. ft. **2.** (1) (a) $1' = 12''$, $12 \times 3 \times 4 = 144$ (cu. in.), (b) $3'' = \frac{1}{4}'$ and $4'' = \frac{1}{3}'$, $1 \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ (cu. ft.); (2) (a) $6'' = \frac{1}{2}'$, $\frac{1}{2} \times 1\frac{1}{2} \times 4 = 3$ (cu. ft.), (b) $1\frac{1}{2}' = 18''$ and $4' = 48''$, $6 \times 18 \times 48 = 5184$ (cu. in.); (3) (a) $8'' = \frac{2}{3}'$, $2 \times 1\frac{1}{2} \times \frac{2}{3} = 2$ (cu. ft.), (b) $2' = 24''$ and $1\frac{1}{2}' = 18''$, $24 \times 18 \times 8 = 3456$ (cu. in.). **3.** (1) (a) $10'' = \frac{5}{6}'$, $2 \times \frac{5}{6} \times 3 = 5$ (cu. ft.), (b) $2' = 24''$ and $3' = 36''$, $24 \times 10 \times 36 = 8640$ (cu. in.); (2) (a) $9'' = \frac{3}{4}'$, $\frac{3}{4} \times 1\frac{1}{3} \times 1 = 1$ (cu. ft.), (b) $1\frac{1}{3}' = 16''$ and $1' = 12''$, $9 \times 16 \times 12 = 1728$ (cu. in.); (3) (a) $4' = 48''$, $48 \times 15 \times 18 = 12{,}960$ (cu. in.), (b) $15'' = 1\frac{1}{4}'$ and $18'' = 1\frac{1}{2}'$, $4 \times 1\frac{1}{4} \times 1\frac{1}{2} = 7\frac{1}{2}$ (cu. ft.). **7.** 162 cu. ft.; 63 cu. ft.; 18 cu. ft. **8.** 864 cu. in.; 6048 cu. in. **9.** (1) 24 ft. = 8 yd., 18 ft. = 6 yd., 9 ft. = 3 yd., $8 \times 6 \times 3 = 144$ (cu. yd.); (2) 144 × \$3.25 = \$468.

Page 314 **2.** 64 cu. in.; 1 cu. yd.; 216 cu. ft.; $\frac{1}{8}$ cu. in.; 8 cu. yd.; 343 cu. in. **3.** 125 cu. yd.; 8 cu. ft.; 27 cu. in.; $\frac{8}{27}$ cu. ft.; $\frac{1}{64}$ cu. yd.; 729 cu. ft. **4.** 343; 729; 512; 1000; 1331. **5.** $5 \times 3 \times 4 = 60$ (cu. ft.); $4 \times 4 \times 4 = 64$ (cu. ft.); so the box 4 ft. on each edge holds more. **6.** $3 \times 3 \times 3 = 27$ (cu. ft.), 27 cu. ft. ÷ 1 cu. ft. = 27 (boxes); $2 \times 3 \times 4 = 24$ (cu. ft.), 24 cu. ft. ÷ 1 cu. ft. = 24 (boxes); so 3 more boxes can be packed in the one 3 ft. on each edge. **7.** (1) $9 \times 7 \times 5 = 315$

(cu. ft.), 315 cu. ft. ÷ 35 cu. ft. = 9 (tons); (2) 9 × 7 × 4 = 252 (cu. ft.), 252 cu. ft. ÷ 35 cu. ft. = 7.2 (tons); (3) 9 × 7 × 3 = 189 (cu. ft.), 189 cu. ft. ÷ 35 cu. ft. = 5.4 (tons). **8.** 13 in. − 2 in. = 11 in.; 28 × 18 × 11 = 5544 (cu. in.); 5544 cu. in. ÷ 231 cu. in. = 24 (gal.). **9.** (1) 35 × 30 × 12 = 12,600 (cu. ft.); (2) 12,600 cu. ft. ÷ 200 cu. ft. = 63 (pupils).

Page 315 **1.** 39 ft. = 13 yd.; 24 ft. = 8 yd.; 9 ft. = 3 yd.; 13 × 8 × 3 = 312 (cu. yd.); 312 cu. yd. ÷ 4 cu. yd. = 78 (loads). **2.** (1) $7 \times 4 \times 4\frac{1}{2} = 126$ (cu. ft.), 126 × 168 lb. = 21,168 lb.; (2) 21,168 lb. ÷ 2000 lb. = 10.584 (tons), which is about $10\frac{1}{2}$ tons; (3) it could not be carried on a 5-ton truck. **3.** $6\frac{1}{4} \times 168$ lb. = 1050 lb. **4.** 18 in. = $1\frac{1}{2}$ ft.; 12 in. = 1 ft.; 8 in. = $\frac{2}{3}$ ft.; $1\frac{1}{2} \times 1 \times \frac{2}{3} = 1$ (cu. ft.); $62\frac{1}{2}$ lb. + 15 lb. = $77\frac{1}{2}$ lb.

Pages 316–320

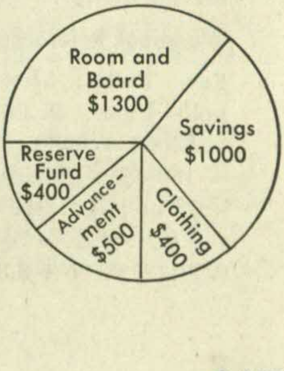
Aim: To review the problems and computation of the year's work

Workbook Reference: Arithmetic Workshop, Book 7, pages 129–131

Key: Page 316 **1.** Since the floor is 18 ft. long, 3 strips of 6-foot linoleum can be laid side by side to cover the length; then each strip must go across the width of the floor, which is 14 ft.; 3 × 14 ft. = 42 ft. **2.** 42 ft. of linoleum 6 ft. wide contains 252 sq. ft., or 28 sq. yd.; 28 × \$2.15 = \$60.20. **3.** (1) $34\frac{1}{8} \div 10\frac{1}{2} = 3\frac{1}{4}$, or 3.25; (2) $34\frac{1}{8} \div 10\frac{7}{8} = 3.137$; yes. **4.** $C = 3\frac{1}{7} \times 10\frac{1}{2}$, or 33 (ft.); 9 in. = $\frac{3}{4}$ ft.; 33 ft. ÷ $\frac{3}{4}$ ft. = 44 (bulbs). **5.** $A = \frac{1}{2}bh$, $A = \frac{1}{2} \times 4 \times 3 = 6$ (sq. in.); no. **6.** $2\frac{1}{2} \times 2$ mi. = 5 mi. **7.** (1) $60 \times 30 \times 4\frac{1}{2} = 8100$ (cu. ft.); (2) 8100 cu. ft. ÷ 100 cu. ft. = 81 (hundreds), 81 × \$.30 = \$24.30.

Page 317 **1.** \$1200 × $\frac{5}{100} \times \frac{1}{2}$ = \$30. **2.** \$15,000 − \$13,800 = \$1200; \$1200 ÷ \$15,000 = .08, or 8%. **3.** \$15,500 + \$17,400 + \$16,000 + \$15,000 + \$13,800 = \$77,700; \$77,700 ÷ 5 = \$15,540. **4.** 8 mi. is 4 times as far as 2 mi.; so Sam can walk 8 mi. in 4 × 36 min., or 144 min.; 144 min. = 2 hr. 24 min. **5.** $A = bh$; $A = 242 \times 90 = 21{,}780$ (sq. ft.), $\frac{21780}{43560} = \frac{1}{2}$ (acre). **6.** $7\frac{3}{4}$ hr. + $3\frac{1}{4}$ hr. + $1\frac{1}{2}$ hr. + $2\frac{3}{4}$ hr. = $15\frac{1}{4}$ hr.; 24 hr. − $15\frac{1}{4}$ hr. = $8\frac{3}{4}$ hr. **7.** 12 × \$.55 = \$6.60; 10 × \$.62 = \$6.20; 8 × \$.65 = \$5.20; \$6.60 + \$6.20 + \$5.20 = \$18.00; 12 doz. + 10 doz. + 8 doz. = 30 doz.; \$18.00 ÷ 30 = \$.60. **8.** $3\frac{3}{4}$ is 5 times $\frac{3}{4}$, so $3\frac{3}{4}$ in. is 5 times as long as $\frac{3}{4}$ in.; 5 × 10 mi. = 50 mi. **9.** 1600 cu. ft. ÷ 1000 cu. ft. = 1.6 (thousands); 1.6 × \$1.50 = \$2.40. **10.** 250 mi. ÷ 14 = 17.85 mi., or 17.9 mi.

11. Room and board: $\frac{1300}{3600} = \frac{13}{36}$; $\frac{13}{36} \times 360° = 130°$
Savings: $\frac{1000}{3600} = \frac{5}{18}$; $\frac{5}{18} \times 360° = 100°$
Clothing: $\frac{400}{3600} = \frac{1}{9}$; $\frac{1}{9} \times 360° = 40°$
Advancement: $\frac{500}{3600} = \frac{5}{36}$; $\frac{5}{36} \times 360° = 50°$
Reserve fund: $\frac{400}{3600} = \frac{1}{9}$; $\frac{1}{9} \times 360° = 40°$



Page 318 **1.** 36,906; 39,566; $59\frac{3}{4}$; $65\frac{1}{24}$; 2.650; 37.69. **2.** 2145; $6\frac{1}{4}$; $248\frac{1}{2}$; 4.375; \$810.05. **3.** 54,039,243; 81,755.4; $22\frac{1}{2}$. **4.** 4,605,000; .43350; $63\frac{3}{4}$. **5.** 17,339,520; 48.01832; $7\frac{7}{10}$. **6.** 7053; 975; 6.5. **7.** 3123; 54.3; .8. **8.** 74.7; 124; 2.4. **9.** \$12.90; \$5.50; \$1114. **10.** \$3.91; \$282.60; \$345. **11.** 16; 140; $12\frac{1}{2}$. **12.** $62\frac{1}{2}$; $56\frac{1}{4}$; 20.

Page 319 **1.** Milk: 6 cups; eggs: 6; sugar: $1\frac{1}{2}$ cups; salt: $\frac{3}{8}$ tsp.; cream: $1\frac{1}{2}$ cups; vanilla: $2\frac{1}{4}$ tsp. **2.** 850 ÷ 170 = 5, so 850 mi. is 5 times as far as 170 mi.; 5 × 40 min. = 200 min., or 3 hr. 20 min.; 3 hr. 20 min. after 3:15 P.M. is 6:35 P.M. **4.** \$4.95 ÷ \$27.50 = .18 = 18%. **5.** 1 hr. 18 min. = 1.3 hr.; 14.5 mi. ÷ 1.3 = 11.15 mi., or 11.2 mi. **6.** $\frac{1}{4} \times 18$ ft. = $4\frac{1}{2}$ ft.; to be within 25% of the true length the estimate can be as low as 18 ft. − $4\frac{1}{2}$ ft. or as high as 18 ft. + $4\frac{1}{2}$ ft., or between $13\frac{1}{2}$ ft. and $22\frac{1}{2}$ ft.; so John's estimate is within 25% of the true length. **7.** 36.8 mi. + 17.5 mi. + 29.0 mi. = 83.3 mi.

Page 320 **1.** $12\frac{1}{2}\% = \frac{1}{8}$; $\frac{1}{8} \times \$52 = \$6.50$; $.15 \times \$44 = \$6.60$; so 15% of \$44 is more. **2.** 286,324 + 307,190 + 292,254 = 885,768; 885,768 ÷ 3 = 295,256. **3.** $2\frac{1}{2} \times 5\frac{1}{3} \times 2\frac{1}{10} = \frac{5}{2} \times \frac{16}{3} \times \frac{21}{10} = 28$. **4.** 1.60 × 175 = 280. **5.** 32. **6.** 2 × 35 = 70; 70 − 8 = 62. **7.** $3\frac{3}{4} \times 84 = 315$; $12\frac{7}{8} \times 24 = 309$; so $12\frac{7}{8} \times 24$ is less. **8.** $12\frac{3}{4} + 10\frac{5}{8} + 14\frac{7}{8} + 11\frac{1}{2} + 10\frac{7}{8} = 60\frac{5}{8}$; $60\frac{5}{8} \div 5 = 12\frac{1}{8}$. **9.** 18 ÷ 3 = 6, so 18 candy bars cost 6 times as much as 3 candy bars; 6 × 13¢ = 78¢. **10.** 11.35 − 9.68 = 1.67; 2.5 × 1.67 = 4.175. **11.** 310 = 200 + 110; 200 × \$.06 = \$12.00; 110 × \$.05 = \$5.50; \$12.00 + \$5.50 = \$17.50. **12.** (1) If 75 is $\frac{1}{2}$ of a number, the whole number is twice 75, or 150; (2) if $\frac{5}{8}$ of a number is 75, $\frac{1}{8}$ of the number is $\frac{1}{5}$ of 75, or 15, and $\frac{8}{8}$ of the number is 8 × 15, or 120. **13.** 27 × \$.25 = \$6.75; 77 × \$.05 = \$3.85, 28 × \$.10 = \$2.80, \$3.85 + \$2.80 = \$6.65; so the bank with the quarters contains \$.10 more. **14.** (1) 28 × 12,650 = 354,200; (2) 354,200 ÷ 49 = 7228.57, or 7228.6. **15.** 2.25900. **16.** 621,844. **17.** 3280. **18.** 33.650. **19.** $6\frac{11}{12}$. **20.** $8\frac{1}{2}$. **21.** $4\frac{2}{3}$.

Page 321

Aim: To present Problem Test 10

Suggestion: This is the last problem test in the book. If pupils have been keeping a record of their scores on problem tests, as suggested for the first problem test on page 39 of the text, they now have a complete record of their problem-solving ability.

Workbook Reference: Arithmetic Workshop, Book 7, page 133

Key: **1.** $3\frac{1}{2} \times 5\frac{1}{2} \times 6 = 115\frac{1}{2}$ (cu. in.); $115\frac{1}{2}$ cu. in. is $\frac{1}{2}$ of 231 cu. in., so the can holds $\frac{1}{2}$ gal. **2.** 25% + 30% + 15% + 18% = 88%, 100% − 88% = 12%; .12 × \$4000 = \$480. **3.** 3 roses = $\frac{1}{4}$ doz. roses; $\frac{1}{4} \times \$6.00 = \$1.50$. **4.** $2\frac{1}{2}$ min. = 150 sec.; 150 sec. ÷ 100 = $1\frac{1}{2}$ sec. **5.** 30 × 15 min. = 450 min.; 450 min. = $7\frac{1}{2}$ hr.; $7\frac{1}{2} \times 3\frac{1}{5}$¢ = 24¢. **6.** $C = 3\frac{1}{7} \times 3\frac{1}{2} = 11$ (ft.); 11 ft. + 6 in. = $11\frac{1}{2}$ ft.; $11\frac{1}{2}$ ft. ÷ 3 ft. = $3\frac{5}{6}$ yd.; so she must buy 4 yd. **7.** (1) .10 × \$2.95 = \$.295, or \$.30; (2) \$2.95 + \$.30 = \$3.25. **8.** 41.74 in. ÷ 12 = 3.478 in., or 3.48 in. **9.** \$840

− $750 = $90; $90 ÷ $750 = .12 = 12%. **10.** 27 in. = $\frac{3}{4}$ yd.; $50\frac{1}{4}$ yd. ÷ $\frac{3}{4}$ yd. = 67 (towels).

Pages 322–323

Aim: To provide two pages of diagnostic tests with page references for those pupils who need additional help

Key: Page 322 **1.** 5 hr. 5 min., 13 yd.; 1 hr. 45 min., 3 yd. 22 in. **2.** 70 sq. ft.; $3\frac{3}{4}$ sq. yd. **3.** 5 sq. ft.; 360 sq. in. or $\frac{5}{18}$ sq. yd. **4.** 9 sq. in.; 49 sq. ft.; $12\frac{1}{4}$ sq. yd.; 110.25 sq. ft.; $\frac{1}{16}$ sq. mi. **5.** 288; $3\frac{1}{3}$. **6.** 3; 135. **7.** 24 sq. ft.; $\frac{3}{8}$ sq. yd. **8.** 65 sq. ft.; $31\frac{1}{2}$ sq. in. **9.** 44 in.; 33 in.; $5\frac{1}{2}$ ft.; $37\frac{5}{7}$ ft. **10.** 132 cu. ft.; 64 cu. ft.

Page 323 **1.** $5\frac{5}{8}$; $2\frac{1}{6}$; $1\frac{1}{2}$; $2\frac{5}{12}$; $\frac{2}{3}$. **2.** $7\frac{1}{2}$; $31\frac{1}{2}$; $\frac{3}{8}$. **3.** 6; $6\frac{1}{4}$; 1. **4.** $3\frac{1}{3}$; $\frac{2}{5}$; $10\frac{2}{3}$. **5.** $\frac{3}{16}$; $\frac{2}{7}$; $2\frac{1}{12}$. **6.** 10.6; 1.1; 13.6; 17.2. **7.** 540; 528.4; .2; .09. **8.** $.19; $2.31; $1.64. **9.** $4.20; $6.86; $1.19. **10.** $368.88; $2214; $122.20. **11.** $37\frac{1}{2}$; $26\frac{1}{4}$. **12.** 35% increase; 26% decrease; 100% increase.

Page 324

Aim: To test readiness for Grade 8

Key: **1.** $4\frac{2}{5}$; $84; $11\frac{3}{8}$. **2.** 3.12750; $480; 4. **3.** 5610; 202.09; $4\frac{1}{2}$. **4.** $36; 46; $\frac{3}{4}$. **5.** $4.13; 21.214; $4\frac{5}{12}$. **6.** 24,264. **7.** 75; $9\frac{2}{3}$. **8.** 150; $2\frac{1}{3}$. **9.** $12\frac{1}{2}$; 108. **10.** 88¢ − 40¢ = 48¢; 48¢ ÷ 40¢ = 1.2, or 120%. **11.** 15 × $.17 = $2.55; $8.29 + $3.16 + $10.75 + $2.55 = $24.75; $24.75 ÷ 3 = $8.25; $10.00 − $8.25 = $1.75. **12.** 25% = $\frac{1}{4}$; $\frac{1}{4}$ × $30.00 = $7.50; $30.00 − $7.50 = $22.50. **14.** 27 − 15 = 12 (words); 12 × $6\frac{1}{2}$¢ = 78¢; $1.45 + $.78 = $2.23; 1.10 × $2.23 = $2.453, or $2.45.

Pages 325–341

Aim: To present additional practice pages

Suggestion: These pages are correlated with sections throughout the textbook. At the bottom of many of the text pages you will find a red circle with a white number that refers to a section of these practice pages. You may use these pages for review on any topic or for choosing miscellaneous exercises when making up a test.

Key: Page 325 **1.** 69. **2.** 50. **3.** 65. **4.** 64. **5.** 476. **6.** 494. **7.** 449. **8.** 439. **9.** 704. **10.** 475. **11.** 3484; 3809; 3981; 5549; 42,225; 33,042; 46,363. **12.** 6252; 3565; 5034; 3898; 43,060; 51,653; 58,231. **13.** $4320.06; $2867.75; $3555.14; $2369.51; $3543.75.

Page 326 **1.** 372; 391; 3748; 9266; 39,876; $199.30. **2.** 202; 3008; 1986; 19,488; 377,538; $207.32. **3.** 432; 1924; 1898; 14,956; 27,669; $200.15. **4.** 17; 136; 3478; 42,678; 494,879; $675.55. **5.** 597; 1876; 2585; 1365; 43,325; $368.53. **6.** 6364; 7515; 262,548; 895,752; 86,832. **7.** 1296; 19,552; 45,441; 419,475; 120,540. **8.** 4606; 7280; 531,069; 179,324; 454,346. **9.** 5330; 42,394; 669,460;

163,548; 314,384. **10.** 5320; 71,613; 330,132; 37,275; 440,910. **11.** 9604; 78,750; 333,330; 166,242; 109,782. **12.** 5963; 10,922; 58,310; 129,108; 396,500. **13.** 1131; 14,202; 693,160; 141,680; 621,216. **14.** 390,260; 230,144; 1,639,624; 3,609,375. **15.** 174,640; 6,789,930; 1,056,450; 11,284,260. **16.** 238,659; 419,055; 488,940; 14,312,810. **17.** 40,223; 7,500,444; 1,125,978; 39,190,064. **18.** 252,685; 767,250; 1,951,750; 28,883,200. **19.** 455,058; 4,864,178; 4,994,260; 5,987,080. **20.** 28,080; 759,156; 832,200; 8,711,820. **21.** 249,676; 823,935; 2,843,328; 32,481,000.

Page 327 **1.** 219; 729; 5003; 19,233 R3. **2.** 1161; 958 R5; 10,526; 10,610. **3.** 1235; 405; 15,064; 3321. **4.** 507 R4; 796 R5; 9293 R4; 7060 R2. **5.** 41; 78; 567; 6218. **6.** 16 R21; 95 R7; 739 R13; 2179. **7.** 13; 89; 893; 4238 R25. **8.** 8 R29; 49; 263; 9938. **9.** 14; 77; 789; 7246. **10.** 33; 92 R7; 986 R7; 8661. **11.** 11; 63 R14; 878 R4; 9115 R20. **12.** 9; 95; 398; 9697. **13.** 41 R12; 60 R23; 904 R24; 5023. **14.** 12; 80 R4; 909; 4090 R7. **15.** 11; 64 R9; 412; 3213. **16.** 23; 92 R4; 718 R18; 2697 R33. **17.** 8; 36; 426; 6753. **18.** 21; 62; 983 R6; 6955. **19.** 20 R16; 98; 725; 9141. **20.** 9; 48; 425 R16; 3571. **21.** 14 R7; 85 R24; 256 R29; 9624. **22.** 23; 24 R55; 884; 4619 R21. **23.** 5 R15; 80 R7; 900 R14; 4200 R13. **24.** 8; 50; 504; 7003. **25.** 7; 65; 483; 4807.

Page 328 **1.** $31\frac{3}{7}$; 26; 836; 4296. **2.** 25; 29; 364; 6485. **3.** 24; $46\frac{11}{532}$; 880; $2505\frac{19}{640}$. **4.** 32; $74\frac{19}{256}$; $405\frac{33}{736}$; 7009. **5.** 5; $31\frac{431}{708}$; 719; 7630. **6.** 14; 83 R89; 129 R120; 4216. **7.** 15; 87 R120; 810; 5835. **8.** 23; 21 R300; 627; 7637. **9.** 12; 42; 924 R17; 9000 R219. **10.** 9 R27; 58 R11; 320 R729; 3008 R121. **11.** $10\frac{1}{2}$; $6\frac{2}{3}$; $5\frac{1}{4}$; $14\frac{1}{4}$; 6; 11; $12\frac{3}{5}$; $7\frac{1}{6}$. **12.** $12\frac{1}{2}$; $7\frac{3}{4}$; 7; $7\frac{1}{3}$; 14; $11\frac{1}{3}$; $9\frac{1}{3}$; 13. **13.** $10\frac{2}{5}$; $11\frac{2}{3}$; $3\frac{4}{5}$; $14\frac{1}{4}$; $7\frac{3}{5}$; $6\frac{3}{4}$; $11\frac{1}{8}$; $9\frac{3}{5}$. **14.** $9\frac{2}{3}$; $11\frac{1}{4}$; 9; $12\frac{3}{8}$; $9\frac{1}{2}$; 16; $8\frac{1}{2}$; $3\frac{15}{16}$. **15.** $14\frac{1}{6}$; $4\frac{1}{5}$; $8\frac{1}{8}$; 13; $13\frac{1}{4}$; $16\frac{1}{2}$; $9\frac{1}{4}$; $12\frac{3}{10}$.

Page 329 **1.** $10\frac{1}{8}$; $2\frac{1}{8}$; $11\frac{7}{16}$; $7\frac{1}{10}$; $6\frac{1}{4}$; $8\frac{13}{16}$; $40\frac{3}{8}$; $16\frac{5}{6}$. **2.** $6\frac{1}{2}$; $7\frac{1}{2}$; $7\frac{7}{16}$; $14\frac{1}{5}$; $3\frac{1}{6}$; $11\frac{1}{2}$; $25\frac{3}{8}$; $17\frac{1}{8}$. **3.** $10\frac{1}{4}$; $16\frac{5}{12}$; $9\frac{1}{16}$; $4\frac{2}{5}$; $3\frac{1}{4}$; $14\frac{1}{16}$; $24\frac{9}{10}$; $11\frac{1}{8}$. **4.** $2\frac{5}{6}$; $12\frac{1}{10}$; $9\frac{1}{4}$; $5\frac{5}{6}$; $11\frac{5}{12}$; $2\frac{1}{12}$; $27\frac{1}{6}$; $10\frac{1}{3}$. **5.** $11\frac{1}{6}$; $16\frac{7}{8}$; $11\frac{3}{16}$; $11\frac{5}{8}$; 15; 18; $70\frac{1}{4}$; 17. **6.** $2\frac{5}{8}$; $4\frac{3}{5}$; $3\frac{1}{2}$; $1\frac{1}{4}$; $3\frac{5}{16}$; $3\frac{3}{8}$; $10\frac{1}{6}$; $6\frac{1}{4}$. **7.** $6\frac{5}{12}$; $\frac{3}{4}$; $2\frac{3}{4}$; $2\frac{2}{5}$; $\frac{1}{2}$; $2\frac{1}{2}$; $6\frac{3}{8}$; 4. **8.** $4\frac{2}{3}$; $2\frac{3}{10}$; $5\frac{3}{5}$; $4\frac{5}{12}$; $1\frac{5}{16}$; $4\frac{3}{8}$; $5\frac{1}{4}$; $7\frac{1}{3}$. **9.** 3; $1\frac{1}{2}$; $2\frac{3}{16}$; $1\frac{1}{2}$; $7\frac{1}{2}$; $2\frac{1}{2}$; $6\frac{1}{3}$; $1\frac{1}{2}$. **10.** $6\frac{2}{5}$; $3\frac{3}{8}$; $1\frac{1}{5}$; $3\frac{3}{8}$; $6\frac{1}{12}$; $6\frac{3}{10}$; $2\frac{1}{12}$; 4. **11.** $8\frac{3}{8}$; $3\frac{5}{8}$; $4\frac{1}{6}$; $2\frac{1}{2}$; $2\frac{1}{2}$; $3\frac{1}{4}$; $\frac{3}{4}$; $1\frac{1}{3}$.

Page 330 **1.** $3\frac{1}{3}$; $7\frac{2}{5}$; $5\frac{1}{2}$; $1\frac{1}{10}$; $2\frac{7}{12}$; $6\frac{5}{16}$; $9\frac{1}{6}$; $11\frac{1}{4}$. **2.** $\frac{1}{6}$; $5\frac{1}{4}$; $6\frac{1}{3}$; $\frac{5}{16}$; $1\frac{3}{10}$; $\frac{7}{12}$; $7\frac{5}{8}$; $4\frac{5}{6}$. **3.** $3\frac{1}{2}$; $4\frac{1}{2}$; $\frac{3}{8}$; $4\frac{1}{2}$; $\frac{7}{12}$; $2\frac{5}{8}$; $1\frac{5}{6}$; $3\frac{5}{8}$. **4.** 3; $2\frac{1}{2}$; $\frac{1}{2}$; $1\frac{3}{5}$; $1\frac{2}{5}$; $1\frac{9}{16}$; $1\frac{3}{4}$; $4\frac{2}{3}$. **5.** $3\frac{1}{3}$; $1\frac{5}{8}$; 3; $3\frac{1}{2}$; $1\frac{1}{2}$; $2\frac{5}{8}$; $2\frac{1}{2}$; $4\frac{1}{4}$. **6.** 4; 4; 4, 10; 4, 20. **7.** 2; 5; 6, 14; 6, 22. **8.** 10; 10; 5, 15; 14, 38. **9.** 12; 3; 16, 36; 9, 39. **10.** 16; 4; 3, 27; 14, 34. **11.** $1\frac{2}{3}$; $\frac{7}{8}$; $1\frac{7}{10}$; $4\frac{2}{3}$; $\frac{13}{16}$; $2\frac{3}{5}$; $6\frac{9}{10}$; $\frac{5}{8}$. **12.** $1\frac{3}{4}$; $\frac{1}{2}$; $5\frac{7}{8}$; $5\frac{7}{10}$; $\frac{9}{16}$; $3\frac{5}{16}$; $\frac{11}{12}$; $2\frac{5}{8}$. **13.** $3\frac{1}{2}$; $5\frac{5}{8}$; $6\frac{7}{12}$; $1\frac{2}{3}$; $5\frac{1}{2}$; $3\frac{13}{16}$; $\frac{2}{3}$; $2\frac{7}{10}$. **14.** $7\frac{7}{8}$; $\frac{5}{6}$; $\frac{7}{8}$; $2\frac{3}{5}$; $\frac{11}{16}$; $1\frac{5}{12}$; $6\frac{7}{8}$; $3\frac{7}{8}$.

Page 331 **1.** $5\frac{5}{12}$; $7\frac{7}{24}$; $10\frac{17}{20}$; $10\frac{11}{24}$; $6\frac{11}{20}$; $11\frac{1}{12}$; $27\frac{23}{24}$; $14\frac{5}{12}$. **2.** $10\frac{13}{24}$; $12\frac{11}{24}$; $9\frac{17}{20}$; $4\frac{11}{24}$; $14\frac{13}{15}$; $9\frac{5}{12}$; $32\frac{13}{24}$; $7\frac{23}{24}$. **3.** $8\frac{1}{12}$; $1\frac{17}{24}$; $2\frac{9}{20}$; $1\frac{19}{40}$; $1\frac{13}{24}$; $3\frac{7}{24}$; $1\frac{5}{12}$; $5\frac{1}{12}$. **4.** $2\frac{5}{12}$; $\frac{7}{12}$; $6\frac{17}{24}$; $1\frac{9}{20}$; $5\frac{7}{15}$; $4\frac{23}{24}$; $15\frac{7}{12}$; $1\frac{7}{12}$. **5.** $1\frac{1}{12}$; $7\frac{7}{24}$; $4\frac{1}{20}$; $2\frac{7}{24}$; $2\frac{13}{15}$; $3\frac{7}{12}$; $6\frac{5}{24}$; $5\frac{23}{24}$. **6.** $14\frac{7}{12}$; $6\frac{19}{24}$; $14\frac{7}{20}$; $7\frac{1}{24}$; $10\frac{8}{15}$; $14\frac{1}{12}$; $28\frac{19}{24}$; $14\frac{5}{12}$. **7.** $8\frac{1}{12}$; $13\frac{5}{12}$; $11\frac{3}{8}$; $17\frac{19}{30}$; $11\frac{19}{20}$;

$5\frac{7}{8}$; $38\frac{7}{12}$; $15\frac{7}{8}$. **8.** $12\frac{1}{4}$; $17\frac{1}{12}$; $7\frac{91}{120}$; $10\frac{11}{24}$; $16\frac{14}{15}$; $18\frac{7}{12}$; $45\frac{3}{8}$; $15\frac{5}{24}$. **9.** $7\frac{23}{24}$; $9\frac{11}{24}$; 23; $18\frac{29}{30}$; $10\frac{3}{4}$; $18\frac{1}{4}$; $46\frac{3}{8}$; $16\frac{1}{4}$. **10.** $15\frac{3}{8}$; $21\frac{13}{24}$; $19\frac{13}{24}$; $23\frac{9}{20}$; $19\frac{3}{8}$; $17\frac{3}{4}$; $63\frac{13}{24}$; $18\frac{1}{8}$.

Page 332 **1.** 3; 24; 42; 27; 4. **2.** $5\frac{1}{4}$; $6\frac{7}{8}$; $10\frac{2}{3}$; $11\frac{7}{10}$; $4\frac{1}{6}$. **3.** $7\frac{1}{2}$; $15\frac{3}{4}$; $6\frac{1}{4}$; $11\frac{1}{4}$; $3\frac{3}{4}$. **4.** $2\frac{4}{5}$; $8\frac{1}{4}$; $5\frac{5}{8}$; $14\frac{1}{6}$; $1\frac{1}{3}$. **5.** 2; 10; 35; 9; $3\frac{1}{2}$. **6.** $2\frac{5}{8}$; $6\frac{2}{3}$; $22\frac{1}{2}$; $9\frac{4}{5}$; $5\frac{1}{4}$. **7.** $25\frac{1}{2}$; $24\frac{3}{4}$; $16\frac{2}{3}$; $36\frac{5}{6}$; $40\frac{1}{2}$. **8.** 60; 51; 64; 50; 22. **9.** $7\frac{1}{2}$; $32\frac{1}{2}$; $22\frac{1}{2}$; $37\frac{1}{2}$; $41\frac{1}{3}$. **10.** $11\frac{1}{4}$; $36\frac{2}{3}$; $68\frac{3}{4}$; $24\frac{3}{8}$; $46\frac{2}{3}$. **11.** 105; 90; 33; 42; 22. **12.** $37\frac{1}{2}$; $94\frac{1}{2}$; $87\frac{1}{2}$; $94\frac{1}{2}$; $110\frac{1}{4}$. **13.** $\frac{4}{9}$; $\frac{15}{32}$; $\frac{3}{20}$; $\frac{3}{8}$; $\frac{3}{16}$. **14.** $\frac{3}{5}$; $\frac{3}{5}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{4}$. **15.** $\frac{2}{3}$; $\frac{15}{32}$; $\frac{2}{15}$; $\frac{3}{8}$; $\frac{5}{16}$. **16.** $\frac{9}{16}$; $\frac{1}{16}$; $\frac{5}{8}$; $\frac{11}{16}$; $\frac{1}{8}$. **17.** $\frac{1}{10}$; $\frac{3}{4}$; $\frac{1}{5}$; $\frac{3}{8}$; $\frac{1}{4}$. **18.** $\frac{5}{6}$; 3; $2\frac{5}{8}$; $\frac{7}{10}$; $\frac{5}{6}$. **19.** 2; $3\frac{1}{2}$; $7\frac{1}{2}$; 1; $\frac{5}{8}$. **20.** $\frac{15}{16}$; $\frac{7}{12}$; $1\frac{7}{48}$; $1\frac{5}{6}$; 6. **21.** $3\frac{3}{4}$; 7; $\frac{2}{3}$; 1; $6\frac{7}{8}$. **22.** 18; 9; $3\frac{1}{30}$; $3\frac{3}{20}$; $10\frac{5}{12}$. **23.** $3\frac{11}{15}$; $32\frac{1}{2}$; $17\frac{1}{2}$; 18; 9.

Page 333 **1.** 12; 40; 36; 80; $10\frac{1}{2}$. **2.** 12; 16; $12\frac{1}{2}$; $16\frac{2}{3}$; 10. **3.** $9\frac{1}{3}$; $17\frac{3}{5}$; $17\frac{1}{7}$; $21\frac{2}{3}$; 24. **4.** $1\frac{11}{24}$; $1\frac{11}{64}$; $10\frac{2}{3}$; $1\frac{1}{24}$; $\frac{5}{6}$. **5.** $1\frac{1}{6}$; $\frac{3}{4}$; $\frac{1}{2}$; 9; 8. **6.** $2\frac{1}{2}$; $2\frac{2}{9}$; $\frac{9}{10}$; $\frac{12}{35}$; $\frac{4}{9}$. **7.** $3\frac{1}{2}$; 20; 16; 62; 2. **8.** 5; $2\frac{1}{2}$; 10; $3\frac{1}{2}$; 4. **9.** $1\frac{13}{32}$; $3\frac{15}{16}$; $1\frac{13}{15}$; $43\frac{1}{5}$; 9. **10.** 9; 4; $1\frac{3}{4}$; $6\frac{1}{4}$; $3\frac{3}{4}$. **11.** $3\frac{1}{3}$; $5\frac{5}{8}$; 6; $4\frac{2}{5}$; $3\frac{1}{3}$. **12.** $\frac{1}{12}$; $\frac{5}{16}$; $\frac{3}{8}$; $\frac{5}{18}$; $\frac{1}{6}$. **13.** $\frac{1}{8}$; $\frac{3}{16}$; $\frac{1}{16}$; $\frac{1}{16}$; $\frac{5}{12}$. **14.** $\frac{3}{20}$; $\frac{2}{25}$; $\frac{5}{48}$; $\frac{3}{64}$; $\frac{1}{10}$. **15.** $2\frac{1}{10}$; $\frac{14}{15}$; $\frac{9}{32}$; $1\frac{1}{10}$; $\frac{3}{16}$. **16.** $\frac{2}{3}$; $1\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{3}{5}$. **17.** $\frac{9}{10}$; $\frac{5}{6}$; $\frac{5}{12}$; $\frac{3}{10}$; $\frac{3}{4}$. **18.** $2\frac{2}{5}$; $10\frac{2}{3}$; $6\frac{2}{5}$; $5\frac{5}{8}$; $1\frac{1}{5}$. **19.** $2\frac{2}{3}$; 8; $3\frac{3}{4}$; 6; $3\frac{3}{5}$. **20.** $\frac{3}{16}$; $\frac{8}{15}$; $\frac{5}{12}$; $\frac{9}{20}$; $\frac{3}{8}$. **21.** $\frac{1}{6}$; $\frac{6}{25}$; $\frac{1}{4}$; $\frac{3}{20}$; $\frac{1}{14}$. **22.** $5\frac{5}{8}$; $2\frac{1}{2}$; $3\frac{3}{8}$; 2; $1\frac{5}{16}$. **23.** $1\frac{1}{2}$; $3\frac{1}{8}$; $2\frac{1}{4}$; $3\frac{1}{8}$; $\frac{1}{4}$.

Page 334 **1.** 6.91; 145.4; 21.01; 1.831; 14.086; 19.022. **2.** 262.7; 16.63; 20.55; 2.510; 32.846; 40.703. **3.** 55.2. **4.** 33.16. **5.** 3.78. **6.** 42.43. **7.** 21.578. **8.** 27.63. **9.** 220.51. **10.** 172.500. **11.** 30.2250. **12.** 43.2; 48.9; 7.67; .58; 3.747; 6.038. **13.** 66.8; 24.7; 7.62; .25; 2.257; 5.159. **14.** 81.5; 39.4; 5.09; .35; 7.639; 1.628. **15.** 17.4; 2.25; \$7.63; \$13.55. **16.** 87.38; 24.95; \$11.46; \$16.13. **17.** .675; 1.71; \$10.11; \$7.72. **18.** .319; .81; \$16.74; \$4.25.

Page 335 **1.** 19.08; 379.4; 29.16; 229.5; 29.792. **2.** 9.27; .531; 403.2; 49.4; 698.04. **3.** 6.328; 3.930; 17.10; 175.5; 4.032. **4.** 9.21; 28.83; 319.2; 58.20; 171.72. **5.** .378; 63.21; 3.36; 21.08; 2.349. **6.** 3.42; 1.176; 3.23; .1300; 31.154. **7.** 3.68; 1.350; 3.478; 2.812; 436.48. **8.** 2.76; .760; 1.200; .3519; .26676. **9.** .196; .693; 1.312; .873; 24.063. **10.** .259; 2.914; 15.54; 3.698; .48608. **11.** .486; 19.44; .744; .1653; 28.014. **12.** 4.15; 3.57; 2.952; 12.42; 6.3674. **13.** 5.92; 5.642; 15.51; .455; 3.675. **14.** .0184; .0168; .0172; .0400; .00086. **15.** .0048; .0042; .0054; .0036; .01500. **16.** .0582; .0372; .0092; .0351; .0070. **17.** .0065; .0132; .0072; .0184; .6240. **18.** .0056; .048; .119; .0600; .04016. **19.** 2.36; 8.64; 5.96; 4.68; .358. **20.** 14.5; 37.6; 30.7; .112; 9.46. **21.** .213; .317; 2.76; 5.43; 2.71. **22.** 3.12; 77.2; 31.2; 76.2; 2.76. **23.** .375; 1.68; 38.7; 2.31; 40.2. **24.** 5.94; .173; 5.81; 6.75; 3.67.

Page 336 **1.** 42.125; 6.75; 18.75; 21.25; 8.6875. **2.** 138.5; 13.64; 12.5; 22.8; 7.25. **3.** 178.75; 14.5; 4.5625; 6.75; 17.5. **4.** 13.68; .9625; .24375; 3.125; .58. **5.** 4.3375; .324; .8625; .85; .48125. **6.** 7.54; 1.45; 1.225; .0975; 1.924. **7.** 5.7625; .912; .675; .425; .475. **8.** 44.4; 40.3; 12.9; 14.3; 2.9. **9.** 3.4; 243.3; 27.9; 4.3; 5.2. **10.** 4.7; 7.6; 2.1; 5.7; 3.9. **11.** 25.9; 132.1; 1.5; .2; 1.1. **12.** 209.75; 228.33; 16.47; 5.88; 16.19. **13.** 21.83; 1.38; 4.41; 5.36; 22.74. **14.** 12.96; .37; .63; 11.86; 16.25. **15.** .75; .8; .65; .875; .1875; .7; .84; .74; .5625; .35. **16.** .43;

.78; .61; .86; .70; .21; .37; .46; .76; .92. **17.** .67; .11; .68; .89; .30; .70; .94; .38; .67; .87.

Page 337 **1.** 283; 137; 368; 31. **2.** 394; 17; 413; 32. **3.** 4.53; 14.2; 6.54; 87.4. **4.** 4.31; 2.64; 12.6; 6.43. **5.** 3.91; 67.3; 5.48; 27.5. **6.** 105; 9.42; 40.8; 13.6. **7.** 4.375; 32.9; 74.9; 3.15. **8.** 28.3; 209.3; 2.1; 1086.9; 310.7. **9.** 43.3; 715.5; 2.5; 76.9; 2.3. **10.** 138.6; 10.3; 27.0; 660.7; 42.4. **11.** 96.67; 1119.44; 7.82; 8.23; 37.10. **12.** 57.78; 1.09; .65; 487.10; 9.44. **13.** 2.14; 81.22; 9.55; 8.46; 23.18. **14.** .02; .013; .0022; .031; 160. **15.** .04; 74.2; .021; .0032; .005. **16.** .075; .012; .0096; .076; .07. **17.** .07; 3780; .0027; 4670; 850. **18.** 54; 1735; 670; 2730; 930.

Page 338 **1.** 12; 18; 16; $48. **2.** 11; 18; 13; $28.50. **3.** 21; 15; 28; $160. **4.** 18; 12; 27; $496. **5.** 13.5; 40; 34; $38.50. **6.** 31; 13; $180; $2.40. **7.** 21; 54; $77.50; $3.60. **8.** 51; 43; $37.50; $1.70. **9.** 12; 66; $108; $.41. **10.** 48; 9; $210; $3.99. **11.** 72; $318; $21.50. **12.** 27; $161; $8.05. **13.** 76; $230; $47.05. **14.** 17; $180; $57.54. **15.** 18; $462; $7.22. **16.** 21; $106; $16.37. **17.** $1.25, $11.25; $4.25, $8.50; $75.00, $425.00. **18.** $3.90, $11.70; $1.80, $12.60; $70.60, $211.80. **19.** $3.20, $9.60; $9.30, $15.50; $31.15, $124.60. **20.** $8.00, $8.00; $8.45, $16.90; $193.25, $193.25. **21.** $7.20, $16.80; $5.75, $40.25; $21.56, $194.04. **22.** $3.75, $15.00; $6.55, $13.10; $73.05, $170.45. **23.** $10.60, $31.80; $4.80, $8.00; $43.90, $131.70.

Page 339 **1.** $.41; $1.01; $8.75. **2.** $.36; $1.36; $5.88. **3.** $.76; $2.91; $2.60. **4.** $.57; $.91; $24.53. **5.** $.74; $4.80; $2.48. **6.** $100; $300; $100. **7.** $66; $90; $550. **8.** $130; $62.50; $63. **9.** $260; $183; $146. **10.** $34; $36; $204. **11.** $2173.50; $675; $55.61. **12.** $684.60; $296; $40.87. **13.** $688.50; $1163.75; $133.24. **14.** $196.00; $532.80; $80.63. **15.** $982.50; $413.75; $29.38.
16. 35%; 87½%; 36%; 25%; 12%. **17.** 44%; 33⅓%; 65%; 35%; 12½%.
18. 33⅓%; 45%; 25%; 75%; 26%. **19.** 75%; 24%; 50%; 37½%; 20%.
20. 95%; 45%; 65%; 83⅓%; 12%. **21.** 48%; 75%; 66⅔%; 75%; 20%.
22. 66⅔%; 28%; 60%; 83⅓%; 24%.

Page 340 **1.** 19%; 2%; 25%; 42%; 29%; 30%; 25%. **2.** 9%; 45%; 1%; 6%; 52%; 27%; 40%. **3.** 33%; 8%; 15%; 36%; 29%; 48%; 100%. **4.** 7%; 5%; 17%; 25%; 39%; 1%; 35%. **5.** 125%; 100%; 175%; 300%; 166⅔%. **6.** 200%; 200%; 140%; 110%; 200%. **7.** 220%; 250%; 120%; 200%; 400%. **8.** 150%; 133⅓%; 250%; 500%; 200%. **9.** 40% increase; 40% increase; 20% increase; 5% decrease. **10.** 33⅓% increase; 100% increase; 20% decrease; 14% decrease. **11.** 15% decrease; 25% decrease; 16⅔% decrease; 33⅓% increase. **12.** 12½% decrease; 15% decrease; 100% increase; 125% increase. **13.** 20% increase; 12½% increase; 33⅓% decrease; 25% decrease. **14.** 12½%; 12½%; 10%; 15%. **15.** 10%; 12½%; 12½%; 6%. **16.** 12½%; 20%; 20%; 8%. **17.** 20%; 25%; 16%; 12%. **18.** 25%; 20%; 12%; 14%. **19.** 16⅔%; 20%; 18%; 18%. **20.** 12½%; 12½%; 12%; 20%. **21.** 10%; 16⅔%; 20%; 16%.

Page 341 **1.** \$12; \$5.50; \$11.88. **2.** \$10.50; \$13.75; \$8.66. **3.** \$6.00; \$9.24; \$4.40. **4.** \$36; \$17.50; \$4.04. **5.** \$5.00; \$1.40. **6.** \$4.95; \$1.65. **7.** \$5.25; \$2.45. **8.** \$1.10; \$8.50. **9.** \$7.02; \$1.00. **10.** 9 ft. 2 in.; 7 da. 1 hr.; 10 ft. 10 in.; 7 hr. 25 min. **11.** 12 ft. 10 in.; 7 lb. 5 oz.; 13 qt.; 21 yd. 17 in. **12.** 6 in.; 4 ft. 6 in.; 3 bu. 2 pk.; 3 yd. 22 in. **13.** 1 ft. 8 in.; 3 bu. 3 pk.; 3 qt. 1 pt.; 7 gal. 1 qt. **14.** 30 ft. 10 in.; 24 lb. 8 oz.; 10 ft.; 13 hr. **15.** 11 ft. 8 in.; 21 bu.; 38 lb. 8 oz.; 36 hr. 15 min. **16.** 21 ft. 6 in.; 30 da. 8 hr.; 57 bu.; 16 hr.

Pages 342–343

Aim: To present ready-reference pages of tables of measure

Pages 344–345

Aim: To present "Suggestions to Teachers"

Audio-Visual Aids

Films: The following 16 mm. motion pictures are listed by producer or distributor. The film catalogues of these firms, which will be sent to you on request, give the objectives of these films and outline briefly the material covered by them.

Coronet Films, Coronet Building, 65 East South Water Street, Chicago 1, Illinois.

Angles and Their Measurement
Arithmetic: Estimating and Checking Answers
Arithmetic: Understanding the Problem
Brushing Up On Division
Brushing Up On Multiplication
Geometry and You
Graphing Linear Equations
How to Find the Answer
Meaning of Pi, The
Number System and Its Structure, The
Percent in Everyday Life
Principles of Scale Drawing
Ratio and Proportion in Mathematics
Story of Weights and Measures
Symbols in Algebra
Triangles: Types and Uses
Volume and Its Measurement

Delta Film Productions, Inc., 1821 University Avenue, St. Paul 4, Minnesota.

Solids in the World Around Us
Surface Areas of Solids I
Surface Areas of Solids II
Volumes of Cubes, Prisms, and Cylinders
Volumes of Pyramids, Cones, and Spheres

Encyclopaedia Britannica Films, Inc., 1150 Wilmette Avenue, Wilmette, Illinois.

Weights and Measures
What Are Decimals?
What Are Fractions?

Film Associates of California, 11014 Santa Monica Boulevard, Los Angeles 25, California.

Adventure in Science: The Size of Things
Big Numbers . . . Little Numbers
Sets, Crows, and Infinity

Johnson Hunt Productions, Film Center, La Cañada, California.

Introduction to Fractions
How to Add Fractions
How to Subtract Fractions
How to Change Fractions
How to Multiply Fractions
How to Divide Fractions
Decimal Fractions
Percentage

Modern Learning Aids, A Division of Modern Talking Pictures, Inc., 3 East 54 Street, New York 22, New York.

Addition and Multiplication
Introduction to Fractions
Manipulating Fractions
Measuring Angles
Operating with Decimals
Subtraction and Division
Whole Numbers

NET Film Service, Indiana University, Audio-Visual Center, Bloomington, Indiana.

Understanding Numbers (7 Films)

Filmstrips: The following filmstrips are listed by producer or distributor. These firms will send you on request catalogues and sales information which will provide you with descriptions and brief outlines of the material covered by these filmstrips.

Don H. Parsons Associates, Instructional Materials and Equipment, 1415 Westwood Boulevard, Los Angeles 24, California.

The Language of Sets
Open Sentences
The Number Line
Negative Numbers
The Closure, Commutative, and Associative Properties

DuKane Corporation, St. Charles, Illinois.

Area and Volume:
- Measuring the Squares
- Studies in Square Inches and Square Feet
- Problems in Area
- Introduction to Volume
- Using the Cubic Inch
- Problems in Volume

Filmstrip House, 347 Madison Avenue, New York 17, New York.

Dividing with Fractions
Early Counting
Early Measuring
Geometric Figures
Multiplying with Fractions

Society for Visual Education, Inc., 1345 Diversey Parkway, Chicago 14, Illinois.

Using and Understanding Numbers — Decimals and Measurements:
- Meaning and Reading of Decimals
- Addition and Subtraction of Decimals
- Multiplication of Decimals
- Division of Decimals
- Changing Fractions to Decimals — Decimals to Fractions
- Advancing in Linear Measurements
- Advancing in Quantity Measurements

Text-Film Department, McGraw-Hill Book Company, 330 West 42 Street, New York 36, New York.

History of Measures Series:
- History of Area Measure
- History of Our Calendar
- History of Telling Time
- History of Linear Measure
- History of Weight and Volume Measure
- History of Our Number System

Equipment and Supplies: Instruction in mathematics is made more effective by the use of charts, models, measuring instruments, devices, and other learning aids. Many of these should be made by the teacher and the pupils. When pupils construct their own aids, they are engaging in valuable learning activities. It is also desirable, however, for teachers to make use of some of the excellent mathematics equipment that can be purchased. From the firms listed below you can obtain protractors, plastic triangles, chalkboard drawing instruments, parallel rulers, steel tapes, geometric surfaces and solids, slide rules, and other equipment. These firms are also sources of mathematics supplies such as graph paper and large cardboard protractors. You can obtain from them some of the materials you will need for the construction of homemade models and devices. Most firms will be glad to provide you with literature describing their products and with the addresses of their nearest offices or distributors.

Milton Bradley Company, 74 Park Street, Springfield 2, Massachusetts
J. L. Hammett Company, Kendall Square, Cambridge 42, Massachusetts
Keuffel and Esser Company, 303 Adams Street, Hoboken, New Jersey
Lapine Scientific Company, 6001 South Knox Avenue, Chicago 29, Illinois
Math-Master Labs Incorporated, Box 310, Big Springs, Texas
F. A. Owen Publishing Company, Dansville, New York
Pickett and Eckel, Inc., 1109 South Fremont Avenue, Alhambra, California
W. M. Welch Manufacturing Company, 1515 North Sedgwick Street, Chicago 10, Illinois
Yoder Instruments, East Palestine, Ohio

More information on films, filmstrips, and other audio-visual aids can be obtained from National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C.